ASTR 407/507 Homework 2

Due: Wednesday Jan 17, 10 AM in class or under Prof's door NO EMAIL submissions
Late Penalty : -30% if turned in by 4 PM the following Friday, -60% if by 5 PM the following Monday

Your answer must be *clearly explained* and *neatly presented*, or points will be deducted. Although you may work with other students to understand, your write-up must be done independently.

1. **Effective potential of the Kepler problem.** Consider a test particle orbiting a central mass M, and define GM=μ . Work in plane polar coordinates (r,θ).

a. Express the velocity vector in term of the two polar unit vectors $\hat{r}$ and $\hat{\theta}$.

b. Compute the magnitude of v^2 in term of the time derivatives of *r* and θ, and the show that the total energy $E=\frac{1}{2}\dot{r}^2+U(r,L)$ where *U* is the `effective potential' which has no depence on anything related to the angular coordinate because the angular momentum (per unit mass) *L* has been used to eliminate it. This has reduced the 2-dimensional motion to a 1-D potential, with turning points like in 1st year physics, that confines the particle.

c. Take μ=1 and plot U(r) for the cases of L=1 and L=3/4 (thus, the effective potential's shape depends on L!), on a single diagram (go to r=5 and use a range of +/-1 for the potential). For both of these cases, identify and measure approximately off your graph what the circular orbit distance is. Return to the analytics of the problem and derive the exact value of *r* that corresponds to the circular orbit of the two cases.

d. Bound orbits have E<0, and thus a horizontal E<0 line on this plot characterizes the motion. Consider the L=1 case and the E=-0.3 energy orbit for that case. Explain
on your diagram how to read off the perihelion and aphelion distances, and measure them roughly off the diagram.

e. Looking at the equation given in (b), realize that perihelion and aphelion correspond to places where $\dot{r}=0$ (that is, the turning points). Using that E= - μ/2a , derive a quadratic equation for the value of *r* of the turning points and show that the two solutions are nothing but *q=a(1-e)* and Q=a(1+e). For the L=1, E=- 0.3 case in (d), show that *q* and *Q* match what you found on the graph, and thus derive *a* and *e* for this orbit from E and L.

2. **Resonances.** The phenomenon of mean-motion resonance can occur when repeating orbital configurations result, due to the fact that periods are simple low order fractional multiples (larger or smaller than one) of a planet. The asteroid belt has a set of locations (in particular, semimajor axes) where this is true.

a. First, show that if a time unit of years and a length unit of au are used, then the value of μ=GM must be $\mu= 4\pi^2$ au^3/yr^2.

b. Compute the semimajor axes which correspond to orbital periods that are 1/3, 1/2, and 2/3 that of Jupiter. Then take the MPCxyz.txt file from Assigment 1 and use the vis-viva equation to compute the semimajor axes (in au) of all the minor planets in the file and compile those with 2<a<5.5 au into a histogram that you should plot, using a semimajor axis histogram bin no larger than 0.1 au, labelling these 3 resonant locations.

3. **Text problem 2-1.** For its part (b), the ball is not thrown `straight up' (avoiding the case of a purely radial orbit). GRADUATE STUDENTS WILL SUBSTITUTE THE GRAD PROBLEM (distributed separately) FOR THIS ONE.