

# Wide-field tracking with zenith-pointing telescopes

Paul Hickson

*The University of British Columbia, Dept. Physics & Astronomy, 6224 Agricultural Road, Vancouver, BC V6T1Z1, Canada*

in original form 2001 February 15

## ABSTRACT

Equipped with a suitable optical relay system, telescopes employing low-cost fixed primary mirrors could point and track while delivering high-quality images to a fixed location. Such an optical tracking system would enable liquid-mirror telescopes to access a large area of sky and employ infrared detectors and adaptive optics. Such telescopes could also form the elements of an array in which light is combined either incoherently or interferometrically. Tracking of an extended field requires correction of all aberrations including distortion, field curvature and tilt. A specific design is developed that allows a 10-metre liquid-mirror telescope to track objects for as long as 30 minutes and to point as far as 4 degrees from the zenith, delivering a distortion-free diffraction-limited image to a stationary detector, spectrograph, or interferometric beam combiner.

**Key words:** instrumentation: adaptive optics – instrumentation: interferometry – telescopes

## 1 INTRODUCTION

The past decade has seen the introduction of large optical telescopes employing primary mirrors that are either fixed or have restricted pointing ability. Examples include fixed zenith-pointing liquid-mirror telescopes (Borra et al. 1989, Hickson et al. 1994, Potter & Mulrooney 1997, Hickson et al. 1998) and the Hobby-Eberly Telescope (HET) which has a tilted primary mirror that can rotate in azimuth only. The simplification provided by limiting the motion of these mirrors provides a very significant reduction in the cost per square meter which ranges from about one to almost two orders of magnitude for the case of liquid mirrors. This comes at a price of limited pointing ability for these telescopes. The HET, which has a segmented spherical primary mirror, employs a movable spherical aberration corrector, and camera or fiber feed, and can access a large area of sky and track objects for up to several hours. Liquid-mirrors, on the other hand, are parabolic and have a strictly vertical axis. All astronomical liquid-mirror telescopes have so far operated only in a fixed zenith-pointing mode, using either high speed imaging (Potter & Mulrooney 1997) or drift scanning detectors (Hickson et al. 1994) to compensate for the sidereal image motion induced by the rotation of the Earth. This has restricted these telescopes to short exposure times, typically less than two minutes duration, and allowed them to access only a small fraction (less than 0.5%) of the sky.

The low cost per unit area of liquid-mirror telescopes makes them attractive for surveys of faint objects. However, the zenith-pointing limitation introduces significant restrictions when these telescopes are used to observe faint ob-

jects: Infrared observations cannot easily be made because infrared arrays have multiplexed readouts and cannot drift-scan like CCDs. Unless the readout rate is very rapid, which introduces noise and data rate problems, the images will be degraded by the sidereal drift. For drift-scanning detectors, the integration time is limited by the size of the detector - it is the time taken for an image to cross the detector, typically a minute or two. A third restriction is the limitation to imaging. Conventional spectroscopic observations are not practical because of the rapid sidereal image motion. The use of adaptive optics is essentially precluded because the star images that are necessary for phase reference are moving. A laser directed at the zenith could provide a fixed reference source, but a natural guide star is still required to provide wavefront tilt information.

A natural way to overcome these limitations would be to employ an optical system which could direct the light received by the primary mirror to a fixed location. This system would necessarily move as required to compensate for the Earth's rotation for some useful period of time. Such an optical tracking system could in principle deliver an optical beam to a stationary detector, spectrograph, adaptive optics system or other instrument. It could also allow the light from many individual telescopes to be brought to a single location and combined either coherently or incoherently, as in the proposed Large-Aperture Mirror Array (LAMA) of 18 ten-meter telescopes (see [www.astro.ubc.ca/lmt/lama](http://www.astro.ubc.ca/lmt/lama)). Clearly, an optical tracking system will be an essential component of any multi-aperture optical/infrared interferometer that employs fixed primary mirrors.

The need for tracking is closely related to the desir-

ability of accessing fields away from the zenith, thereby increasing the area of sky available to liquid mirror telescopes. In this case, the primary consideration is the correction of field aberrations, such as coma and astigmatism, that occur when a paraboloid is used to focus oblique rays. The problem was first considered by Richardson & Morbey (1988) who found an optical design that allows pointing up to 7.5 degrees from the zenith, while maintaining good image quality over a small, but useful, instantaneous field of view. Borra (1993) concluded that such a correction might be feasible at zenith angles as large as 45° Wang et al. (1994), Moretto et al. (1995) and Borra et al. (1995) subsequently presented designs that give good image quality at large zenith angles using actively warped mirrors.

While these studies have shown that it is possible to correct the off-axis aberrations of a parabolic primary mirror observing at high zenith angles, there is an additional issue that has not yet been addressed. In order to image and track an extended field of view and track, a telescope must not only provide sharp images of celestial objects – it must also accurately preserve the geometry of the field. In other words, the positions of objects in the field must not change during the exposure. This leads to three additional requirements: First, the image scale must not vary significantly over the full range of zenith angle. If this were not the case, images would move radially towards or away from the field center during the exposure causing a smearing of the images. Second, distortion must be carefully controlled or images will again be smeared during the exposure. If large zenith angles are involved, atmospheric dispersion and refraction must be continuously compensated. Third, any curvature and tilt of the focal plane must be eliminated or at least held constant. The optical designs published to date do not satisfy these conditions and are therefore not suitable for imaging an *extended* field of view for long exposure times.

An additional interesting development is the prospect of combining light from several primary mirrors at a single focus. If this is done incoherently, the image brightness increases in proportion to the total area of the primary mirrors. By combining the light coherently (interferometrically), it is possible to achieve much higher resolution and sensitivity. Several prototype optical interferometers are operational and have demonstrated phase locking and interference at optical wavelengths (Young et al. 1998). Interferometric systems are being built for the Keck and VLT telescopes and other projects are planned (von der Luehe et al. 1997, Angel et al. 1998, Booth et al. 1999). A prerequisite for interferometry using fixed primary mirrors is that the light intercepted by each mirror be collimated and delivered to a beam combiner, an optical system that brings all the light to a common focus in a specific geometrical configuration. Hence, it is of interest to consider the feasibility of tracking systems that can deliver a collimated beam to a fixed location.

Interferometry imposes additional requirements on any tracking system. As is well known, the optical path lengths from the various apertures to the combined focus must be kept equal to within a tolerance of order  $c/\delta\nu$ , where  $\delta\nu$  is the optical bandwidth. Furthermore, for interferometric imaging, the Abbe's condition must be preserved (the sine of the angle of incidence of all rays arriving at the focus

must be strictly proportional to the radial position of these rays in the entrance pupil), and the lateral and longitudinal geometry of the exit and entrance pupils must match in accordance with the appropriate pupil magnification factors. Once a suitable design is found for the tracking system, these additional factors can be accommodated by careful design of the path-length equalization and beam-combining systems.

In this paper, we first analyze the restrictions that optical tracking imposes on the optical design. We show that this leads to a natural solution based on spherical symmetry. A practical design is then developed that allows a 10-meter liquid-mirror telescope to point and track over an 8 degree diameter area of sky. Such a design could be suitable for the LAMA optical array.

## 2 DESIGN CONSIDERATIONS

It is convenient to divide the optical system into two parts: a stationary axisymmetric system (the telescope), and a second optical system (the relay system) which relays light from the stationary system to the final focus, and has moving elements to provide tracking. This division is always possible because the primary mirror alone forms a minimal stationary axisymmetric system. The advantage of this approach is that the two systems can be analyzed and designed individually using conventional techniques. They are then coupled to form the complete optical design. As we shall see, the requirements of tracking place constraints on the ways that the two systems can be coupled.

The telescope receives light from some angular region of the sky centred on the telescope axis and produces an image of this region on its focal surface. This region will be called the *accessible field of view*. The tracking system intercepts light from a portion of the telescope focal surface and transfers it to a final fixed focus, where an image is formed. The portion of the focal surface imaged by the relay system at any given time, and the corresponding angular region of the sky, will be called the *instantaneous field of view*. The telescope focal surface may be real or virtual. In the latter case, light is intercepted by the relay system before reaching the telescope focus.

It is not necessary that images formed at the focal surface be free from aberrations that affect image quality (spherical aberration, coma and astigmatism), as these could in principle be corrected by the relay system. However, it simplifies the design of the relay system if field-dependent aberrations such as coma and astigmatism are corrected by the telescope. As we shall see, tracking imposes additional requirements on the focal surface curvature, tilt and distortion.

The relevant geometry is illustrated in Fig. 1. Here  $S$  denotes the focal surface of the telescope. This surface is assumed to be spherical with radius  $R_F$  (departures of the focal surface from a sphere can be treated as higher order aberrations).  $C$  denotes the centre of curvature of the focal surface and  $V$  marks the intersection of this surface with the symmetry axis of the telescope.  $P$  indicates the location of the telescope exit pupil. The centre of the instantaneous field of view is denoted by  $I$ . The line connecting this point with  $P$  corresponds to the path of the principal ray (the ray

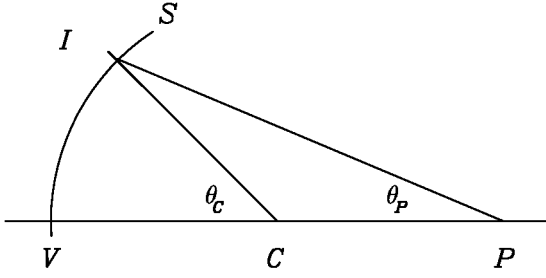


Figure 1. Geometry of the focal surface.

that passes through the centre of the pupil). Let  $\theta_P$  denote the angle between the principal ray and the axis.

## 2.1 Focal plane tilt

Consider now the relay system which will deliver the light from a region centred at  $I$  to a fixed focal plane. Let  $F$  denote the image of  $I$  in this plane.  $F$  is thus the centre of the instantaneous field of view in the final focal plane. The actual relay system may contain flat mirrors which serve only to change the direction of the light beam. Such mirrors have no effect on the properties of the final image, so for the purpose of optical design and analysis one can ignore these mirrors and consider only the elements with optical power. Thus the beam can be considered to pass from  $I$  to  $F$  without any redirection by flat mirrors. The straight line connecting points  $I$  and  $F$  will be called the axis of the relay system. For simplicity, we shall assume that the final focal plane is perpendicular to this axis. Generalization to a focal plane of arbitrary orientation is possible but will not be considered here.

Let us now suppose that the optical elements of the relay system are rotationally symmetric about this axis. This assumption restricts the types of relay systems that we can consider, but allows us to make a general analysis of the optical design problem. It will now be apparent that the axis of the tracking system must be normal to the focal surface at the point  $I$ . Any departure from this condition will result in a tilt of the final focal plane. Such departures are amplified by the longitudinal magnification factor  $m_L = m_T^2$ , where  $m_T$  is the transverse magnification of the relay system. Thus, *the axis of the relay system must pass through the centre of curvature of the telescope focal surface.*

## 2.2 Field curvature and distortion

Curvature of the focal surface, considered alone, is not a problem as it is constant over the accessible field of view and can therefore easily be removed by the relay system (the necessary and sufficient condition for a flat field is that the Petzval sum of all optical elements in the telescope plus relay system vanish). However, tracking imposes strict limitations on distortion, which in turn restricts field curvature, as will now be discussed.

Distortion is a non-linear variation in image scale on the telescope focal surface. In tracking systems, even a small amount of distortion will lead to significant differential image motion within the instantaneous field of view. This in

turn will result in a radial elongation of images during an exposure. To avoid this, the mapping of the sky onto the focal surface must be strictly linear. To quantify this, let  $\theta_O$  be the field angle of an object in the sky, measured with respect to the telescope axis, and  $\phi_O$  be the azimuth angle of this object with respect to this axis, measured from an arbitrary fiducial direction. Let  $\theta_C$  be the angle between the image of this object and the axis, at the centre of curvature of the focal surface (see Fig. 1), and  $\phi_C$  be the azimuth angle of the image. By symmetry,  $\phi_C = \phi_O$ . Linearity requires that

$$\theta_C = m\theta_O \quad (1)$$

where  $m$ , the angular magnification, is a constant. Now, consider the tangential magnification at this point. The angular displacement on the sky resulting from a small change  $d\phi_O$  is  $\sin\theta_O d\phi_O$ . The corresponding angular displacement of the image on the focal surface is  $\sin\theta_C d\phi_C$ . These must be related by the same magnification factor  $m$ , therefore

$$\sin\theta_C = m \sin\theta_O \quad (2)$$

Clearly, equations (1) and (2) can only both be satisfied if  $m = 1$ , which gives

$$\theta_C = \theta_O \quad (3)$$

Thus, tracking imposes the requirement that *the angle subtended at the centre of curvature by any point on the focal surface must be equal to the field angle of the corresponding object.*

This leads to a condition for the radius of curvature of the focal surface, as follows. From elementary optics, the image scale on the focal surface  $ds/d\theta_O$  is equal to  $f$ , the effective focal length of the telescope. Now, we also have the geometric identity  $ds/d\theta_C = R_F$ . Equation (2) then requires that

$$R_F = f. \quad (4)$$

Thus to eliminate distortion, *the radius of curvature of the focal surface must equal the effective focal length of the telescope.*

These conditions are satisfied at the focus of a concave primary mirror, regardless of its aspheric shape or conic constant. By symmetry, a spherical primary mirror with radius of curvature  $R_1$  has a focal surface that is concentric with the centre of curvature of the primary mirror and has radius  $R_F = R_1/2$ , equal to the primary mirror focal length. A point on this surface subtends an angle at the centre of curvature exactly equal to the field angle in the sky of paraxial rays that focus to this point. The same is true for a non-spherical primary mirror - the aspheric shape introduces aberrations but does not change the Gaussian image parameters, which are determined only by the curvature. Therefore, the distortion conditions (Eqns. 2 and 3) are exactly satisfied. To satisfy the focal-plane tilt condition (Section 2.2), a necessary and sufficient condition is that the axis of the tracking system be constrained to always pass through the center of curvature of the primary mirror. For a spherical primary mirror, the tracking system need only correct the spherical aberration of the primary mirror, which is independent of field angle, in order to achieve good image quality. For a parabolic primary mirror, the aberrations depend on field angle and are therefore more difficult to correct in the

tracking optics. However, even in this case it is possible to obtain excellent image quality over a wide accessible field of view, as will be shown in Sections 3 and 4.

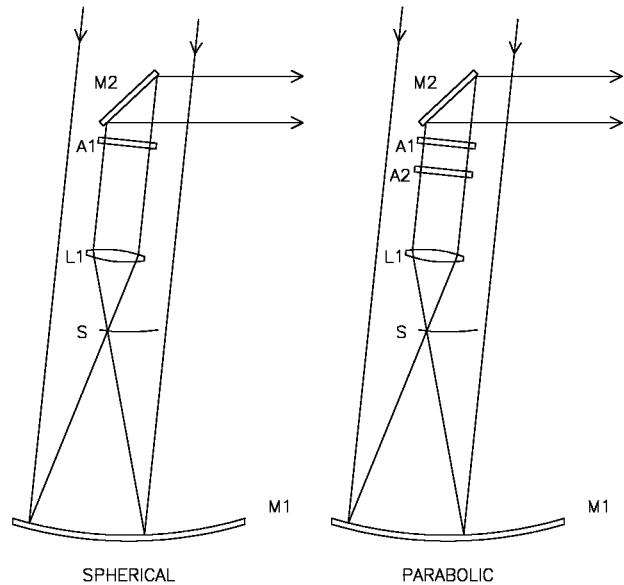
### 3 CONCEPTUAL DESIGN OF TRACKING OPTICS FOR SPHERICAL AND PARABOLIC PRIMARY MIRRORS

The conceptual design of the tracking system is simplest and most easily illustrated for the case of a spherical primary mirror. Therefore, we begin by discussing this case. We then extend the analysis to the case of a parabolic primary mirror, which is of particular importance to liquid-mirror telescopes, which necessarily have a parabolic form.

#### 3.1 Spherical primary mirror

Let us suppose that the primary mirror is spherical. The tracking optics must collimate the beam received from the primary mirror, and transmit it to a fixed location where it is brought to a focus. For simplicity, we shall assume that the tracking optics deliver a fixed, aberration-free, parallel beam, which is subsequently refocussed by additional optical elements. Since focussing of a fixed parallel beam is a simple task, accomplished by a small telescope or “beam combiner”, we do not consider the refocussing system further, but direct our attention to the tracking system itself. Since this system images points on the focal surface to parallel rays, it is simply a telescope operating in reverse. Tracking is accomplished by moving this “tracking telescope”, to follow the motion of images on the focal surface, keeping the axis aligned with the center of curvature. The angle that the axis of the tracking telescope makes with the vertical is thus equal to the field angle (the zenith angle of the centre of the instantaneous field of view of the telescope). In order to keep the direction of the beam constant, a flat mirror can be placed at the centre of curvature and rotated by half the field angle.

The system is illustrated schematically in the left panel of Fig. 2, in which the tracking telescope is represented by a simple lens. In principle, any wide-field telescope design that has a focal plane curvature matched to that of the focal surface (ie  $R_1/2$ ) can be used for the tracking telescope. However, we must also correct the considerable spherical aberration of the primary mirror. This might be incorporated into the optical design of the tracking telescope, or it could be done by adding an aspheric corrector element, centered on the axis of the tracking telescope, as shown in the figure. Where should such a corrector be located? In the Schmidt design, an aspheric corrector is placed at the centre of curvature of the primary mirror, in order that its location does not introduce a preferred direction. In this way, the correction that it introduces is nearly independent of field angle. In practice, the corrector itself defines a preferred direction, which ultimately limits the field of view of the Schmidt system. With this in mind, one might think that the best location for the corrector shown in Fig. 2 would be at a position conjugate to the centre of curvature (ie. at a real image of the centre of curvature). However, in our system, the corrector moves with the tracking telescope, so there is no preferred



**Figure 2.** Conceptual tracking systems. For the spherical system shown on the left, light reflected from the spherical mirror M1 reaches a focus on the spherical surface S, then is refocussed by lens L1 to produce a parallel beam that is directed by the flat mirror M2 to a fixed location. The only aberration in this system is spherical, which is removed by the aspheric lens A1. For a parabolic primary, another aspheric lens A2 is placed at the image of M1 formed by L1. It introduces a change in optical path length equivalent to the difference between the spherical and parabolic mirrors, making primary mirror appear spherical.

direction on the sky no matter where we place the corrector along the tracking telescope axis. Therefore, the only real consideration in its location is the optical performance of the tracking telescope itself. Unlike the Schmidt design, there is no limitation on the available field of view.

The above discussion has introduced the concept of tracking optics for a fixed spherical primary mirror. Rather than present detailed optical designs, we now proceed to the case of a parabolic primary, which is particularly relevant for large liquid-mirror telescopes.

#### 3.2 Parabolic primary mirror

The discussion of the preceding section applies also to this case, but with an additional complication. A parabolic primary mirror does not introduce spherical aberration, but instead we have to deal with a variety of field-angle-dependent aberrations such as coma, astigmatism and higher-order effects. The simplest way to deal with these is to *optically convert the parabolic primary mirror to a spherical shape*. Once this is done, tracking can be accomplished exactly as for the spherical case. We shall find, however, that the process of converting the parabola to a sphere introduces a preferred direction, which ultimately limits the accessible field of view, as in the case of the Schmidt telescope.

A parabolic primary mirror can be optically converted to a sphere by placing immediately in front of it an aspheric lens that introduces an optical path length difference (OPD) equal to the OPD between the parabola and a sphere of

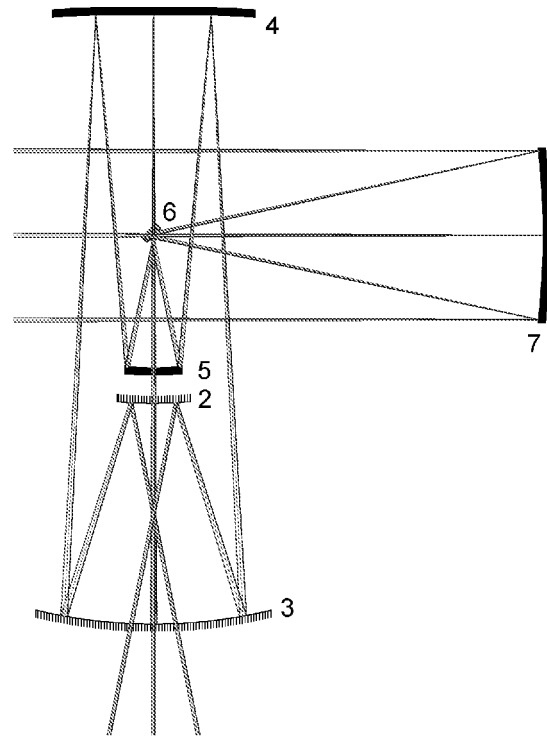
the same radius of curvature. For large mirrors such a lens would be impractical, but the same effect can be obtained by locating a similar lens at an image of the primary mirror formed by the tracking telescope. In other words, *we place an aspheric corrector at a location conjugate to the primary mirror*. To within an arbitrary additive constant, the optical thickness of this lens is equal to half the difference in surface height between the parabola and a sphere, at the corresponding point on the primary mirror. This is illustrated schematically in the right panel of Fig 2.

It is necessary that this aspheric corrector be exactly conjugate to the primary mirror, both longitudinally and laterally. As the tracking telescope moves, the position of the primary mirror with respect to the axis of the moving telescope changes. It is therefore necessary to move the aspheric corrector, with respect to the axis of the tracking telescope, to maintain its position and orientation conjugate to the primary mirror. In summary, for the case of a parabolic primary mirror, we must add an additional aspheric corrector that is located conjugate to the primary mirror and moves with respect to the axis of the tracking system.

#### 4 A PRACTICAL ACHROMATIC DESIGN

In order to see how well this concept works in practice, several trial optical designs were investigated using CODE-V optical ray-tracing software. The simple refractive aspheric plates described above produce good results only at the design wavelength. Chromatic aberration from these strong aspheric surfaces causes severe image degradation at other wavelengths. It is unlikely that even a colour-compensated system, employing two or more glasses, would give sufficient chromatic correction. Therefore, attention was directed to all-reflecting designs. The most suitable design that was found is described in this section. The investigation was not exhaustive, and little attempt was made at optimization, so this design should be considered as illustrative only. Even so, the results are very encouraging. This design is capable of providing sub-arcsecond image quality at zenith angles of up to four degrees. With adaptive optics, it can produce diffraction-limited image quality over an instantaneous field of view limited only by atmospheric anisoplanatism. Unlike previous optical designs, it is explicitly free from variations in distortion, focal-plane tilt and image scale over the entire range of zenith angles. Such a system would allow exposure times as long as 40 minutes.

The design of the tracking optics is illustrated in Fig. 3. Details of the optical system are given in Table 1. Light from the primary mirror focal surface enters a two-mirror telescope, formed by  $M2$  and  $M3$ , that resembles a Cassegrain system. An aspheric mirror,  $M4$ , is placed at the the image of the primary mirror produced by the  $M2 - M3$  system. A pair of relay mirrors,  $M5$  and  $M6$  directs the light to a second aspheric mirror  $M7$ , which produces the final collimated beam. The mirror  $M4$ , which converts the parabolic primary mirror to a sphere, moves laterally as the telescope tracks in order to remain conjugate to the primary mirror. The mirror  $M6$  is flat and rotates about its axis as the system tracks in order to keep the reflected beam horizontal. The aspheric mirror  $M7$ , which removes the spherical aberration, produces a horizontal collimated beam.



**Figure 3.** Parabolic tracking system at zero zenith angle. Light reflected from the telescope primary mirror (not shown) enters the system from below. A collimated beam is produced and directed to a fixed location where it can be brought to a focus. Numbers labeling the optical surfaces correspond to those in Table 1

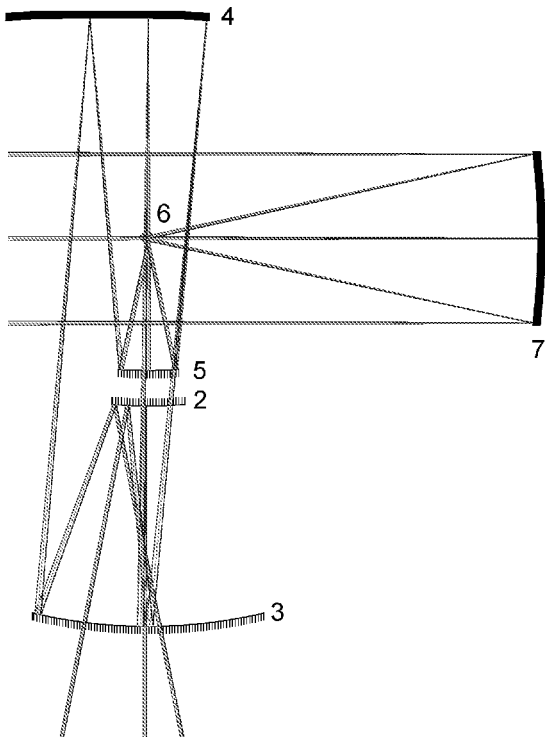
During tracking,  $M2$ ,  $M3$  and  $M5$  rotate about the centre of curvature of the primary mirror, maintaining an optical axis that passes through the centre of curvature, as shown in Fig 4.  $M4$  moves with this group, but also moves transversely with respect to the axis, in order to maintain its position conjugate to the primary mirror. The flat mirror  $M6$  rotates by half the angle of the  $M2$ - $M3$ - $M5$  axis so that the reflected beam is horizontal. The  $M6$ - $M7$  axis remains horizontal, as does the collimated beam produced by  $M7$ . The stop in this design is chosen to be at  $M7$ , which forms the exit pupil of the beam compressor. The diameter of the output beam is 0.39 m, which corresponds to a beam compression factor, and angular magnification, of 25.6.

The performance of this system is shown by Table 2 which lists RMS spot diameters, for three positions in the instantaneous field, as a function of zenith angle. The spot diameters range from 0.3 to 1.0 arcsec, over a 1 arcmin diameter field of view up to a zenith angle of four degrees. Such a system would allow a low-latitude liquid-mirror telescope to access up to 7% of the sky.

While the instantaneous field of view of of this system is quite small by the standards of wide-field astronomy, it is quite well-suited for very-large ground-based telescopes. Such telescopes will achieve maximum sensitivity only by the use of adaptive optics (AO) to compensate for atmospheric seeing. AO systems employ a deformable mirror, located at or near a pupil in the telescope beam, to correct wavefront phase errors introduced by atmospheric turbu-

**Table 1. Parabolic tracker optical specification**

Surface	radius (mm)	axial sep. (mm)	diameter (mm)	conic const.	remarks
1	-5000.00	25250.000000	10000	-1.000000	primary mirror
2	-960.00	500.000000	160	-15.390077	
3	1140.00	1380.132573	520	-0.402904	
4	$\infty$	800.000000	262	0.000000	aspheric
	A = 0.20217832E-08, B = -.16411329E-14, C = 0.42870284E-19, D = -.34827625E-24 decenters: 1°: 23.221 mm, 2°: 46.509 mm, 3°: 69.836 mm, 4 deg: 92.754 mm				
5	-979.496000	300.000000	134	2.096362	aspheric
	A = 0.64528091E-09, B = 0.28359035E-12, C = -.12423697E-15, D = 0.19904456E-19				
6	$\infty$	883.208421	48	0.000000	45 deg lat
7	-1604.997119	$\infty$	390	0.000000	aspheric
	A = 0.49209044E-09, B = 0.10372729E-14, C = -.43318379E-19, D = 0.53379419E-24				



**Figure 4.** Parabolic tracking system at 4-degree zenith angle. The entire unit rotates about the centre of curvature of the telescope primary mirror in order to follow the images (this rotation is not shown in the figure). Aberrations are controlled by translating the aspheric mirror M4 (shown at the top of the figure), keeping it conjugate to the parabolic primary mirror. Numbers labeling the optical surfaces correspond to those in Table 1

lence, thereby restoring the quality of the image. These mirrors can change their shape on millisecond timescales and can have several hundred independently-controlled degrees of freedom. The required drive signals are determined by measuring the residual phase error, after correction by the deformable mirror, in the light from a reference star (either a natural star or a laser-illuminated spot in the Earth's atmosphere) by means of a wavefront sensor. The wavefront phase correction applied by the AO system also restores the quality of the images in an area surrounding the reference

star within a characteristic (isoplanatic) angle. This angle depends on the strength and altitude of the turbulence and on the wavelength of light. It is typically less than an arc-minute in the near-infrared and is smaller still at visible wavelengths.

It is therefore of interest to investigate the potential of the tracking system when adaptive optics is employed. AO systems can in principle correct aberrations introduced by the telescope optics, but only at a given point in the field. To give good performance with an AO system, the tracking optics design must therefore minimize *variations* in aberrations across the field. To simulate an adaptive optics system, an additional reflecting surface was added to the system described by Table 1. This surface was taken to be a general x-y polynomial of the tenth degree, which simulates the effect of a deformable mirror. The polynomial coefficients were optimized, at each zenith angle, in order to minimize the spot diameters, in the same manner that an AO system optimizes the shape of a deformable mirror in order to minimize the diameter of the image of a reference star. The conic and aspheric constants of the tracking system elements were reoptimized at a zenith angle of 2 degrees, then held constant. The specifications of this AO-optimized system are given in Table 3, and the performance is summarized in Table 4. In addition to the RMS spot diameters, Table 4 lists the RMS wavefront error, and the Strehl ratio that would be achieved with a perfect AO system and no atmosphere. The Strehl ratio (the ratio of the central intensity of the PSF to that of an unaberrated PSF) is typically about 90%, throughout the field, at zenith angles up to 2°, which indicates that the system is diffraction limited. At zenith angles greater than 2°, there is a noticeable drop in Strehl ratio towards the edge of the field. However, even at a zenith angle of 4°, the fall in Strehl ratio is comparable to that expected due to atmospheric anisoplanatism. This shows that over almost all of its range, the instantaneous field of view in which near-diffraction-limited image quality can be achieved will be limited by atmospheric anisoplanatism, rather than optical aberrations.

## 5 CONCLUSIONS

We have shown in this paper that a telescope with a fixed primary mirror can track celestial objects for an extended period of time. To accomplish this requires more than just a

**Table 3. AO-optimized tracker optical specification**

Surface	radius (mm)	axial separation (mm)	diameter (mm)	conic constant	remarks
1	-5000.00	25250.000000	10000	-1.000000	primary mirror
2	-960.00	500.000000	160	-1.253120	
3	1140.00	1380.132573	520	-0.004374	
4	$\infty$	800.000000	262	0.000000	aspheric
	A = 0.19630681E-08, B = 0.35801621E-15, C = 0.50652415E-19, D = -.14431399E-24 decenters: 1°: 24.200 mm, 2°: 48.052 mm, 3°: 71.166 mm, 4°: 93.303 mm				
5	-979.496000	300.000000	134	4.310945	aspheric
	A = 0.65349064E-09, B = 0.36582659E-14, C = 0.19443600E-17, D = -.16005167E-21				
6	$\infty$	883.208421	48	0.000000	45 deg tilt
7	-1604.997119	258.000000	390	0.000000	aspheric
	A = 0.46139661E-09, B = 0.82228387E-15, C = -.13956758E-19, D = 0.19158183E-24				
8	$\infty$	$\infty$	385	0.000000	XY-polynomial

**Table 2. Performance of the parabolic tracker**

Zenith Angle (°)	0	1	2	3	4
RMS spot dia. (arcsec)					
-0.5 arcmin	0.62	0.73	0.79	1.03	1.03
0.0 arcmin	0.30	0.31	0.36	0.68	0.69
+0.5 arcmin	0.62	0.42	0.36	0.79	0.96

**Table 4. Performance of the AO-optimized tracker**

Zenith Angle (deg)	0	1	2	3	4
RMS spot dia. (mas)					
-0.5 arcmin	16.3	13.3	10.8	23.9	43.4
0.0 arcmin	13.2	13.9	16.9	22.9	30.8
+0.5 arcmin	16.3	15.8	17.2	28.4	43.5
RMS OPD (nm)					
-0.5 arcmin	110	100	123	228	335
0.0 arcmin	73	79	104	149	147
+0.5 arcmin	110	99	75	198	334
Strehl Ratio ( $\lambda = 2 \mu\text{m}$ )					
-0.5 arcmin	0.89	0.91	0.86	0.60	0.33
0.0 arcmin	0.95	0.94	0.90	0.80	0.80
+0.5 arcmin	0.89	0.91	0.95	0.68	0.33

corrector system that produces sharp images. As well, there must be no variations in the relative positions of these images in the field, throughout the tracking range. This requirement places additional constraints on the optical system that have been outlined in Section 2. We have established these conditions can be satisfied by a system of moving optics (the tracking system), providing that tracking is accomplished by rotating this system about the center of curvature of the fixed primary mirror. The tracking system is simplest in the case of a spherical primary mirror, where pointing and tracking over very large zenith angles is possible, although in practice this would be limited by the effects of atmospheric refraction and dispersion. A parabolic primary mirror can also be accommodated, by means of a conjugate aspheric corrector, although the range of zenith angle is more restricted in this case. The designs presented here are illustrative of the potential of the concept. More analysis is required to explore the ultimate limits of such systems. In particular, by specifically employing a wide-field

telescope design in the tracking system, it may be possible to significantly increase the instantaneous field of view.

Large ground-based optical telescopes can achieve maximum sensitivity for point-like sources only if they operate near the diffraction limit (the well-known  $D^4$  advantage). Adaptive optics is required for this and is rapidly becoming an essential part of modern observatories. Current AO systems provide wavefront compensation over as many as several hundred independent modes. They achieve diffraction-limited images in the K-band, and give significant improvements in image quality at shorter wavelengths (eg. Roddier, Northcott & Graves, 1991, Rigaut et al 1998, Wizinowich et al 2000). Future systems will have as many as  $10^3$  modal degrees of freedom and will provide near-complete correction into the visible spectral region. At present, atmospheric isoplanatism restricts the useful field of view of adaptive-optics systems to less than one arcmin. Future multi-conjugate AO systems should increase this limit, but not likely by an order of magnitude. For this reason, we have focussed on optical tracking designs that are matched to this atmospheric limit. With adaptive optics, the design presented here would allow a very large liquid-mirror telescope to achieve diffraction-limited resolution and sensitivity, access as much as 7% of the sky, and track fields for up to 30 min. By providing a collimated beam to a fixed location, it would also allow light from many primary mirrors to be combined, either incoherently or coherently, or to be directed to large fixed instruments.

Because the AO correction is provided by a separate deformable mirror, it makes no difference whether the telescope primary mirror is made of glass or liquid mercury. An AO system will work as well with a liquid-mirror telescope as with a conventional telescope, provided that the telescope can track a reference star. The costs of current AO systems are comparable to those of other major instruments such as infrared array detectors, and are probably comparable to the cost of the tracking optics described in this paper. While this cost is significant, it is not a dominant factor in the overall budget of a large (10 metre or more) liquid-mirror telescope.

## ACKNOWLEDGMENTS

This work was supported by grants from the Natural Sciences and Engineering Research Council of Canada.

## REFERENCES

- Angel J. R. P., Hill J. M., Strittmatter P. A., Salinari P., Weigelt G., 1998, in Reasenberg R. D., ed., Proc. SPIE Vol. 3350, *Astronomical Interferometry*, p 881
- Booth A. J. et al., 1999, in Unwin S., Stachnik R., eds, ASP Conf. Ser. Vol. 194, *Working on the Fringe: Optical and IR Interferometry from Ground and Space*, p. 256
- Borra E. F., 1993, *A&A*, 278, 665
- Borra E. F., Content R., Drinkwater M. J., Szapiel S., 1989, *ApJL*, 346, L41
- Borra E. F., Moretto G., Wang M., 1995, *A&AS*, 109, 563
- Hickson P., Borra E. F., Cabanac R., Chapman S. C., de Lapparent V., Mulrooney M., Walker G. A. H., 1998, in Stepp L. M., ed, Proc. SPIE Vol 3352, *Advanced Technology Optical/IR Telescopes VI*, p. 226
- Hickson P., Borra E. F., Cabanac R., Content R., Gibson B. K., Walker G. A. H., 1994, *ApJL*, 436, 201
- von der Luehe O., Derie F., Koehler B., Leveque, S. A., Paresce F., Verola M., 1997, in Ardelberg A. L., ed, Proc. SPIE Vol. 2871, *Optical Telescopes of Today and Tomorrow*, p. 498
- Moretto G., Lemaitre G. R., Bactivelane T., Wang M., Ferrari M., Mazzanti S., di Biagio B., Borra E. F., 1995, *A&AS*, 114, 379
- Richardson E. H., Morbey C. L., 1988, in Robinson L. B., ed, *Instrumentation for Ground-Based Optical Astronomy, Present and Future. The Ninth Santa Cruz Summer Workshop in Astronomy and Astrophysics*, Springer-Verlag, New York, p. 720
- Potter A. E.; Mulrooney M., 1997, *Adv. Space Res.*, 19, 213
- Rigaut F., Salmon D., Arsenault R., Thomas J., Lai O., Rouan D., Véan J. P., Gigan P., Crampton D., Fletcher J. M., Stilburn J., Boyer C., Jagourel, P., 1998, *PASP*, 110, 152
- Roddiier F., Northcott M. J., Graves J. E., 1991, *PASP*, 103, 131
- Wang M., Moretto G., Borra E. F., Lemaitre G., 1994, *A&A*, 285, 344
- Wizinowich P., Acton D. S., Shelton C., Stomski P., Gathright J., Ho K., Lupton W., Tsubota K., Lai O., Max C., Brase J., An J., Avicola K., Oliver S., Gavel D., Macintosh B., Ghez A., Larkin J., 2000, *PASP*, 112, 315
- Young J. S. et al., 1998, in Reasenberg R. D., ed, Proc. SPIE Vol. 3350, *Astronomical Interferometry*, p. 746