5.6 Several datasets, one test

The significance level p is an integral of a probability distribution function f:

$$\int_{\alpha}^{\infty} f(x)\,dx = p$$

where α is a critical value. For a particular set of data, α is a statistic and p is random. Applying the change of variable rule

$$\mathrm{prob}(\alpha)\,d\alpha = \mathrm{prob}(p)\,dp$$

and remembering that f is the probability distribution of α, it follows that p is uniformly distributed between zero and one. (Notice this will not be true if the null hypothesis isn't true, as then f will not be the probability distribution of α.)
Now

$$\log W = \sum_{i=1}^{n} \log p_i$$

is a sum of n random variables; $\log p_i = u$ is distributed like e^{-u} (for $u < 0$).
To find the distribution of the sum, we need the Fourier transform of this; it is proportional to $1/(k-\imath)$. The convolution theorem tells us that a sum of n terms will have a transform like $1/(k-\imath)^n$. This transform can be inverted for integer n (see tables, or the indispensable *MATHEMATICA*) and yields the required distribution of $x = -\log W$. This is

$$\frac{1}{\Gamma[n]} x^{n-1} e^{-x}.$$

The results on the mean and variance follow easily from direct integrations. To see the Gaussian form, expand the log of the distribution function in a Taylor series around n, to second order.