ASTRONOMY 102

Problem Set #3 **Due: Wednesday 19th March 2008, by 5 p.m., Hennings 312 slot**

Instructor: Douglas Scott **(Must be handed in on time, otherwise marks will be deducted)**

Answer all three questions, although only one question will be chosen at random for grading

Note: please indicate your answers clearly, e.g. by highlighting or drawing a box around each one – this will please the person marking and assigning grades! Also, pay attention to the level of precision which is required

1. Stephen Hawking for a day! One unexpected (theoretical) property of black holes was discovered by Stephen Hawking in the early 1970s – namely that isolated black holes give off very weak blackbody radiation and would eventually evaporate.

(a) You can derive the Hawking temperature and evaporation rate of a black hole in an approximate way using the following argument. Start with the approximation that the energy of a particle moving at near the speed of light is $E = p\,c$, where p is momentum, and that the average energy of a photon coming from a blackbody of temperature T is kT (where k is Boltzmann's constant). Now use the 'Heisenberg Uncertainty Relation', which says that if you fix particles to be within some size Δx, then the momentum is constrained such that $p\,\Delta x = h$, where h is Planck's constant. So now take Δx to be the Schwarzschild Radius, ($R_{\rm S}$) for particles trapped in a black hole, recalling that $R_{\rm S} = 2GM/c^2$, where M is the mass of the black hole. Putting all this together, show that the temperature of particles constrained to be inside the black hole $T_{\rm H} \propto (1/M)$.

(b) By putting in the constants (k, h, G and c, but ignoring factors like 2 and π if you like, for this approximate argument), calculate the temperature of a black hole of mass: (i) 1 kg; (ii) 10^{10}kg; and (iii) $1\,{\rm M}_\odot$. For each case also state the type of electromagnetic radiation at which the emission peaks.

(c) By keeping in the constants (but you can ignore 2s and πs again) find the approximate evaporation time for each of the black hole masses in (b).

2. New data on the Galactic Centre!

(a) There is a star near the centre of our Galaxy which has recently been observed in a complete orbit with period of 15 years and major axis of 0.18 arcsec. Assuming that the Galactic Centre is 8 kpc away, calculate the semi-major axis in metres. Use this value to estimate the mass of the central object (actually the sum of the masses, but you can ignore the small mass of the star; you can also assume that the orbit is almost in the plane of the sky, so there are no inclination effects).

(b) The orbit is actually quite elliptical, but for simplicity we will assume it is circular. What fraction of the speed of light is the orbital speed of the star? How much bigger is this than the orbital speed of Mercury around the Sun also measured as a fraction of the speed of light (from which we can measure some relativistic effects if we make very precise observations)?

(c) If the central object is a black hole, how many Schwarzschild Radii away is the star orbiting? How much closer is this than $R/R_{\rm S}$ for Mercury in the Sun's gravitational field? Would you expect orbital effects of General Relativity to be detectable for this star?

2. Round and round!

(a) You observe a certain spiral galaxy, which is nearly edge-on, and notice that it has 2 prominent gas clouds, one about equally distant on each side of the centre of the galaxy. With a radio telescope you detect the hydrogen '21 cm' emission from these clouds. You measure a wavelength of 21.1417 cm and 21.1371 cm for the emission from the 2 clouds (where you expect the wavelength to be precisely 21.1061 cm for an object at rest). Use this to estimate the velocity at which the galaxy as a whole is moving away from us, and the rotation speed of these clouds around the centre of this galaxy.

(b) By estimating the distance to the galaxy and the angular positions of the clouds you estimate that they are both about 22 kpc from the centre of the galaxy. Use this information, together with your answer for part (a) to estimate the mass of the galaxy contained within the orbits of the clouds.

(c) Assuming that these clouds have existed for the entire age of the Universe, i.e. about 14 billion years (actually not a good assumption, but let's ignore that!), how many times have they rotated all the way round this galaxy?