2.5 Bayesian reasoning...

We are given initially

$$prob(+|disease) = 0.9$$

Bayes:

$$prob(+|disease)prob(disease) = prob(disease|+)prob(+)$$

or

$$prob(disease|+) = \frac{prob(+|disease)prob(disease)}{prob(+)}$$

Find the denominator from normalization

$$prob(disease|+) + prob(no disease|+) = 1$$

giving the final result

$$prob(disease|+) = \frac{prob(+|disease)}{prob(+|disease) + prob(+|no \ disease) \left(\frac{prob(no \ disease)}{prob(disease)}\right)}$$

depending on the false-alarm rate and the general incidence of the problem. e.g. if prob(disease)=0.1 and the false-alarm rate prob(+|no disease)=0.01

$$prob(disease|+) = 0.91.$$

On the other hand, if the false-alarm rate were 0.1, then

$$prob(disease|+) = 0.5.$$

Think of the second case this way. Take 1000 people; 100 of them (1000×0.1) have the disease. Of these, 90 or (100×0.9) give a positive test. However, another 90 people $((1000-100)\times0.1)$ give a positive test without having the disease. So, given a positive test, a fraction 90/(90+90) actually have the disease.