ASTR 200   Homework 1

Due: Thursday Sept 14, 5 PM   in ASTR 200 box outside Henn 312.   NO EMAIL submissions
Late Penalty : -30% if turned in by 4 PM the following Friday, -60% if by 5 PM the following Monday

General: See course web page for general instructions about Homework standards.
In particular, your answer must be *clearly explained* and *neatly presented*, or points will be deducted.

1. The Small Angle approximation. [15 points]
(a) In lecture 2 it was stated that the fractional error in the small angle approximation is ~10^{-17} for an angle of one arcsecond. Prove this, where the fractional 'error' E is E = (approx – exact)/(exact) .
(b) What fractional error occurs when the angle is 1 arcminute? When the angle is 0.5 degrees?

2. Planetary Structure [30 points]
Although we mentioned in the first class that the Earth is actually a slightly flattened oblate spheroid (bulging slightly at the equator, but less than one percent), for the purposes of this question assume the Earth is perfectly spherical, with radius R=6370 km and mass M=5.98 x 10^{24} kg. Work to three significant figures throughout. Recall that the density is just the mass divided by the volume that the material occupies.

a) Here you will consider a two-component model of the Earth, with a central 'core' and a surrounding 'mantle'. From measurements of rocks near the surface and an understanding of how they will compress, it is thought at a rough density of 4.50 g/cc (cc = cubic centimeter) is correct for the mantle. [For reference, recall that the density of water is 1.0 g/cc = 1000 kg/cubic meter]. From studies of seismic wave propagation, the Earth's core is estimated to be about 3380 km in radius. Given this information, what would the density of the core need to be in this model? (Hint: the mass of the mantle would be a uniform Earth with the mantle density minus mass of the spherical center which will be replaced by the core). Careful with conversions of mass and length units.

b) Consider instead a model where the density of the Earth is continuously changing from 3.40 g/cc at the surface and rises linearly as one descends to the center, with a constant slope m. In algebra: $\rho(r)=\rho_c - mr$, so that the density drops from the core density (when r=0) to the value at r=R. Because the mass dM of an infinitesimal spherical shell of thickness dr will be $dM=\rho(r)4\pi r^2 dr$, one can perform a one-dimensional integral to compute the planet's total mass, which must be that of the Earth. Using this information, determine what the density at the Earth's center would have to be in this model.

3. Synodic periods [15 points].

The synodic period is the time interval between repetition of the same two-planet configuration, under the approximation that planetary orbits are nearly circular; for example, between two oppositions.
Asteroid 154660 is named Kavelaars after astronomer JJ Kavelaars who works in Victoria, and discovered by David Balam (another Canadian astronomer who worked in Victoria). Its semimajor axis is 1.92 AU and its orbit is approximately circular. It was last near opposition on Dec 1/2016; when (what month and year) will the next opposition occur?