8.1 Source counts and luminosity function

The counts of objects on the sky, the surface densities as a function of intensity, are the fundamental and simplest data from sky surveys at all frequencies. In integral form the counts are the so-called 'log N - log S' curve at radio wavelengths, and the 'log N - log m' curve at optical wavelengths, where N is the surface density, S is the (observed) flux density and m is the (observed) magnitude.

For simplicity, consider initially a simple Euclidean uniformly-filled universe in which the volume density of objects in units of Mpc^{-3} or whatever, as a function of luminosity P - the 'luminosity function' - is $\rho(P)$ per unit of luminosity dP. The flux S observed from an object at distance R is

$$S = P/R^2, \tag{1}$$

and the radius out to which an object of luminosity P has a flux $> S$ is

$$R(S) = (P/S)^{1/2}. \tag{2}$$

The number of objects of power P which a survey detects at flux-density S in the shell R to $R + dR$ is

$$\begin{aligned} dN(P,S) &= \rho(P)dV(R) && (3)\\ &= \rho(P)4\pi R^2 dR && (4)\\ &= -2\pi\rho(P)P^{3/2}S^{-5/2}dS && (5) \end{aligned}$$

This is the differential source count for a given luminosity P.

Hence the integral count, the number of objects seen on the sky with luminosity P and with flux $>S_0$ is

$$\begin{aligned} N(P,>S_0) &= \rho(P)\int_0^{R(S)} 4\pi R^2 dR && (6)\\ &= \frac{4\pi}{3}\rho(P)P^{3/2}S_0^{-3/2}, && (7) \end{aligned}$$

and when the contributions of objects of all luminosities are considered,

$$\begin{aligned} N(>S_0) &= \int_0^\infty \rho(P)P^{3/2}dP S_0^{-3/2} && (8)\\ &= const.S_0^{-3/2}, && (9) \end{aligned}$$

Then if the luminosity function happens to obey a power law of slope $-\beta$, ignoring constants we get

$$\begin{aligned} N(>S_0) &= \int_0^\infty P^{-\beta}P^{3/2}dP S_0^{-3/2} && (10)\\ &= \int_{P_1}^{P_2} P^{(3/2-\beta)}dP S_0^{-3/2} && (11) \end{aligned}$$

if the power law has finite limits (as it must to avoid infinite numbers of objects). In addition if $\beta \sim 1.5$ as it appears in some regions, we get simply

$$N(>S_0) = const.(P_2 - P_1).S_0^{-3/2} \tag{12}$$

In this simple geometry, we see that the slope of the source count is $-3/2$ in the integral form, or $-5/2$ in the differential form, *independent of the slope or even the shape of the luminosity function.*

It is critical to emphasize that for real-universe calculations, the foregoing analysis is essentially useless by equation 4, *because of the severe effects of relativistic cosmology.* It is vital at the outset to replace R by D, the appropriate relativistic co-moving distance measure, and to replace dV by the appropriate relativistic co-moving volume element. As soon as this is done, the equations(4) and on become invalid, not even good approximations. In particular the differential count becomes a different curve in the dN vs S plane for each luminosity P. This set of curves in the integral plane each reaches a steepest slope of -3/2 at $S_0 = \infty$ with the curves progressively shallower as luminosity P is increased. The total predicted source count is given by the luminosity function dictating the proportion in which curves for each power should be added to the total.

The simplistic analysis from non-relativistic universes contributed in large measure to one of the greatest controversies in modern astronomy - the Steady-State versus Big-Bang argument of the 1960's. The Hoyle vs Ryle arguments about radio source counts were largely along the lines of whether or not the observed source count was a power-law of slope -3/2, or whether it was a steeper function than this, implying an increase of space density at earlier cosmic epoch. Only a relatively small range of flux density was available from the early radio surveys. And only a few 10s of redshifts were available for the brighter radio galaxies, so that the protagonists did not realize that the median redshifts of the samples of radio sources they were counting exceeded 1. (Added to this was the controversy over whether redshifts had any relation to cosmic distance.) Nevertheless nobody seemed to carry out the calculation to show simply how much relativistic cosmology (or Steady-State cosmology) affected the source-count, how much flatter than -3/2 the observed slope really was required to be if the Universe was uniformly filled with radio sources. The answer was an initial slope much closer to -1.0 than -1.5; and no radio survey gave a result anything like this.

This is not a book about cosmology; but it is nevertheless a recommended exercise is to calculate the source count using relativistic geometry to examine this issue. The formalism will then allow analyses of space density using any of the luminosity-function techniques described. Distances and volume elements for relativistic cosmologies are readily available, in e.g. the useful compilation by D. Hogg (astro-ph/9905116).

To see what many years of radio surveys did for the observed shape of the source counts, look at the compilation presented by Wall (1994) in Austr. J. Phys. 47, 625. The very great difference in shape between observed counts and those calculated for uniformly-filled relativistic universes attests to the need for *cosmic evolution*, the need to change the shape of the luminosity function with cosmic epoch. Recent studies have shown that the form of cosmic evolution derived from radio-source counts in the 1960s matches closely the form of star-formation evolution discovered with deep optical surveys in the late 1990s.