ASTRONOMY 102

Problem Set #4 **Due: Monday 14th April 2008, by 5 p.m., Hennings 312 slot**
Instructor: Douglas Scott **(Must be handed in on time, otherwise marks will be deducted)**
Answer all three questions, although only one question will be chosen at random for grading

Note: please indicate your answers clearly, e.g. by highlighting or drawing a box around each one, and also staple your sheets together – this will please the person marking and assigning grades! Also, pay attention to the level of precision which is required

1. Predicting the existence of the CMB

(a) Gamow, Alpher and collaborators 'predicted' the temperature of the CMB in the 1940s through a remarkably simple argument, a version of which you will now reproduce! The basic idea is that conditions in the early Universe must have once been very similar to those inside stellar nuclear reactors (and that's why helium exists everywhere today). The first part is to find an estimate for the average density of the Sun, and convert that to an equivalent number of hydrogen atoms per cubic metre (you can assume the Sun is all hydrogen if you like).

(b) Now you want to estimate the temperature required to rapidly cook hydrogen into helium. Stars do this at a relatively low temperature because they have a very long time to do it! But for the early Universe you need a temperature which is equivalent to that of the $\gamma-$rays produced in the nuclear reactions. So we want $k\,T = E$, with k being Boltzmann's constant again) and E comes from converting hydrogen to helium (0.7% of the rest mass, remember Assignment 1?). What temperature does this correspond to in Kelvin?

(c) The average density of 'baryons' (i.e. protons and neutrons) today is about $1\,\mathrm{m}^{-3}$. What was the 'scale factor' of the Universe when the average density was like that of the Sun (from part (a))?

(d) If the temperature of the Universe at that time was the same as you calculated in part (b), then using this same scale factor, what is the observed temperature today?

2. Hotter and denser!

(a) The Cosmic Microwave Background photons are observed from the last time they scattered off matter in the Universe, which happened at a redshift of approximately 1100. Given that the CMB temperature today is 2.725 Kelvin, and using Wien's Law, what was the peak wavelength at the time of last scattering? What part of the electromagnetic spectrum is this in?

(b) By what factor was everything in the Universe closer together when the light left the CMB 'last scattering surface' compared with today?

(c) By considering how much volumes have changed, estimate the average density of matter at that time. You can assume that the average matter density in today's Universe is $\Omega_{\mathrm{m}} = 0.25$ times the critical density.

(d) What was the temperature of the Cosmic Microwave Background at that time (assuming it is 2.725 Kelvin today)? Hence calculate the effective mass density of radiation (ρ_{rad}) then?

(e) If the 'dark energy' density today has Ω_Λ=0.75, and it is simply vacuum density, then what was its effective mass density at the last scattering time?

(f) Was the Universe matter-dominated, radiation-dominated, or dark-energy dominated at that time? At what redshift would it go from being radiation-dominated to matter-dominated?

3. A very big number!

(a) The Planck epoch, t_{Pl} is the earliest time that we can think about in the history of the Universe. [Its approximate value is found by asking when the quantum mechanical wavelength of the observable Universe was equal to its Schwarzschild Radius, and is given in the Glossary of the textbook]. It can be formed by making the dimension of time out of fundamental constants, leading to the expression: $t_{\mathrm{Pl}} = \sqrt{G\hbar/c^5}$. How long is this in seconds? [Note: $\hbar$ is Planck's constant divided by 2π].

(b) Using the Planck time, what is the Planck length, l_{Pl}, i.e. the distance light travels in a Planck time? Give your answer in metres.

(c) The Planck mass can be thought of as the mass of a black hole with Schwarzschild radius l_{Pl}. What is the Planck mass in kg?

(d) The Planck density can be defined as one Planck mass per cubic Planck length. This is an extremely high density, above which our ideas of gravity and quantum mechanics break down. Estimate the value of the Planck density ρ_{Pl} in $\mathrm{kg\,m}^{-3}$. The Planck density is the simplest (but wrong!) guess for what the density of the vacuum might be.

(e) The 'cosmological constant', Λ, can be interpretted as the energy density of the vacuum (actually $\Lambda = 8\pi G\rho_{\mathrm{vac}}$). We know that the actual density of the vacuum today is given by $\Omega_\Lambda \simeq 0.75$, i.e. about 0.75 times the critical density. Given a value for the critical density today, calculate the ratio of ρ_{Pl} to ρ_{vac}. [The fact that this is an extremely large ratio is one of the great cosmological mysteries of our time].