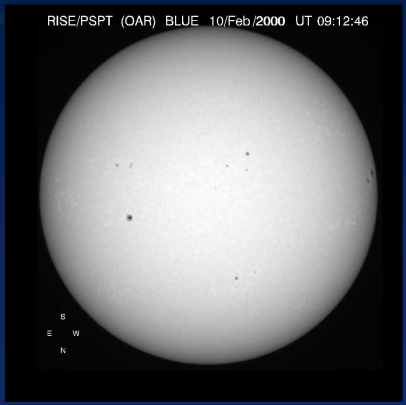
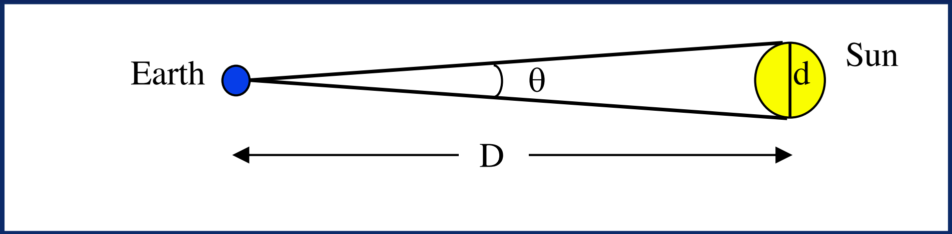
Stellar Radii – Section 13.4, P 318-321

The Sun - resolved as a disk of diameter 0.53°



Calculation of the Sun's radius



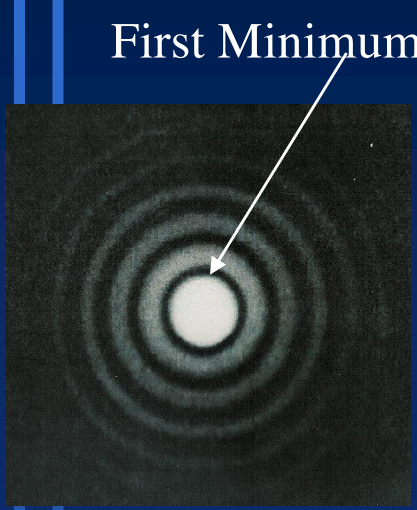
θ(rad) = d/D = d/1AU = 0.53(2π/360°) = 0.0093, hence d = 0.0093AU = 0.0093 x 1.496 x 10^8 km = 1.392 x 10^6 km or R_{sun} = 6.96 x 10^5 km

Can we apply same principles to the stars?

e.g. α Cen (like Sun) D = 1.3 pc = 2.7 x 10^5 AU; if $R_{cen} = R_{sun}$
$\theta = 2R_{sun}/D$ = 0.0093 AU/2.7 x 10^5 AU = 3.3 x 10^{-8} rad = 0.007 arcsec
(angular diameter of a dime 150 km away!)

Can we resolve this small angle with a telescope?

Stellar radii



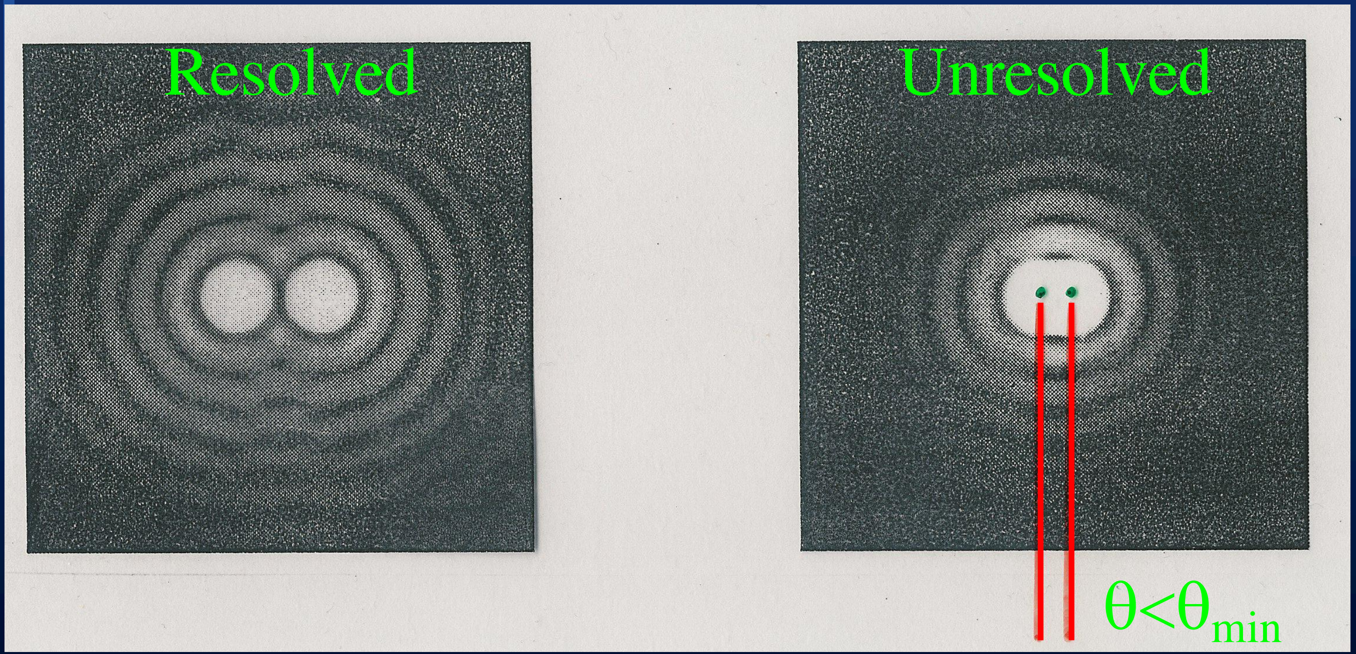
Airy disk - diffraction pattern produced by a circular aperture (telescope mirror).

First minimum usually taken as resolution limit of telescope

Angular distance from centre to first ring $\theta = 1.22\ \lambda/D$

λ is wavelength of observation, D diameter of telescope, θ in radians

Usually say 2 objects resolved if angular separation > λ/D



Stellar Radii

Example: resolution of human eye

Pupil aperture = 1 cm;

visible light λ = 500 nm = 5 x 10^{-5} cm (same units)

$\theta_{diffraction} = \lambda/d = 5 \times 10^{-5}$ cm / 1 cm

$= 5 \times 10^{-5}$ rad = 0.003° = 10 arcsec

If aperture > 10 cm, then $\theta_{diffraction}$ = 1 arcsec

This is about the limit imposed by the atmosphere

To resolve α Cen:

θ = 0.007 arcsec = 3.3 x 10^{-8} rad

$\theta > \theta_{min} = 1.22\lambda/D \rightarrow D > 1.22\lambda/\theta$

In visible light, λ = 500 nm = 5 x 10^{-7} m

Therefore need D > 1.22 (5 x 10^{-7} m)/(3.3 x 10^{-8}) = 18.6 m

Compare with world's largest telescope, Keck = 10 m

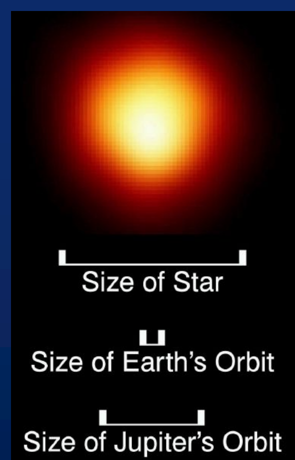
Ways to Measure Stellar Radii

Method 1: Using Blackbody Laws:

$L = 4\pi R^2 \sigma T^4$

L is known if distance is known, T is obtained from spectrum or colour, solve for radius R

Two examples →



There is a tremendous range of stellar radii spanning 7-8 orders of magnitude.

Example - Betelgeuse:

$L = 4\pi R^2 \sigma T^4$ →

$L/L_{sun} = (R/R_{sun})^2 \, (T/T_{sun})^4$

Sun: $M_V = +4.8$, Betel: $M_V = -5.5$

Thus $L_{Betel}/L_{Sun} = 13{,}200$

Sun: T = 5800K, Betel: T = 3500K

Thus $R_{Betel}/R_{Sun} = (L_{Betel}/L_{Sun})^{1/2}$ x $(T_{Betel}/T_{sun})^{-2} = 320$ (better is 930 - why?)

Sirius B $L/L_{Sun} = 0.03$, T = 27,000K

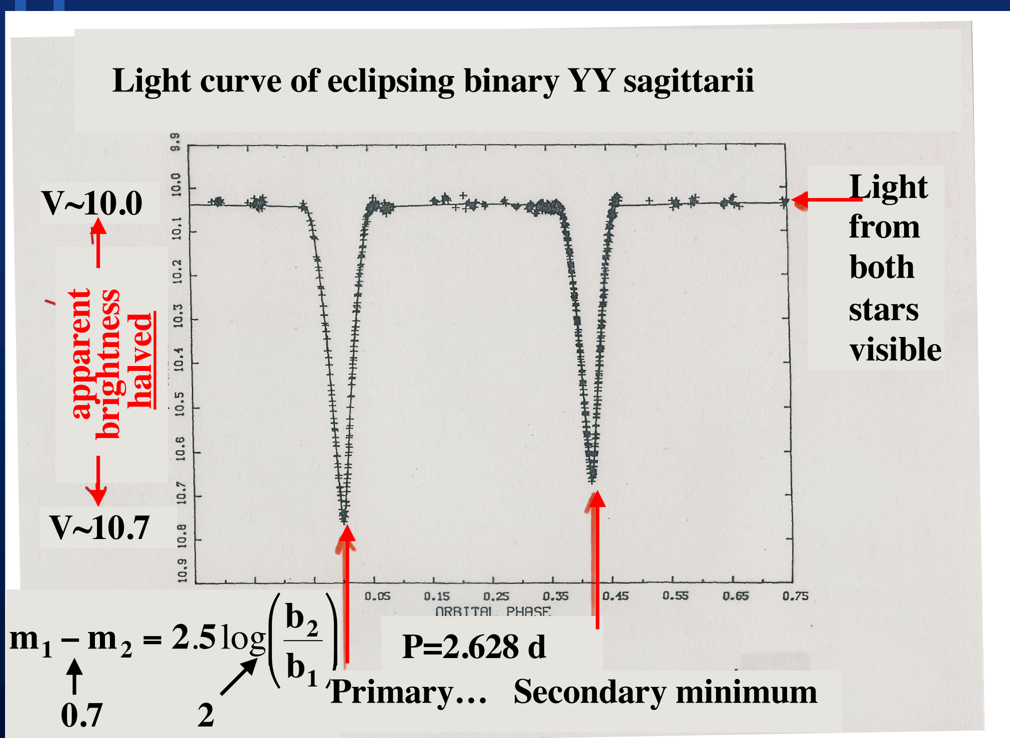
Derive $R_{Sirius\ B} = 0.008\ R_{sun}$

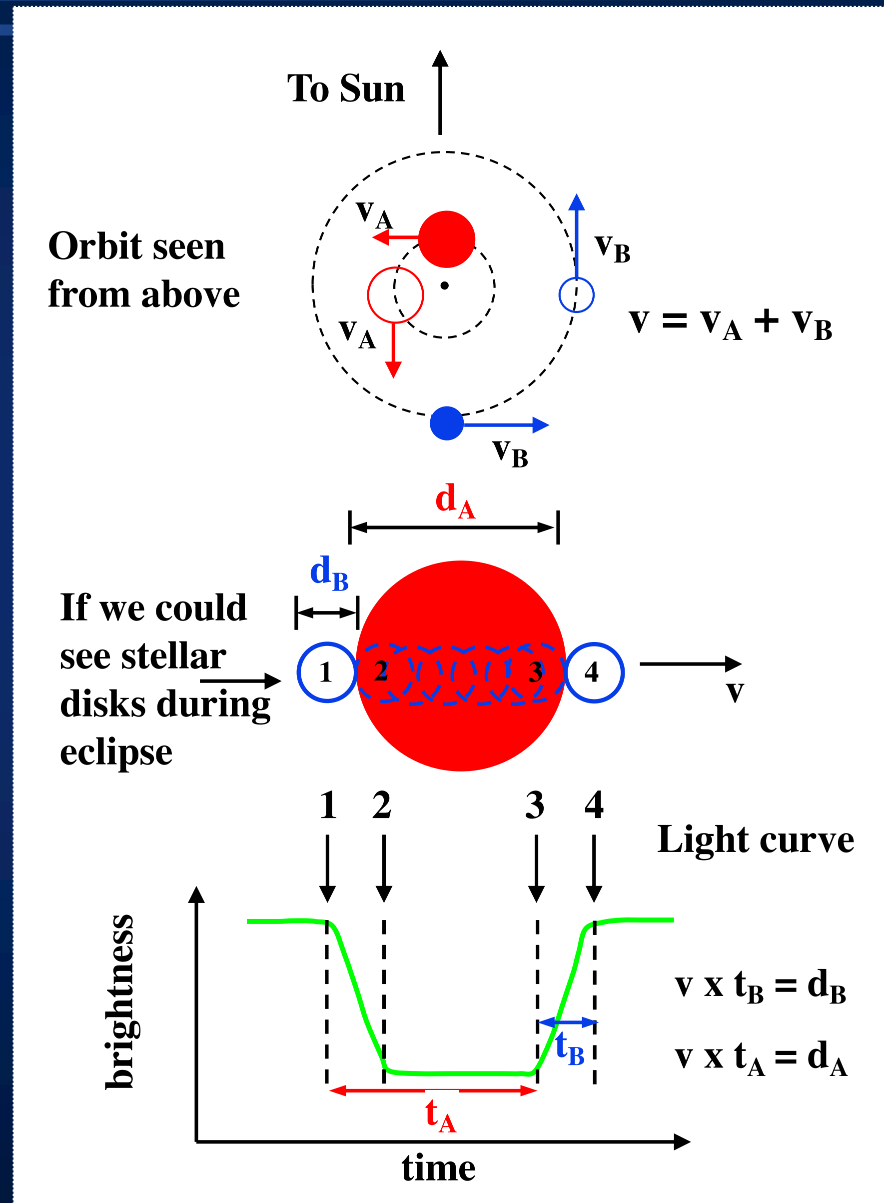
Which is about size of the Earth

Ways to Measure Stellar Radii

Method 2: Using Eclipsing Binaries: Explanation

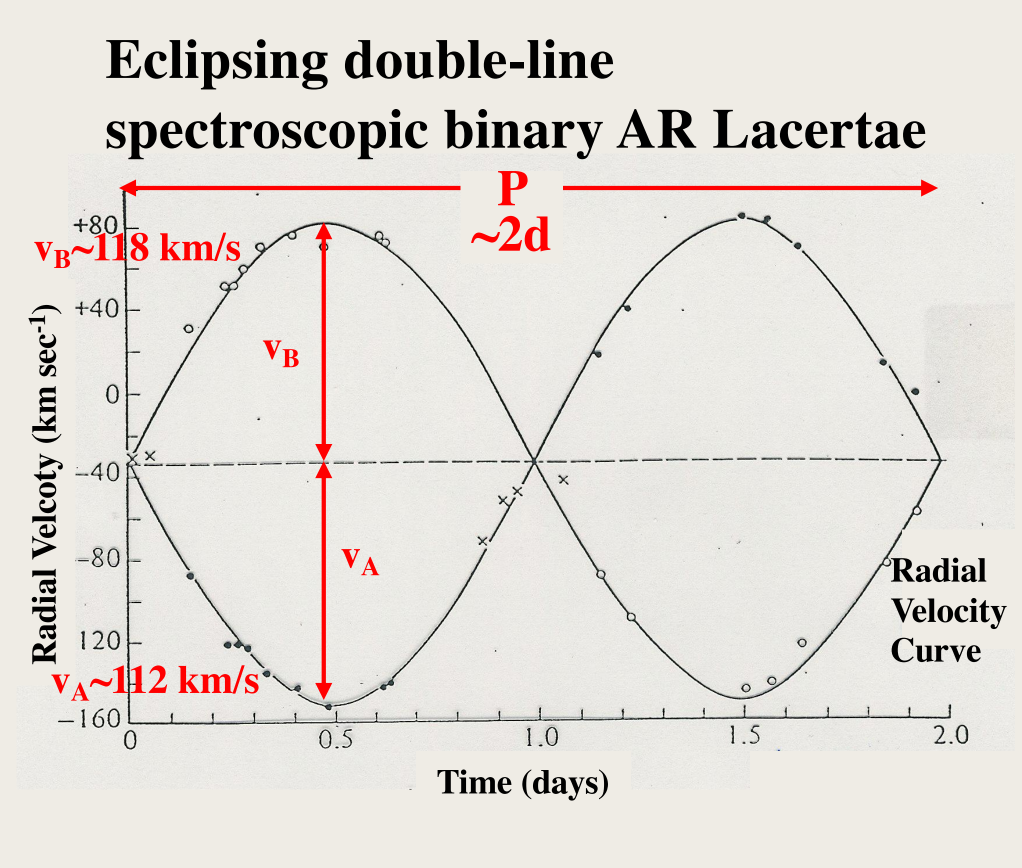
If orbital inclination i ~ 90°, orbital plane is close to line of sight and stars will partially or totally eclipse each other. Duration of eclipses, combined with orbital speeds, gives stellar radii.

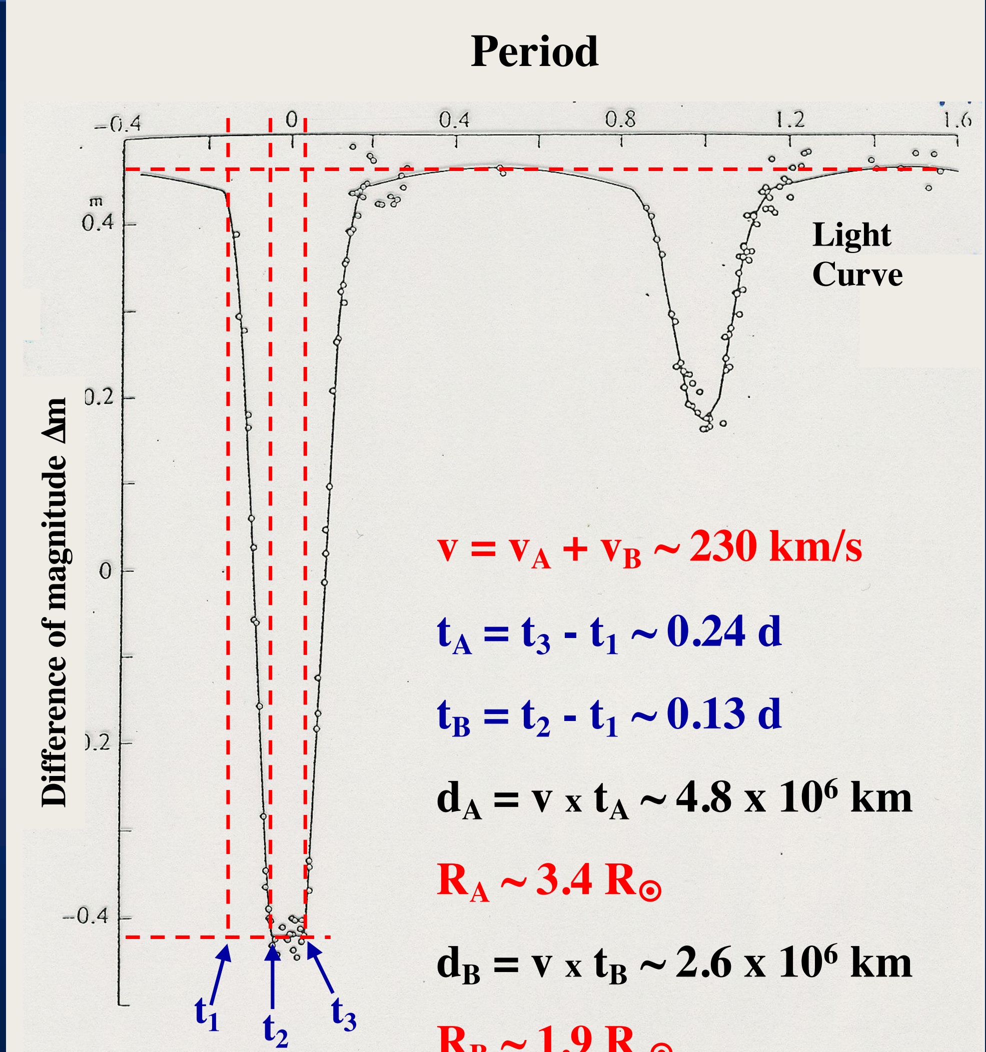




Ways to Measure Stellar Radii

Method 2: Using Eclipsing Binaries:
A Real Example





$v = v_A + v_B \sim 230$ km/s

$t_A = t_3 - t_1 \sim 0.24$ d

$t_B = t_2 - t_1 \sim 0.13$ d

$d_A = v \times t_A \sim 4.8 \times 10^6$ km

$R_A \sim 3.4\ R_\odot$

$d_B = v \times t_B \sim 2.6 \times 10^6$ km

$R_B \sim 1.9\ R_\odot$

Summary Stellar Radii