8.9 Source counts from confusion

Suppose the true flux density from a source is s_0, so the number of sources (per solid angle) we measure near s_0 is $N(s_0)\,ds_0$.
We observe with a beam profile $\Omega(x)$, suitably normalized, where x is the angular distance from the beam centre. This means that if our source is at x from the beam centre, we record a flux density $s(x) = \Omega(x)s_0$. Hence the number of sources we observe near $s(x)$ and near x can be obtained from simply changing variables, and is

$$dN(s,x) = N\left(\frac{s(x)}{\Omega(x)}\right)\frac{ds(x)}{\Omega(x)}\,dx.$$

(By 'dx' here we mean a small solid angle near x.)
To get the final result, we need to integrate over the whole beam:

$$dN(s) = \int N\left(\frac{s(x)}{\Omega(x)}\right)\frac{dx}{\Omega(x)}\,ds$$

which gives the number of sources near the apparent flux density s.
Notice that for a power-law source count, the effect is simply to change the normalization of the source count, but for other forms of the source count the effect of the finite beam is to alter the *shape* of the count. This is not particularly intuitive!