ASTRONOMY 102

Problem Set #2 **Due: Wednesday 13th February 2008, by 5 p.m., Hennings 312 slot**

Instructor: Douglas Scott **(Must be handed in on time, otherwise marks will be deducted)**

Answer all three questions, although only one question will be chosen at random for grading

Notes: please indicate your answers clearly, e.g. by highlighting or drawing a box around each one; the steps in obtaining the answer should also be clearly written out, not just the numerical answer; you should pay attention to the level of precision required; make sure to write your name on your work; and staple any sheets together. Following these guidelines will please the person marking and assigning grades!

1. Seeing double

(a) 70 Ophiuchi is a nearby binary system which can be separated with a modest sized telescope. Find a plot of the orbit of 70 Ophiuchi B around 70 Ophiuchi A in a book, or on the internet. Make a copy of this (either with a printer or a drawing) showing where B is relative to A just now. What is the period in years and the apparent semi-major axis in arcseconds?

(b) For the rest of this question you should adopt the observed quantities given here, which are deliberately *not* precisely the right values for this system (although they should be in the right ball-park), i.e. you should calculate things rather than looking them up! Assume that the parallax of 70 Ophiuchi has been measured to be 0.12 arcsec – what is its distance in parsecs? What is the semi-major axis (in A.U. or metres)?

(c) Using Kepler's 3rd Law (or otherwise) calculate the sum of the masses of the 2 stars and give the answer in solar masses.

(d) It is observed that on the plane of the sky the 'A' and 'B' stars appear to orbit a stationary point, and that the distance of the 'A' star from this point is always 2/3 times that of the 'B' star. With this information determine the separate masses $M_{\rm A}$ and $M_{\rm B}$ in $M_\odot$.

(e) The apparent magnitude of the 'B' star is observed to be 2 magnitudes greater than that of the 'A' star. From the ratio of the luminosities and the ratio of the masses, what mass-luminosity relation is suggested by these 2 stars? (give your answer in terms of $L \propto M^\alpha$ for some α).

2. Like a diamond in the sky!

(a) Typical white dwarfs have a mass of $\simeq 1.0\,{\rm M}_\odot$ and a radius approximately equal to the Earth's. Estimate the density of a typical white dwarf in ${\rm kg\,m^{-3}}$. Approximately how many times denser is this than diamond?

(b) Assuming for simplicity that it consists entirely of carbon nuclei in a sea of degenerate electrons, estimate the total number of carbon nuclei and the total number of electrons in a white dwarf.

(c) If a white dwarf is observed to be genuinely white, what is its approximate surface temperature? [In fact younger (bluer) white dwarfs are much hotter than this, and older (redder) ones are cooler, but the easiest ones to discover are roughly white].

(d) The total thermal energy content can be estimated by assuming that each particle in the white dwarf has an energy of kT, where k is Boltzmann's constant. Show that an estimate of the cooling timescale is $t_{\rm cool} \propto T^{-3}$. What is the cooling time for a typical white dwarf?

3. Star formation on the back of an envelope!

(a) The 'Jeans criterion' is an approximate way of understanding the typical masses involved in star formation. The criterion involves deciding whether a cloud of gas is able to collapse under its own gravity against the resisting force from internal pressure. It turns out that this condition implies that above a certain mass the cloud will collapse, while for lower masses it wouldn't. This characteristic mass, the Jeans mass, depends only on the density and temperature of the gas. The Jeans mass can be determined in the following way. You can assume that self-gravity gives a force of approximately GM^2/R^2, and that ideal gas pressure (which is force per unit area) is $n\,k\,T$, where n is the particle number density (equal to ρ/m, where m is the typical particle mass), k is Boltzmann's constant and T is the temperature. Using all of these approximations, and ignoring constants like 2 and π, set $F_{\rm grav} = F_{\rm press}$ to obtain an expression for the Jeans Mass: $M_{\rm J} \propto T^{3/2} n^{-1/2}$ (you should keep in the dimensionful constants like k, G and m).

(b) In a dense interstellar molecular cloud the typical conditions are $T \simeq 10\,{\rm K}$ and $n \simeq 10^9 {\rm m}^{-3}$. What is the Jeans mass in such a cloud (express your answer in ${\rm M}_\odot$)?

(c) In a large gas cloud in the intergalactic medium (the space between galaxies) typical conditions are more like $T \simeq 10^4\,{\rm K}$ and $n \simeq 10 {\rm m}^{-3}$. What is the Jeans mass in the intergalactic medium? Do you expect individual stars to form between galaxies?