2.11 Toy universes

Let us consider a second universe as well as the one sugested. The second one will have luminosities distributed according to a power law of slope -3, and we observe it with a telescope of sensitivity limit 200 flux units..

We assume a simple Euclidean universe in which the maximum radius $R_{\rm max} = 1.0$, total volume $4\pi/3$, and of course we're at the centre.

First step: distribute say 10^6 sources uniformly throughout this sphere. This does NOT mean uniformly in radius! For equal values of Δr, the volume shells increase with radius as $r^2 dr$. Hence, following the prescription of Equation (2.16), if ran_i is a random variable uniformly distributed between zero and unity, we can get values of r_i to provide the right radial distribution from

$$r_i = \mathrm{ran}_i^{1/3}$$

and these values will run from 0 to 1.0. We can check our derived radii in a number of ways, for example by plotting a histogram of source densities in volume elements $(4\pi/3)[R_2^3 - R_1^3]$ corresponding to equal increments in R as shown in Figure 1.

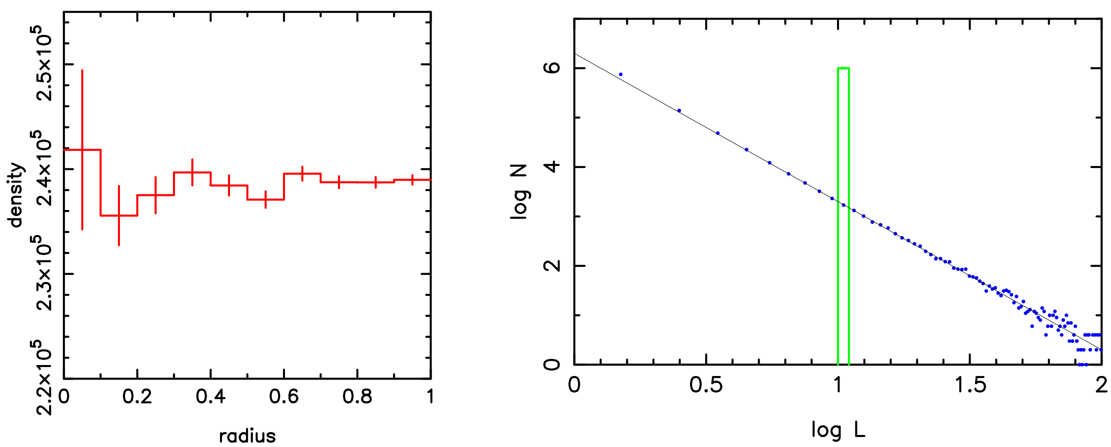

Figure 1: Left: Space density as a function of radius. Error bars correspond to $\sqrt{N}$; the 'near' volume elements are much the smaller, contain far fewer objects than the 'far' ones, and thus have larger error bars. The data are consistent with uniform space density throughout our universe. Right: the two luminosity functions – 1st universe, green histogram, all $l_i = 10$; 2nd universe, blue dots, l_i drawn from a l^{-3} distribution.

We can use the same values of r_i for each universe. If we wish (as it's our universe and we're at the centre), we can ascribe right ascensions and declinations (ϕ_i, θ_i) to each 'galaxy' as well, and use them to plot sky patches of 'survey areas', as shown in Figure 2.10.

The second step is to ascribe luminosities to our sources. The example suggests giving them all an equal luminosity of $l_i = 10$ units and we do this for our 1st universe. For our 2nd universe, we ascribe luminosities according to a power law of slope -3. To do this we call upon equation (2.16) again, and the result is

$$l_i = \mathrm{ran}_i^{-1/2}$$

with luminosities running from 1.0 to infinity, although numbers of course rapidly diminish with increasing luminosity. Figure 1 (right panel) shows a check that a selection of 10^6 objects have luminosities distributed according to our two prescriptions.

Our task in making source counts for these two universes is now straightforward, easily demonstrated with a piece of (readily translatable) Fortran code. For the first universe (for which we have a telescope of flux sensitivity $0.1 * 10 = 1.0$), and using equal bins of 0.05 in $\Delta(log_{10}S)$:

```
do i=1,1000000
  if(s(i).gt.(1.0))then           !flux density limit imposed
    index=20.*alog10(s(i))+1.     !find which count bin flux belongs
    icount(index)=icount(index)+1 !form histogram of count
  endif
enddo
```

This gives us a histogram `icount`, which is the source count.

Our second universe follows the same coding, except that now we have decided on a flux density limit of 200, so that ″1.0″ in the second line is replaced by ″200.0″.

The results for both universes are shown in Figure 2.

So are we done? In principle, but wait – here are three issues. (1) Why should the slopes of the counts be the same for such different universes? (2) How did we get to calculate the perfect fit lines in the source-count picture? And – most important of all – (3) are these universes vaguely realistic?

The last issue first. We have an indication from Figure 1, right panel, that all is not well with our simple model in which we assigned a single luminosity to all our objects. A delta function for a luminosity function cannot be remotely realistic. We know that at any wavelength there is always a spread, and that there are always fewer objects of higher luminosities per unit volume. Even the power law is not ideal, although it follows our general ideas of most luminosity functions. We could have improved the realism with say a Schecter function (Section 3.4) if we were modelling a galaxy universe.

If for our 2nd universe we plot luminosity against distance, and put in a survey flux sensitivity limit, we end up with a sample of sources from our universe which roughly follows a Hubble relation, as shown in the right panel of Figure 3. We can ‘observe’ only 1333 of the total of 10^6 objects in our universe. *Inferring general properties about the objects in our universe from a small sample of them is what we do*, and this diagram has the same general form as Figures 4.1 and 4.3. Thus we have a

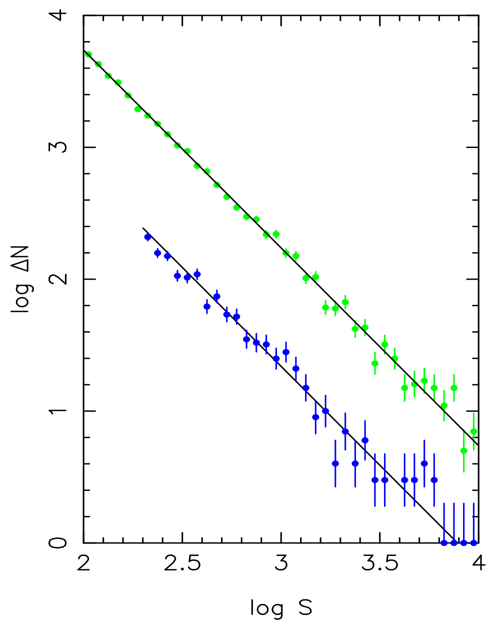

Figure 2: Source counts for our two universes. Green points: single-luminosity universe. Blue points, universe with luminosities distributed according to l^{-3}. Error bars correspond to $\sqrt{N}$ errors where N is the number of fluxes in each bin.

toy universe here which is not totally unrealistic and from which we might hope to make further modifications such as a Friedman geometry, a better luminosity function, some observational uncertainty about our measurements, K-corrections, etc., to yield a simulation which might be of some use to us.

The same cannot be said of our 1st universe. Its equivalent plot is shown in the left panel of Figure 3. Our single luminosity assignment does not yield a Hubble diagram or anything like it. Moreover, our chosen sensitivity imposes no cutoff; the faintest object in our universe has a flux of 10, while our chosen sensitivity is 1.0. It is hardly realistic to expect to see all objects in the universe!

Examine your chosen universe from more than one perspective to make sure that your simulation bears some relation to reality.

The answers to questions (1) and (2) are mechanical, but bear comment. The first question: for any luminosity, the number of sources visible above a flux density of S is obtained by first calculating the volume out to which the objects can be seen and then multiplying this by the source density within that volume. As $S = L/R^2$, $R = \sqrt{L/S}$, this volume is

$$V_i = (4\pi/3)(\sqrt{L_i/S})^3$$

while the luminosity function gives us a space density ρ_i for each L_i.

Thus for each luminosity

$$N_i(S) > \rho_i V_i = C_i S^{-3/2}.$$

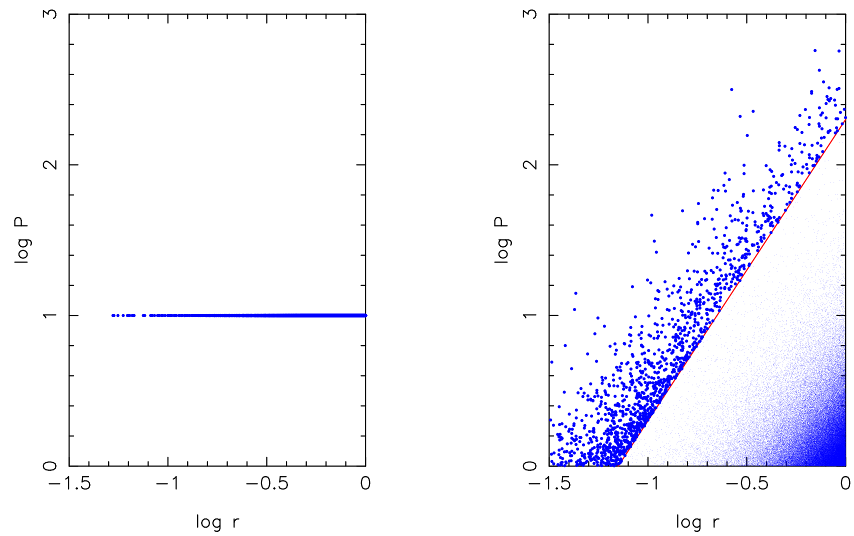

Figure 3: The luminosity (or power P) vs radial-distance plane. Left: 1st universe. All objects have a luminosity of 10, and the flux cutoff for our telescope is assumed as $0.1 \times 10 = 1.0$. Hence all objects in the universe are 'visible'. Every tenth object has been plotted; otherwise the dots would form a solid horizontal line. Right: 2nd universe. The luminosities are distributed at random amongst the objects according to a power law l^{-3}. Our telescope is sensitive down to a flux of 200 units. All 10^6 objects are plotted as very faint dots except those above the sensitivity limit (the red line) – the sample of 1333 objects we can 'see' in order to infer the properties of all 10^6 objects. The number density of faint dots in the figure gives rise to the apparent continuum in the bottom right corner.

Adding this up for all L_i:

$$N(>S) = \sum C_i S^{-3/2}$$

Thus it matters not whether we have one luminosity or a continuum of them described by a luminosity function: in a Euclidean universe, $N(>S) = KS^{-3/2}$. This not the case in a Friedman geometry, because the relation between R and redshift z is not linear (except at tiny z), so that the law is a curve, not a power-law, a different curve for each luminosity that is always (for a uniformly filled universe) flatter than a $-3/2$ law. At infinitely large flux densities however, the slopes at any luminosity are asymptotic to $-3/2$. But by the time we get to anything like high enough fluxes, there are generally too few objects to see the form of the source count, an example of *cosmic variance* (Section 10.6).

As for the second question, consider our 1st universe, 10^6 sources each with $l_i = 10.0$. We know our source-count law is $N(>S) = K_1 S^{-1.5}$ and as our minimum flux is $S_{\min} = 10.0$, $K_1 = 10^6/10^{-1.5} = 3.16 \cdot 10^7$. We want the *differential* source count, i.e. $dN = -1.5 \cdot 3.16 \cdot 10^7 S^{-2.5} dS$ or $dN = -4.74 \cdot 10^7 S^{-2.5} dS$. We have chosen to plot in the conventional logarithmic form, in equal intervals of $log_{10}S$, and remembering that $dS = S \cdot d(logS)/(log_{10}e)$, we get $dN = 4.74 \cdot 10^7 \cdot S^{-1} \cdot S^{-2.5} d(log_{10}S)/(log_{10}e)$, or $dN = 4.74 \cdot 10^7 \cdot S^{-1.5} d(log_{10}S)/0.434$. This plot was made with equal intervals of $\Delta(log_{10}) = 0.05$, so that our final law is

$$\Delta N = 5.46 \cdot 10^6 \cdot S^{-1.5}.$$

I have spelt things out, simply because there are numerous frustrating pitfalls with power-law plots, as emphasized in the text. (Are you sure you'd have remembered that factor of $1/log_{10}e$?)

It is left as an exercise to work out the corresponding law for the second (semi-realistic) universe. Note that the key is to work out the normalization, first for the luminosity function:

$$\int_1^\infty Cl^{-3}dl = 10^6,$$

so that $C = 2.0 \cdot 10^6$. Some further work, in steps of calculating first $N(l, > S)$ and then $N(> S)$, will reveal the final law to be $N(> S) = 4.0 \cdot 10^6 S^{-1.5}$ from which the plot will turn out as

$$\Delta N = 4.0 \cdot 10^6 \cdot S^{-1.5} \cdot 0.05/0.4343 \ .$$

The solid black lines in Figure 2 represent the two curves, of identical slope of course. Note that our 1st universe has a higher count by far. This is because our steep luminosity function for the 2nd universe implies far fewer sources visible with luminosities as high as 10.0. Note also that the sensitivity limit does not change the normalization of the source count at all, because above the flux cutoff, we see the all the sources in the universe. The only implication of the flux cutoff for the source count is to place a lower limit on our plot, as shown in Figure 2 (2nd universe).

(Similar considerations will yield you the black line in Figure 1, right panel – note that this plot is in units of equal Δl.)

What the source count can contribute to our knowledge of the spatial distribution of objects is revealed, to some extent at least, in Chapter 8.