Planetary Interiors

ASTR 407/507

Planetary Interiors

- How do we determine a planet’s bulk structure?
- How do pressure and temperature vary inside a planet?
- How do planets lose heat?

Planetary Mass

- The mass *M* and *bulk* density $<\rho>r$ of a planet are two of its most fundamental and useful characteristics
- These are easy to obtain if something (a satellite, artificial or natural) is in orbit round the planet, thanks to Isaac Newton . . .
  $n^2 a^3 = G(M+m) \sim GM$
- This furnishes planetary mass M

  - If no satellite exists, then mass can still sometimes be obtained in a spacecraft flyby (deflection of trajectory) or long-range gravitational effect of the object.

- The physical size (and thus volume) is usually acquired via direct optical measurement (for small bodies it may be a stellar occultation).
- Then $<\rho> = \text{mass/volume} \sim 3M/(4 \text{ pi } R^3)$
  - (If object non-spherical, use true 'volume')

Bulk Densities

- So for bodies with orbiting satellites (Sun, Mars, Earth, Jupiter etc.) M and $<\rho>$ are trivial to obtain
- For bodies without orbiting satellites, things are more difficult – we must look for subtle perturbations to other bodies' orbits (e.g. the effect of a large asteroid on Mars' orbit, or the effect on a nearby spacecraft's orbit)
- Bulk densities are an important observational constraint on the structure of a planet. A selection is given below:

Object	Earth	Mars	Moon	Mathilde	Ida	Callisto	Io	Saturn	Pluto
R (km)	6378	3390	1737	27	16	2400	1821	60300	1180
ρ (g/cc)	5.52	3.93	3.34	1.3	2.6	1.85	3.53	0.69	~1.9

Data from Lodders and Fegley, 1998

What do the densities tell us?

- Densities tell us about the different proportions of gas/ice/rock/metal in each planet
- But we have to take into account the facts that (1) bodies with low pressures may have high porosity, and that (2) most materials get denser under increasing pressure
- A big planet with the same bulk composition as a little planet will have a higher density because of this self-compression (e.g. Earth vs. Mars)
- In order to take self-compression into account, we need to know the behaviour of material under pressure.
- On their own, densities are of limited use. We have to use the information in conjunction with other data, like our expectations of bulk composition.

Bulk composition (reminder)

Element	C	O	Mg	Si	S	Fe
$\log_{10}$ (No. Atoms)	7.00	7.32	6.0	6.0	5.65	5.95
Condens. Temp (K)	78	--	1340	1529	674	1337

- Four most common refractory elements: Mg, Si, Fe, S, present in (number) ratios 1:1:0.9:0.45
- Inner solar system bodies will consist of silicates (Mg,Fe,SiO_3) plus iron cores
- These cores may be sulphur-rich (Mars?)
- Outer solar system bodies (beyond the snow line) will be the same but with solid H_2O mantles on top

Example: Venus

- Bulk density of Venus is 5.24 g/cc
- Surface composition of Venus is basaltic, suggesting peridotite mantle, with a density ~3 g/cc
- Peridotite mantles have an Mg:Fe ratio of 9:1
- Primitive nebula has an Mg:Fe ratio of roughly 1:1
- What do we conclude?

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- Venus has an iron core (explains the high bulk density and iron depletion in the mantle)
- What other techniques could we use to confirm this hypothesis?

Pressures inside planets

- Hydrostatic assumption (planet has no strength)

$$\frac{dP}{dr} = -\rho g$$

- For a planet of constant density ρ (is this reasonable?)

$$g(r) = \frac{4}{3}\pi G\rho r = g_0 \frac{r}{R}$$

$$P(r) = \frac{1}{2} g_0 \rho R \left(1 - \frac{r^2}{R^2}\right) = \frac{2\pi G}{3} R^2 \rho^2 \left(1 - \frac{r^2}{R^2}\right)$$

- So the central pressure of a planet increases as the *square* of its radius
- Moon R=1800km P=7.2 Gpa, Mars R=3400km P=26 GPa

Pressures inside planets

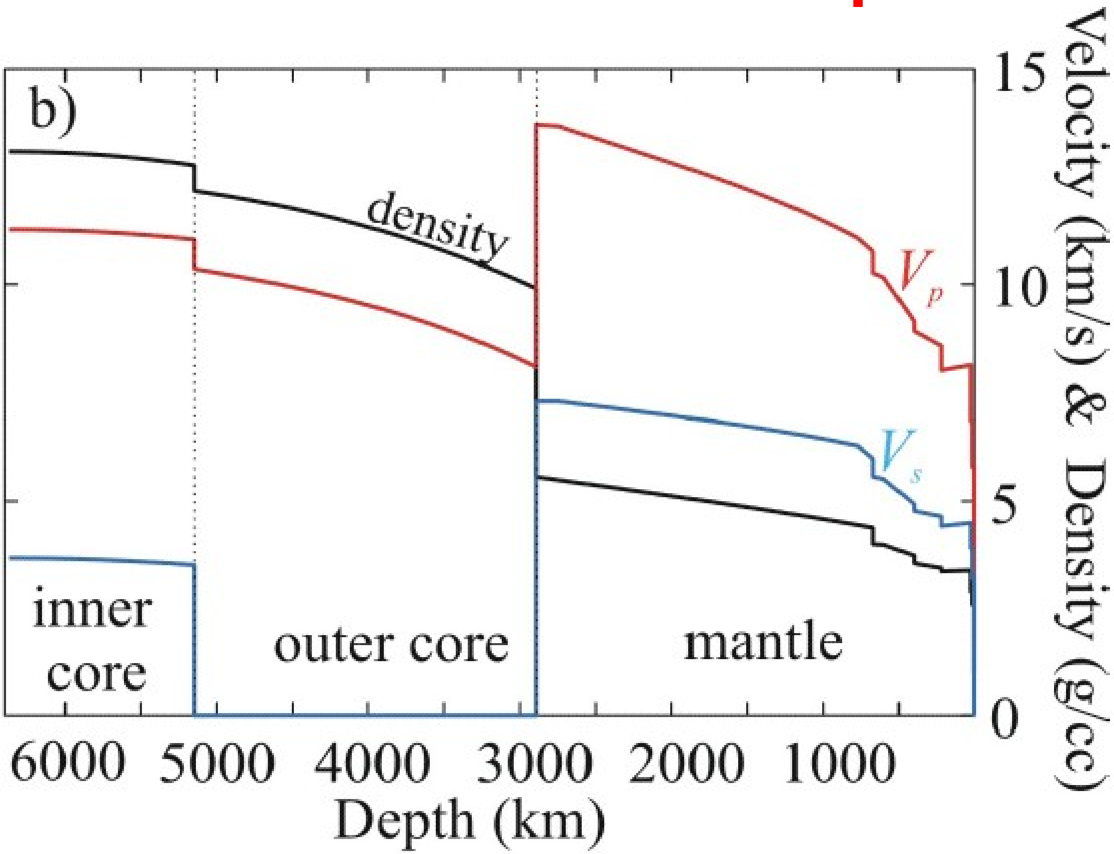
- The pressure inside a planet controls how materials behave
- E.g. porosity gets removed by material compacting and flowing, at pressures ~ few MPa
- The pressure required to cause a material's density to change significantly depends on the bulk modulus of that material

$$\boxed{\frac{d\rho}{\rho} = \frac{dP}{K}}$$

The bulk modulus K controls the change in density (or volume) due to a change in pressure

- Typical bulk modulus for silicates is ~100 GPa
- Pressure near base of mantle on Earth is ~100 GPa
- So change in density from surface to base of mantle should be roughly a factor of 2 (ignoring phase changes)

Real planets



- Which planet is this?
- Where does this information come from?

- Notice the increase in mantle density with depth – is it a smooth curve?
- How does gravity vary within the planet?

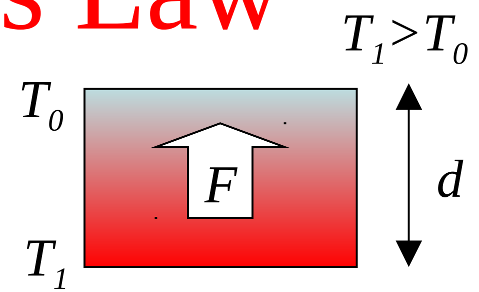
Other techniques

- There are other things we can do
- We can make use of more gravitational information to determine the moment of inertia of a body, and hence the distribution of mass within its interior
  - $I/(MR^2)$ gives information on central concentration
- There are also other techniques
  - Seismology (Earth, Moon)
  - Electromagnetic studies (Earth, Moon, Galilean satellites)

Temperature Structures

- Planets generally start out hot (see below)
- But their surfaces (in the absence of an atmosphere) tend to cool very rapidly
- So a temperature gradient exists between the planet's interior and surface
- The temperature gradient means that the planet will tend to cool down with time

Conduction - Fourier's Law



- Heat flow $\boxed{F = k\frac{(T_1 - T_0)}{d} = k\frac{dT}{dz}}$
- Heat flows from hot to cold (thermodynamics) and is proportional to the temperature gradient
- Here k is the thermal conductivity ($Wm^{-1}K^{-1}$) and units of F are Wm^{-2} (heat flux is *per unit area*)
- Typical values for k are 2-4 $Wm^{-1}K^{-1}$ (rock, ice) and 30-60 $Wm^{-1}K^{-1}$ (metal)
- Solar energy flux at 1 au is ~1300 $W\ m^{-2}$
- Mean subsurface heat flux on Earth is 0.08 $W\ m^{-2}$
- What controls the surface temperature of most planetary bodies?

Specific Heat Capacity C_p

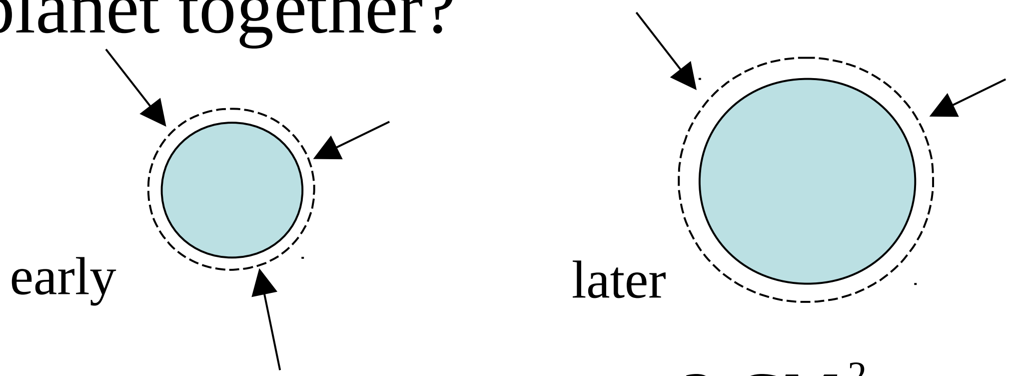
- The specific heat capacity C_p tells us how much energy needs to be added/subtracted to 1 kg of material to make its temperature increase/decrease by 1K
- Units: J kg^{-1} K^{-1}
- Typical values: rock 1200 J kg^{-1} K^{-1} , ice 4200 J kg^{-1} K^{-1}

$$W = mC_p\Delta T$$

- Energy = mass x specific heat capacity x temp. change
- E.g. if the temperature gradient near the Earth's surface is 25 K/km, how fast is the Earth cooling down on average? (about 170 K/Gyr)
- Why is this estimate a bit too large?

Energy of Accretion

- Let's assume that a planet is built up like an onion, one shell at a time. How much energy is involved in putting the planet together?



In which situation is more energy delivered?

Total accretional energy = $\frac{3}{5}\frac{GM^2}{R}$

If all this energy goes into heat*, what is the resulting temperature change?

$$\Delta T = \frac{3}{5}\frac{GM}{C_p R}$$

* Is this a reasonable assumption?

Earth M=6x10^{24} kg R=6400km so ΔT=30,000K

Mars M=6x10^{23} kg R=3400km so ΔT=6,000K

What do we conclude from this exercise?

Accretion and Initial Temperatures

- If accretion occurs by lots of small impacts, a lot of the energy may be lost to space
- If accretion occurs by a few big impacts, all the energy will be deposited in the planet's interior
- Additional energy is released as differentiation occurs – dense iron sinks to centre of planet and releases potential energy as it does so
- What about radioactive isotopes? Short-lived radio-isotopes (^{26}Al, ^{60}Fe) can give out a lot of heat if bodies form while they are still active (~1 Myr after solar system formation)
- A big primordial atmosphere can also keep a planet hot
- So the rate and style of accretion (big vs. small impacts) is important, as well as how big the planet ends up

Cooling a planet

- Large silicate planets (Earth, Venus) probably started out molten – magma ocean
- Magma ocean may have been helped by thick early atmosphere (high surface temperatures)

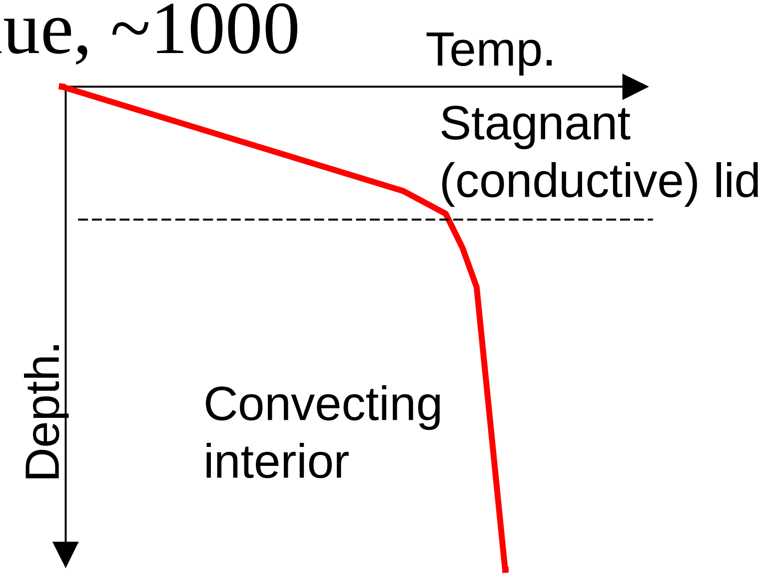
- Once atmosphere dissipated, surface will have cooled rapidly and formed a solid crust over molten interior
- If solid crust floats (e.g. plagioclase on the Moon) then it will insulate the interior, which will cool slowly (~ Myrs)
- If the crust sinks, then cooling is rapid (~ kyrs)
- What happens once the magma ocean has solidified?

Cooling a planet (cont'd)

- Planets which are small or cold will lose heat entirely by conduction
- For planets which are large or warm, the interior (mantle) will be convecting beneath a (conductive) stagnant lid (also known as the lithosphere)
- Whether convection occurs depends if the Rayleigh number *Ra* exceeds a critical value, ~1000

$$Ra = \frac{\rho g \alpha \Delta T d^3}{\kappa \eta}$$

Here ρ is density, g is gravity, α is thermal expansivity, ΔT is the temperature contrast, d is the layer thickness, κ is the thermal diffusivity and η is the viscosity. Note that η is strongly temperature-dependent.



Convection

- Convective behaviour is governed by the Rayleigh number *Ra*
- Higher *Ra* means more vigorous convection, higher heat flux, thinner stagnant lid
- As the mantle cools, η increases, *Ra* decreases, rate of cooling decreases -> *self-regulating* system

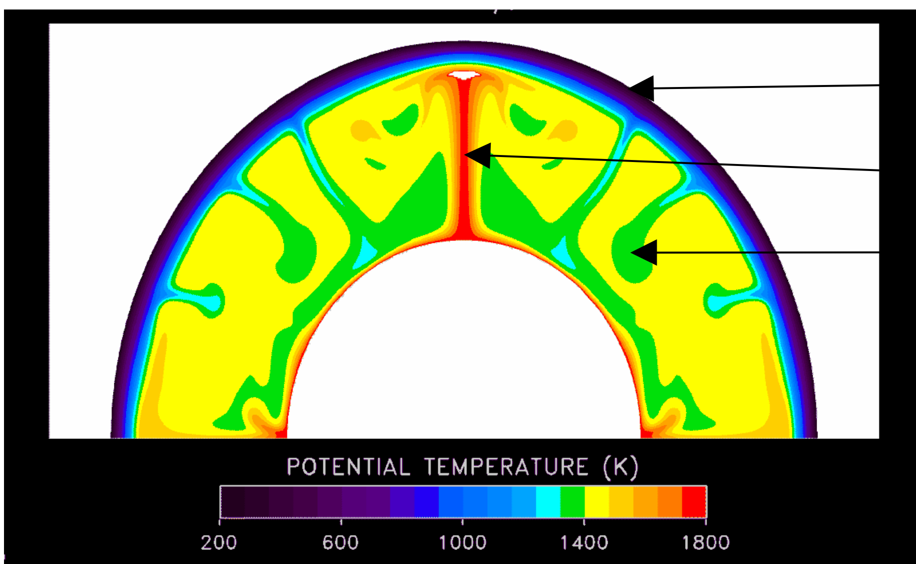


Image courtesy Walter Kiefer, Ra=3.7x10^6, Mars

The number of upwellings and downwellings depends on the balance between internal heating and bottom heating of the mantle