3.2 Simple error analysis

Suppose our two random variables are x and y.
If

$$z = x + y$$

then the change in z due to a small change in x is δx, and similarly for y. Now we make a crucial assumption: that we add up the *squares* of these changes, because the δx etc. are random. We should write them as δX. This assumption is really only appropriate for a Gaussian (see Exercise 3.3). However, we then get

$$\delta Z = \sqrt{\delta X^2 + \delta Y^2}.$$

Because we add up the squares, the result is the same for the error in the difference

$$z = x - y.$$

Note that, depending on context, we interpret δ to mean a variation (as in calculus) or a random error. Using upper and lower case, in the book, can help with this notational problem but takes nearly as much thought as paying attention to context!
The implications in the case of a difference may be rather severe, since z might be close to zero and the the fractional error $\delta Z/Z$ would be large. Differences of noisy quantities should be avoided if possible!
For a quotient

$$z = xy$$

we have

$$\delta(xy) = x\delta y + y\delta x$$

from calculus, taking the Taylor expansion only to first order. Again we identify the (supposed independent) error contributions and add them in quadrature:

$$\delta Z = \sqrt{Y^2\delta X^2 + X^2\delta Y^2}$$

and the result is neater for the fractional error

$$\frac{\delta Z}{Z} = \sqrt{\frac{\delta X}{X} + \frac{\delta Y}{Y}}.$$

For a quotient, we have

$$\delta(x/y) = \frac{-x}{y^2}\delta y + \frac{\delta x}{y}$$

and (remembering not to bother with the minus signs) we can identify the separate error terms and again find the simple relationship between the fractional errors.
As stressed in the book, all of this only works for small, near-Gaussian errors.