ASTR 407/507 – Jan 2018 ASSIGNMENT 4 Due Wed Feb 14, 10 AM

1. Planetary interior models. The most common simple planetary interior model is a two-component model with a dense core and rocky mantle. Take the planet to be spherical with mass *M* and radius *R* (measurable) and *mantle* density ρ (unknown). Take the core density to be *k* times the mantle density, and the core to extend to some fraction *f* of the planetary radius.

a) Write the planetary mass as a function of *f* and *k*. Invert this to give ρ=ρ(M,R,f,k).

b) Write an expression for the planet's total moment of inertia in terms of ρ, R, f, and k.

c) Thus derive an expression for the polar moment of inertia $I/(MR^2)$ for such a two-layer planet, and plot as a function of f for each of the values of k=1.5, 2.5, 4.5.

d) Mercury's value of $I/(MR^2)$ was poorly known until a few years ago, at which point it was determined that the ratio was 0.353 +/- 0.013. Add this range to your graph and explain (i) how this rules out some values of k, and (ii) the physical interpretation of the multi-valued solution (which partially includes why the extremal values are the same).

e) If we take the (nickel-iron dominated core)/(silicate mantle) to be best approximation to Mercury, with a forced density contrast of k=2.5, then for the `largest possible core' solution consistent with the data within 1-sigma uncertainty, what are the mantle and core densities (kg/m^3) and the core radius (km)? Mercury has bulk density 5430 kg/m^3, R = 2440 km, $M=3.30 \times 10^{23}$ kg.

2. Poynting-Robertson drag on cm-scale particles. One can take the expression for the force generated by PR drag and apply the perturbation equations of celestial mechanics to derive the orbital element evolution equations for semimajor axis and eccentrcity:

$$\frac{da}{dt} = -\frac{K}{a}\frac{2+3e^2}{(1-e^2)^{3/2}} \quad \text{and} \quad \frac{de}{dt} = -\frac{5Ke}{2a^2\sqrt{1-e^2}} \quad \text{where} \quad K = \frac{3L}{16\pi c^2 s\rho} \quad \text{contains the solar}$$

luminosity *L*, the speed of light *c*, and the particle radius and density *s* and ρ .

(a) For a particle on an initially circular orbit at semimajor axis a_o, show that the orbital evolution results in collapse of the orbit to the origin in finite time T and derive a scaling law of the form: $T = C_{yr}\left(\frac{s}{1\,cm}\right)^x\left(\frac{\rho}{1\,g/cc}\right)^y\left(\frac{a_o}{1\,au}\right)^z$ where C is a constant expressed in years and *x, y*, and *z* are exponents you should determine.

(b) Compare the collapse timescale in part (a) to that estimated by dividing the initial semimajor axis by the *initial* orbital decay rate. By what factor is it larger or smaller? Give a physical interpretation as to why it is larger or smaller.

3. The tidal evolution of the Lunar orbit. Using the tidal formulation presented in class, *analytically* calculate :

a) how long ago the Moon was at a semimajor axis of 30 Earth radii (assuming that $k_2^T/Q = 0.29/11.7$ for the Earth is *constant* in time).

b) Assuming that the Moon could stay intact, how much more time (back into the past) would it take for the Moon to be at 2.5 Earth radii? (This is where the Moon likely accreted out of an impact-generated disk).

c) Now write a simple numerical integration algorithm (even Euler's method will be fine) to integrate (back in time) the 1st -order orbital evolution differential equation for the case of constant (k_2/Q); reproducing your answer from question 2(b) above. (You should convince yourself that your step size is small enough by showing convergence). Based on that numerical solution, generate a graph of the lunar distance as a function of time into the past.

d) NOW assume that Q varies into the past with an `ad-hoc' time variation that takes into account the fossil records (Q not very different from now at 300 Myr ago) and values of Q like that of Mars (80-ish) for solid-bodies. More specifically, take Q to vary as :

$$Q = 11.7 * (1 + T_{Gyr})^{p}$$

where T is measured in Gyr into the past and *p* is some power. If we take as a constraint that the Moon must have been at roughly 2.5 Earth radii 4.4 Gyr ago, what is the value of *p*? For that particular value of *p*, make a graph of the lunar orbital distance as a function of time into the past. (For this problem we assume that the Moon always had a circular orbit. The time variation is not intended to be realistic...).