3.7 Change of variable

We have $u = \sin(x)$ and the change of variable rule

$$p(u)\,du = 2p(x)\,dx$$

where $p(x) = 1/(2\pi)$. There is a 2 here because two x's can map to the same u. It follows that

$$p(u) = \frac{1}{2\pi}\,\partial x/\partial u$$

or

$$p(u) = \frac{1}{\pi(1-u^2)^{1/2}}$$

which is correctly normalized over $[-1, 1]$.
Let $U_i = \sin\Theta_i$, where Θ is uniform in $[0, 2\pi]$. The distribution of the sum $\sum U_i$ can be found from the convolution theorem (see Exercise 3.4). However, in this case we cannot in general do the necessary inversion of the Fourier transform. It is however possible to do the transforms numerically, using the FFT (Chapter 8). In Figure 1 are numerical estimates of the distribution of sums of 1 to 6 U_i. It is quite remarkable how the bizarre distribution for one variable in the sum is washed out into a Gaussian-like form by the sixth.

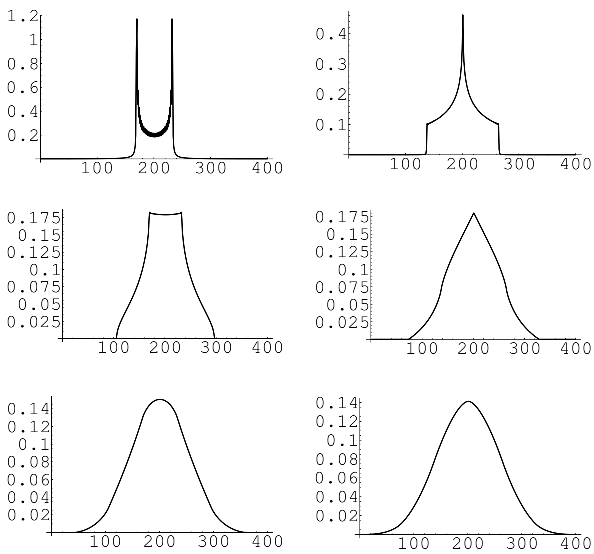

Figure 1: Estimates of the probability distribution of sums of random variables $U_i = \sin\Theta_i$, where Θ is uniform in $[0, 2\pi]$. The units of the x-axis are arbitrary, reflecting how the transform was done numerically.