2.2 Efficient choosing

For a simple strategy, divide the set of nights into equal parts: a "training" set of the first five nights, and a "target set " of the second half. Pick the first night in the second half that is better than any of the nights in the first half. To get the best night of the ten, we need the best night to be in the second half (probability $1/2$). If the second-best night is in the first half (probability $1/2$) we will definitely pick the best one, so the chance of doing so is at least $1/4$.

For an exact solution, suppose the total number of nights is N and the training set is r long. We need to consider two kinds of night: the best one B, and the best one up to, but not including, the one we are at. Call this "second-best" one S.

If B occurs at location $r+1$, we definitely choose it. The probability of this is $1/N$.

If B occurs at $r+2$ (chance $1/N$) we will choose it if S was in the training set. The chance of this is $r/(r+1)$.

Proceeding in this way, we find that the chance of choosing B is

$$\text{prob(B)} = \frac{1}{N} \sum_{i=0}^{N-r+1} \frac{r}{r+i}.$$

This sums to a combination of polygamma functions. If you are *au fait* with these, you will be able to show that the maximum of the probability is for a training set of length close to $r = n/e$, at which point it is about 0.37. A simple numerical summation will give similar results.