ASTR 407/507 Assignment 7 Due : Wed Apr 4, 10 AM

Solar system small-body populations are reasonably represented by power laws between some minimum radius r_{min} and maximum r_{max}. Take the differential power law for the number of objects between r and r+dr to be N(r) dr = C r^{-q} dr. where q>0 is the *differential* size index. One aspect of power laws is that the physical quantities (total number N_{tot}, total mass *M*, total cross sectional area *A*) that apply to the total population between the two limits can often be expressed in terms of *one* of the end-point radii, rather than both, because one limit is negligible (expect for one specific size index where there is a transition and the quantity in question depends on the logarithm of the end-point ratio).

1. Compute simple approximate for each quantity N_{tot}, *M*, and *A*, which depend only on *q*, *C*, the object physical density ρ (taken to be constant) and the limiting radii. In each case neglect one of the limiting radii if you can, specifying which range of q your result in valid for. For each quantity there will be a critical *q* where dependence becomes logarithmic; which *q* is that for these 3 quantities?

2. Construct a overly-simplistic model of the Kuiper Belt from the very largest two objects (Pluto and Eris, taking each to be r=1000 km), by taking a differential q=5 power law from there to down to 50 km radius (your work in question 1 will have shown that the total mass were to diverge if this were to continue to zero radii). At this 50-km `knee' take the size index to switch to q=2 for smaller objects. Here it is easier to work with the cumulative number distribution (Hint: The cumulative N for radii below the knee must 'anchor' to value at the knee, and extend to smaller radii with power law cumulative index -1). This allows one to calculate various quantities, where the results from question 1 will be useful. *Be very careful with units*. Consider two options for the small-than-knee regime : one that ends at r_{min}=1 km (cometary sized) and the second which continues to zero size.
(a) Make a table with columns Big, 50-1, and 50-0 and rows *N (>r)* , *M*, and *A*, into which you insert values for the quantity's value for that portion of the population. Use units of Earth-masses for *M* and km-squared for *A*.
(b) For the *r*=1000 km to zero case, compute the total cross-sectional area of all the objects and express this as the fraction of the Solar System plane's area, assuming the Kuiper Belt is a annulus that occupies the plane from 35 to 50 AU. This is essentially the 'optical depth' for light that is perpendicularly incident on the disk.
(c) Given that the R-band magnitude of the Sun is -26.5, compute the H-magnitude (the solar system `absolute magnitude) of a r=50-km object, if the albedo is 4%. Neglecting phase effects, the apparent magnitude *m* in a given photometric band is related to the H magnitude (in the same band) via
$m = H + 2.5 \log_{10} (d^2 \Delta^2)$
where d and Δ are heliocentric and geocentric distances, in AU. The 4-meter CFHT, 10-meter Keck, and Hubble Space telescope (HST) have rough apparent magnitude limits of 24.5, 25.5, and 27.0 for simple exposures. At what distances do these r=50 km knee objects fade below visibility of those telescopes (that is, how close do such objects have to be in order to be seen) when seen in the direction opposite from the Sun (opposition)?
Can any of the telescopes see r=1 km comet nuclei in the trans-neptunian region?
Approximately how far away can Keck see Pluto?

3. This question is deliberately free-form. Explain your reasoning. Consider spherical meteoroids of radius *r* vertically incident at the top of the atmospheres of Earth, Mars, and Venus. Take the zeroth-order approximation that a meteoroid will stop due to momentum upon traversing its own mass in atmosphere. In fact this is roughly where a large visible bolide produces its `terminal burst' where it breaks into many smaller fragments which then rapidly decelerate.

a) assume first that the atmospheres can treated as isothermal and thin, taking reasonable values for the constant temperature, mean molecular weight, and surface pressure (see text and quote your source). Derive and expression for the minimum radius of a meteoroid that can reach the ground of each of the planets. Compute this value for rocky meteoroids of density 2.5 g/cc and iron meteorites of mass 8 g/cc.

b) Comment on the turnover in the crater diameter distribution of the venerian surface in light of your results in (a).

c) **ASTR 507 students only.** Obtain from the professor the electronic format table containing the 'US standard atmosphere', which is a standard model giving the pressure and number density as a function of altitude. (The local mass density is just the number density times the mean molecular weight). Assuming the entry for height z applies from z to the next entry and that the atmosphere above 1000 km is irrelevant (that is, ignore the z=1000 km row), make a more realistic measurement of the stopping height of a 90-cm radius meteoroid (a largish meteorite-dropper pre-atmospheric size) and compare to an estimate from the approach of part (a).