Basic Properties of Stars

Stellar Masses and the Mass-Luminosity Relation - Sections 13.5 and 13.6

Pages 322 - 332

Kepler's Third Law

$p^2 \propto a^3$ - the square of the periods (p) of the planets are in proportion to the cube of their semi-major axes (a).

If p is in years and a in AU, the proportionality constant = 1, thus $p^2 = a^3$

Generally

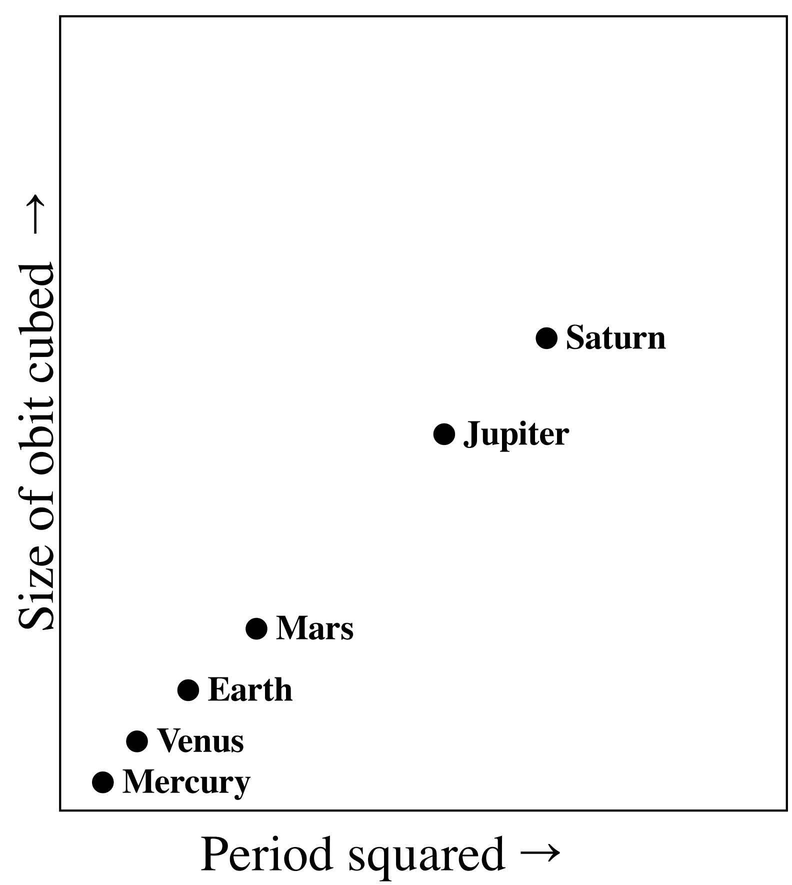
in sec

$$\boxed{p^2 = \frac{4\pi^2}{G(m_1+m_2)}\,a^3}$$

in m

in kg

Gravitational Constant = 6.67×10^{-11} Newton m^2/kg^2



Kepler's observation of planetary orbits based on Tycho Brahe's data

Application Kelper’s 3rd Law

Mass of the Sun: $p^2 = 4\pi^2 a^3/G(M_{sun} + M_{planet})$

$p^2 = 4\pi^2 a^3/GM_{sun}$ if $M_{planet} << M_{sun}$

For Earth’s orbit, P = 1 year = 365.2564 d = 3.256×10^7 s

a = 1 AU = 1.496×10^{11} m

$\rightarrow M_{sun} = 1.989 \times 10^{30}$ kg (if use orbit any other planet same result)

Is approximation $M_{earth} << M_{sun}$ valid?

Consider satellite orbit ($M_{satellite} << M_{earth}$) eg Hubble Space Telescope

Altitude 559 km = 6400 + 559 from Earth’s centre, a = 6.96×10^6 m

Orbital P = 97 min, so P = 5820 sec

$M_{earth} = 5.89 \times 10^{24}$ kg ~ 3×10^{-5} M_{sun}

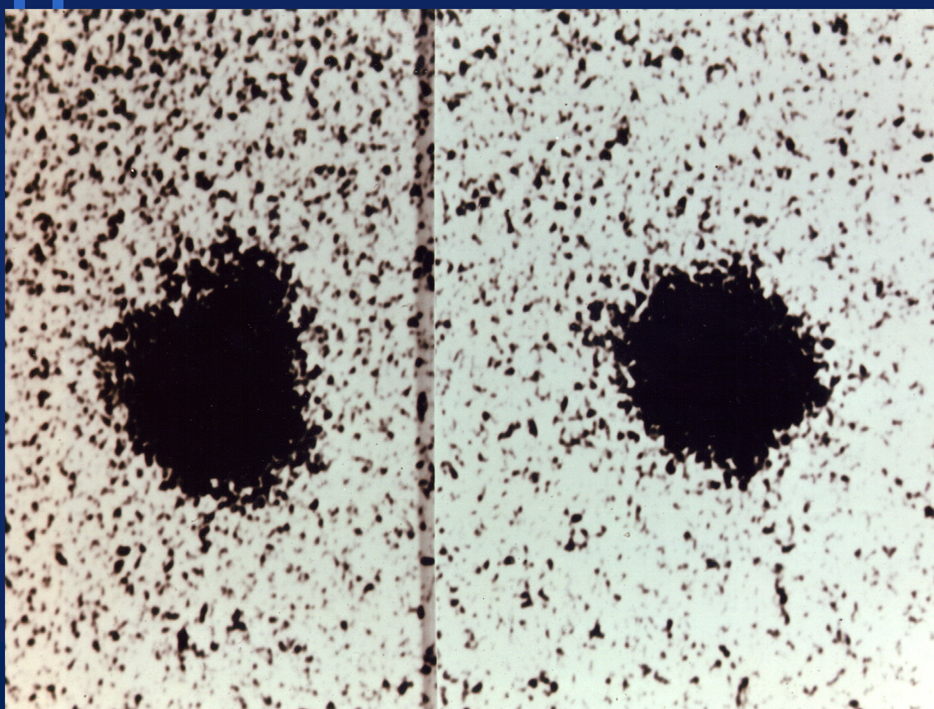
Hence, $M_{earth} << M_{sun}$ is a valid approximation

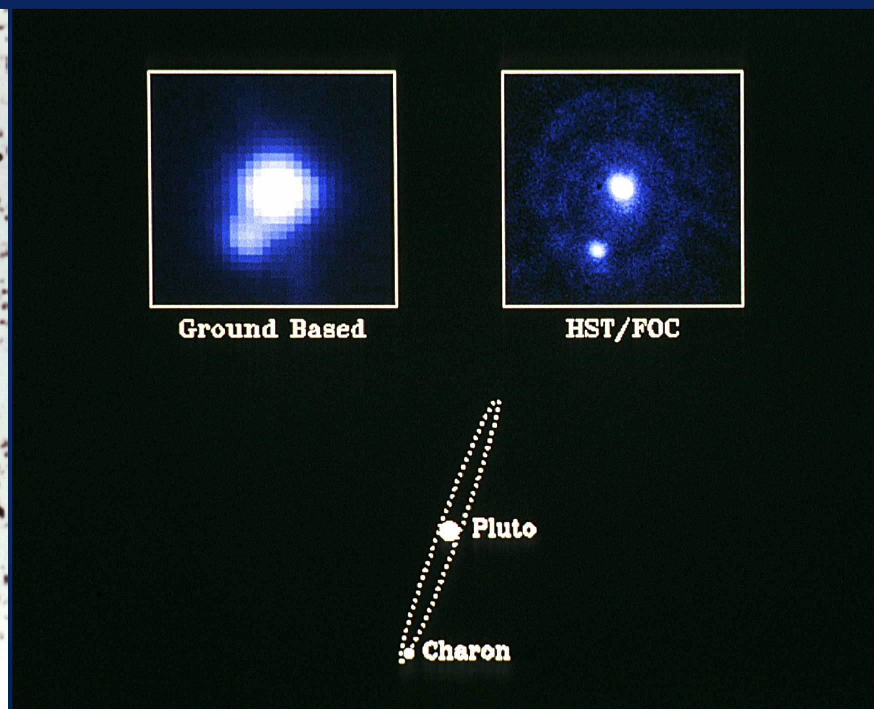
Get masses of planets if they have a Moon – Mercury? Venus?

Part of Reason Pluto got Dumped

M_{earth} = 5.9 x 10^{24} kg; $M_{Jupiter}$ = 1.9 x 10^{27} kg

In 1971, Pluto's moon, Charon, was discovered.

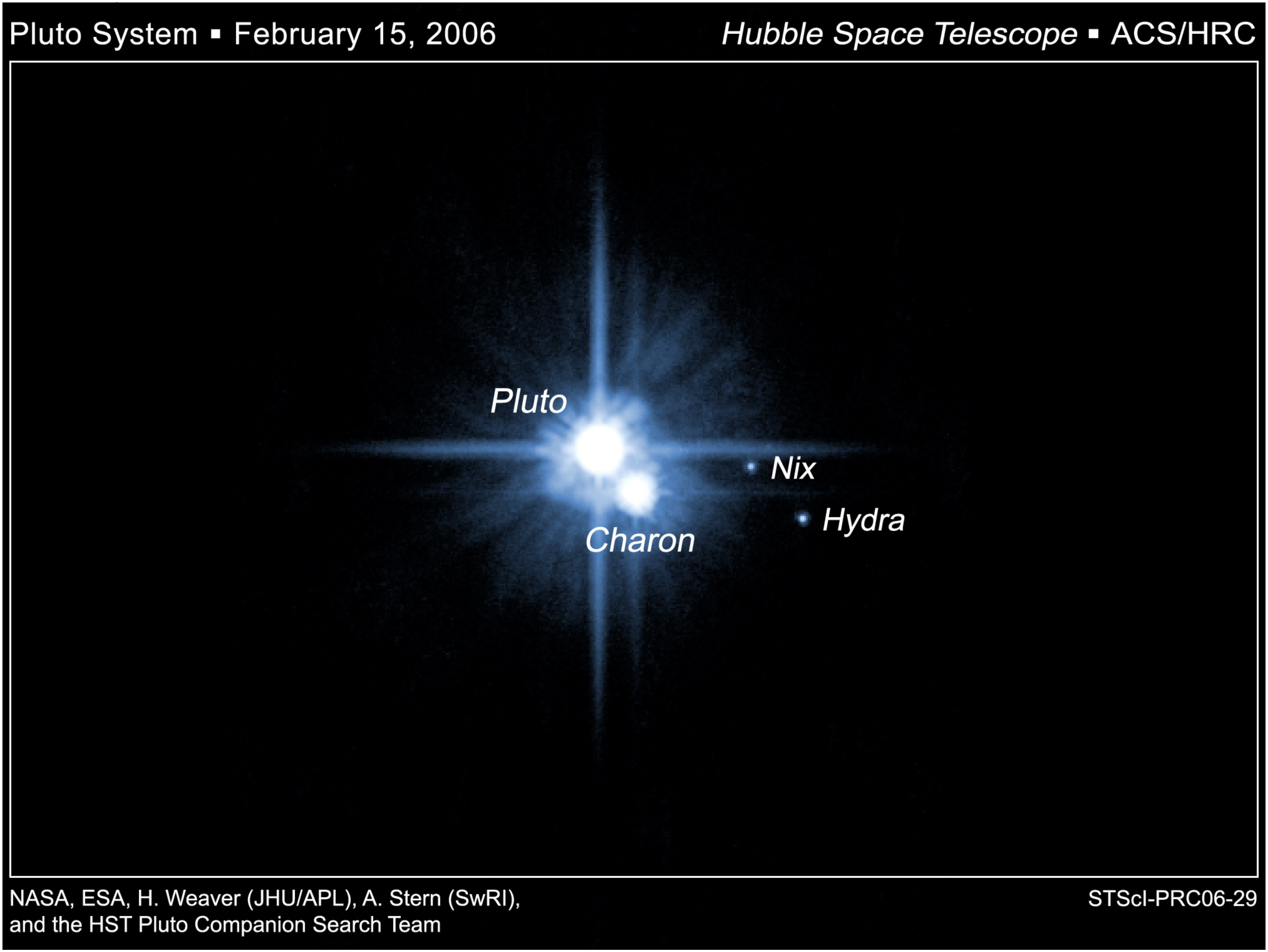




$p^2 = 4\pi^2 a^3 / GM_{pluto}$ if $M_{charon} << M_{pluto}$

Charon's orbit: p = 6.39 d = 5.52 x 10^5 s; a = 1.97 x 10^7 m

$\rightarrow M_{pluto}$ = 1.485 x 10^{22} kg = 0.002 M_{earth}

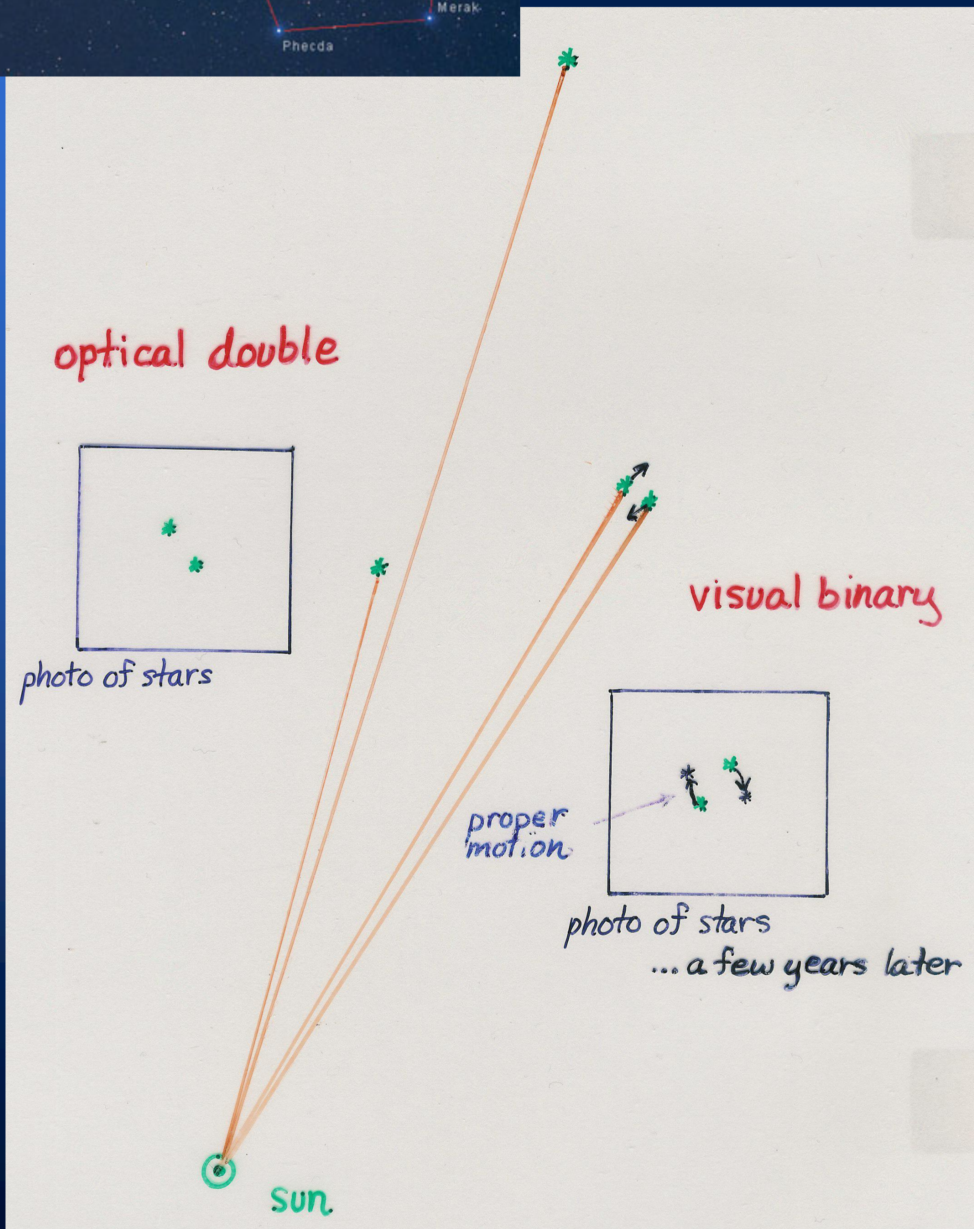

Pluto System ▪ February 15, 2006
Hubble Space Telescope ▪ ACS/HRC
Pluto
Nix
Hydra
Charon
NASA, ESA, H. Weaver (JHU/APL), A. Stern (SwRI), and the HST Pluto Companion Search Team
STScI-PRC06-29

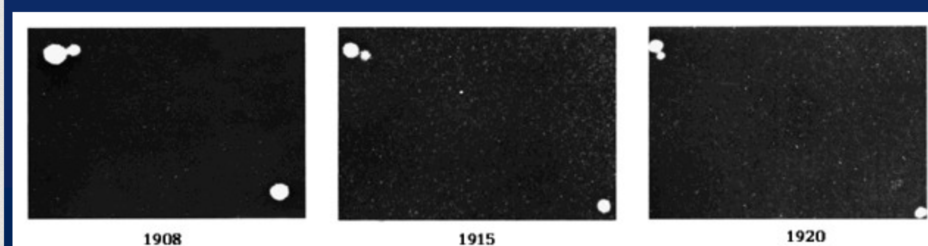
Binary Stars: Visual Binaries

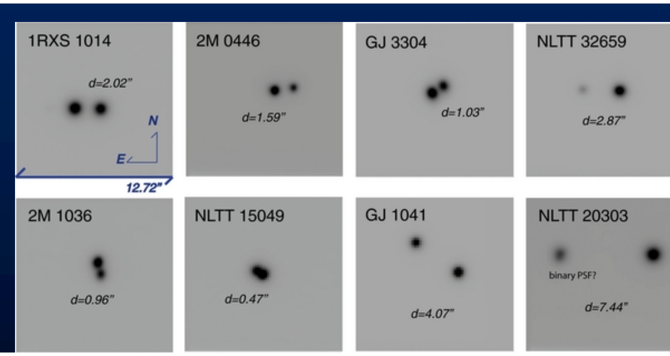
Two stars in binary orbit
stars are resolved in telescope

Distinguish from optical double as detect orbital motion after several years

Since separations are generally large here (a^3 proportional to p^2) periods tend to be long





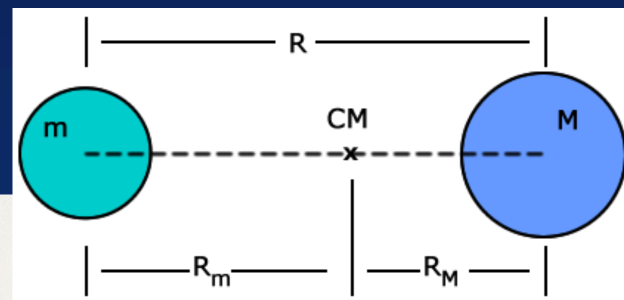


Binary Stars: Visual Binaries

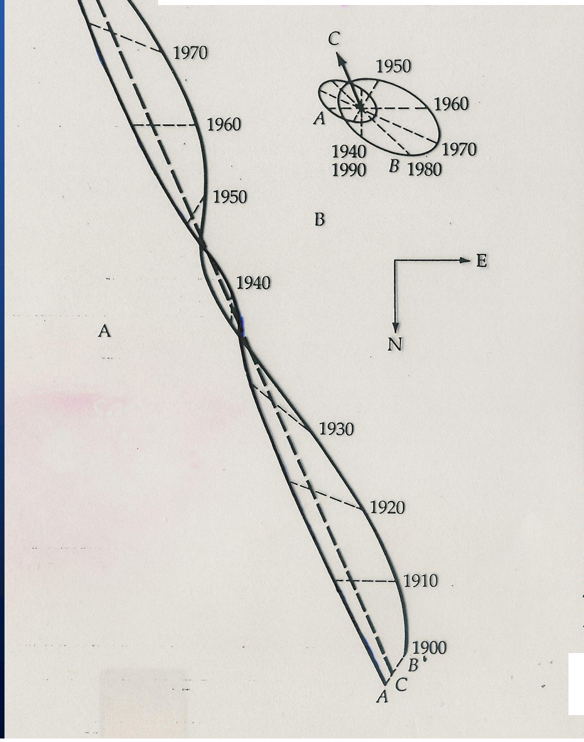
Orbits obey Kepler's Laws BUT M_{star1} not >> M_{star2}

Must use $p^2 = [4\pi^2/G(M_1 + M_2)]a^3$

Stars revolve about a mutual centre of mass



Astrometry of Sirius revealed the presence of a binary companion



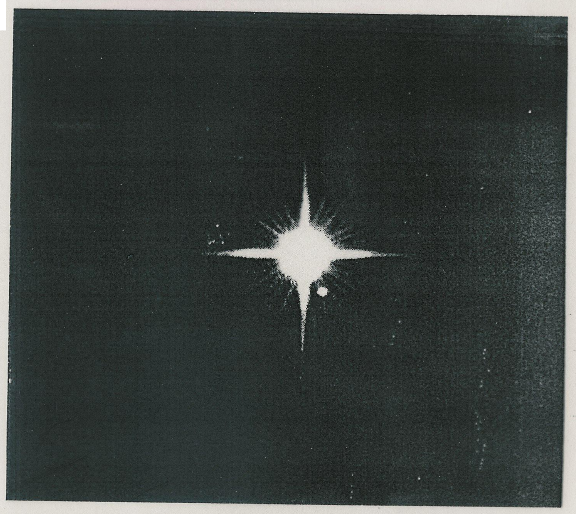
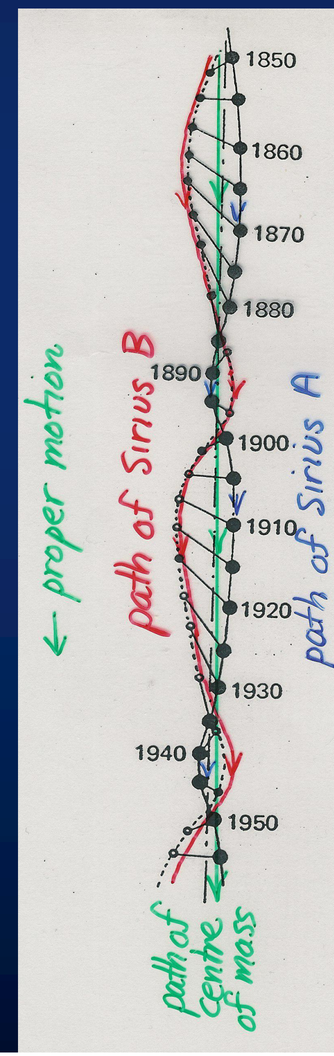


FIGURE 22-9 Sirius and Its White Dwarf Companion Sirius, the brightest-appearing star in the sky, is actually a double star. The secondary star is a white dwarf, seen in this photograph at the five o'clock position, almost obscured by the glare of Sirius. The spikes and rays around Sirius are the result of optical effects within the telescope. (Courtesy of R. B. Minton)

… the first known white dwarf

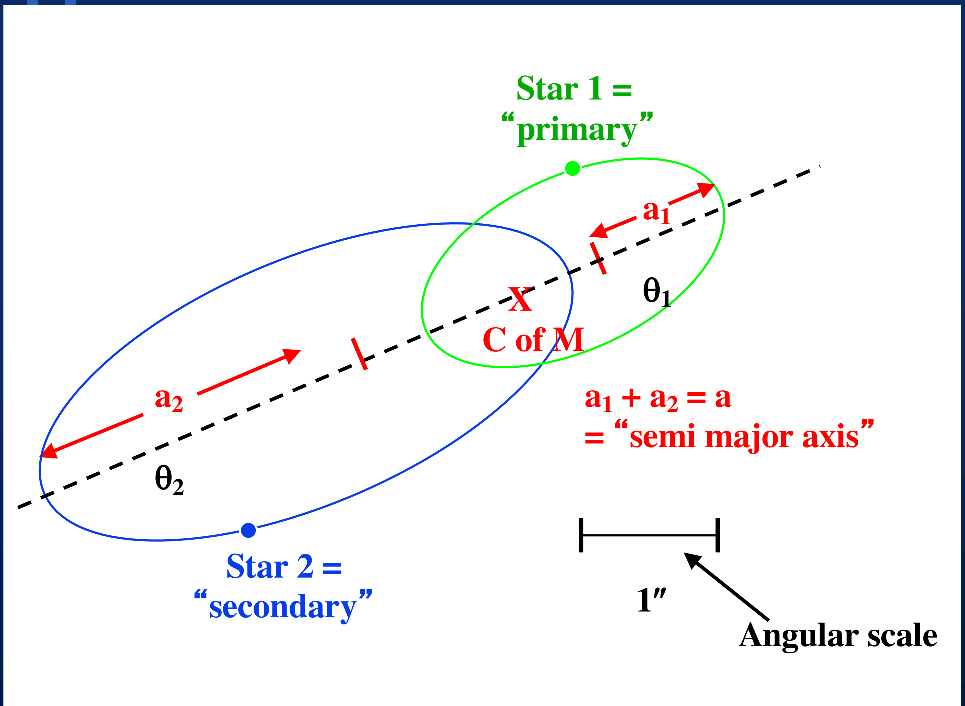
Fig 13.7



Masses from Visual Binaries

Motion of each star with respect to CM can be measured and used to calculate stellar masses

Mass calculation if orbits are seen face-on (in plane sky)



$m_1/m_2 = a_2/a_1$ (semi major axes ellipses)

Can use angular separations θ_1 and θ_2 (gives mass ratio)

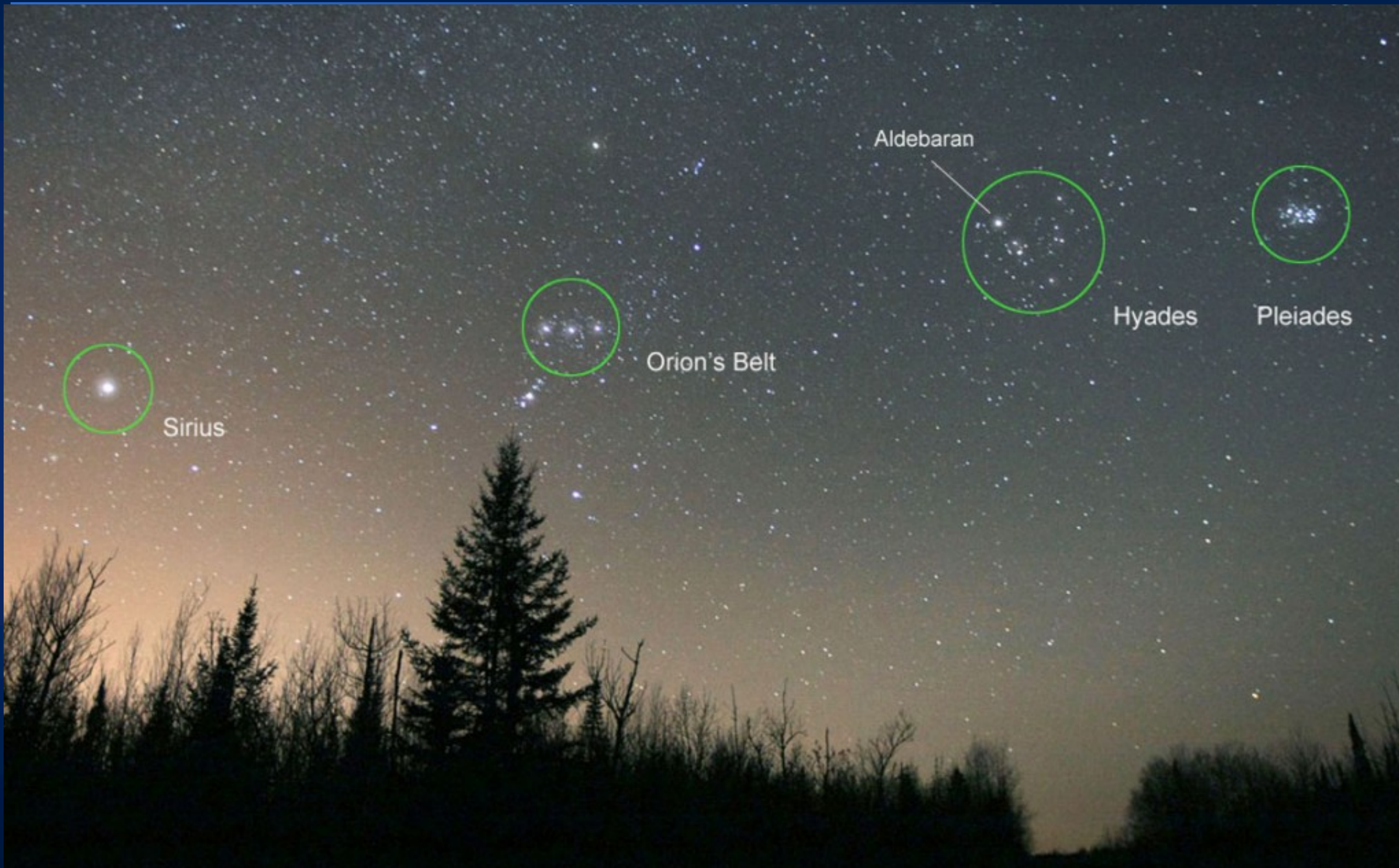
If distance d known, $a = a_1 + a_2$ can be measured from $\theta = \theta_1 + \theta_2 \rightarrow \theta = a/d$ $\rightarrow a = d\theta$.

Then $p^2 = [1/m_1+m_2]a^3$ gives sum of masses m_{tot}

$m_{tot} = m_1 + m_2 = m_1(1 + m_2/m_1)$

m_2/m_1 known so get m_1, and m_2 from $m_{tot} - m_1$ - individual masses known

Sirius



The Sirius System

Visual binary system P = 50.05 years

Parallax = 0.38" (D = 2.63 pc) Assume plane orbit in plane sky

Ratio semi major axes = 2.5 = m1/m2

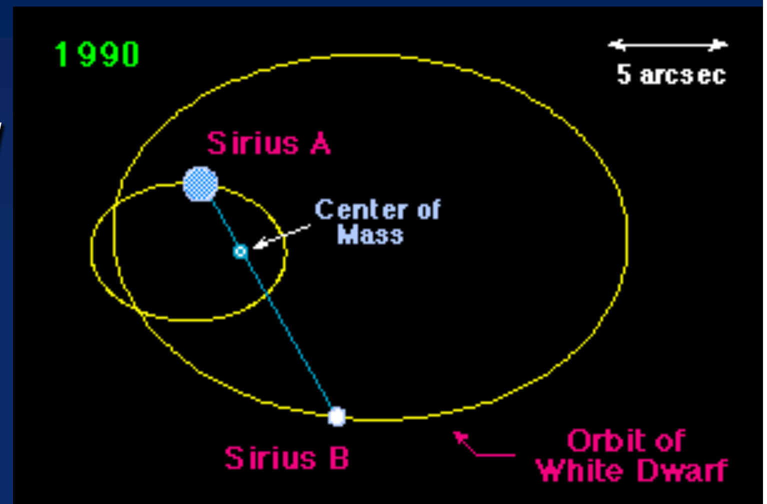
Sum masses = a^3/p^2 (in AU and years)

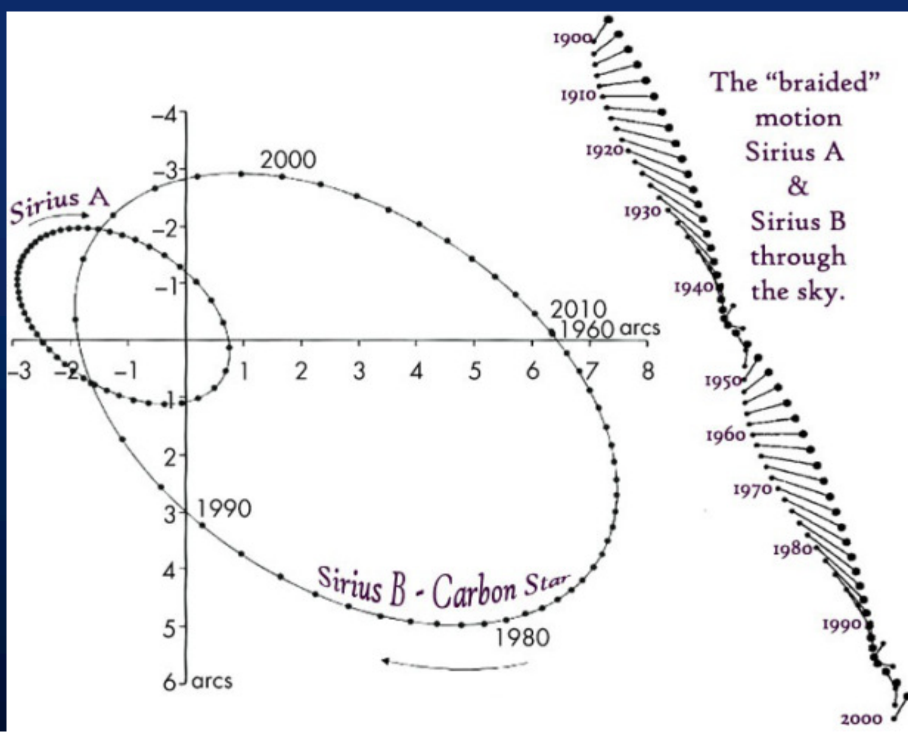
a = a2 + a1 = 5.7" + 2.3"=15.0+6.0 =21.0AU

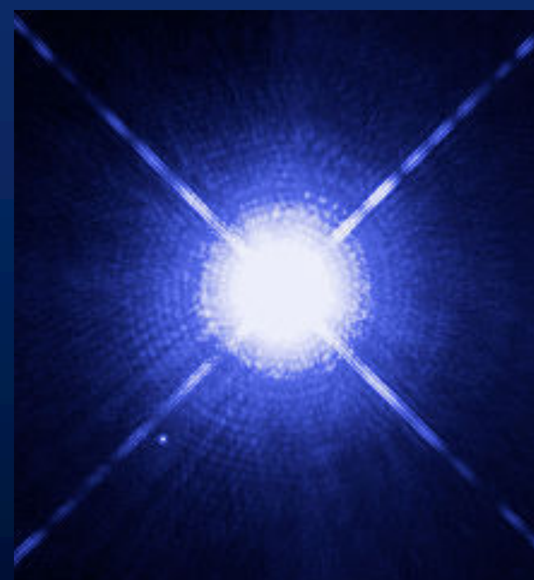
m1 + m2 = $21.0^3/50.1^2$ = 3.7 Solar Masses

m1 =2.6, m2 = 1.1 (better is 2.1 and 1.0)

Numbers not quite right - what is wrong?



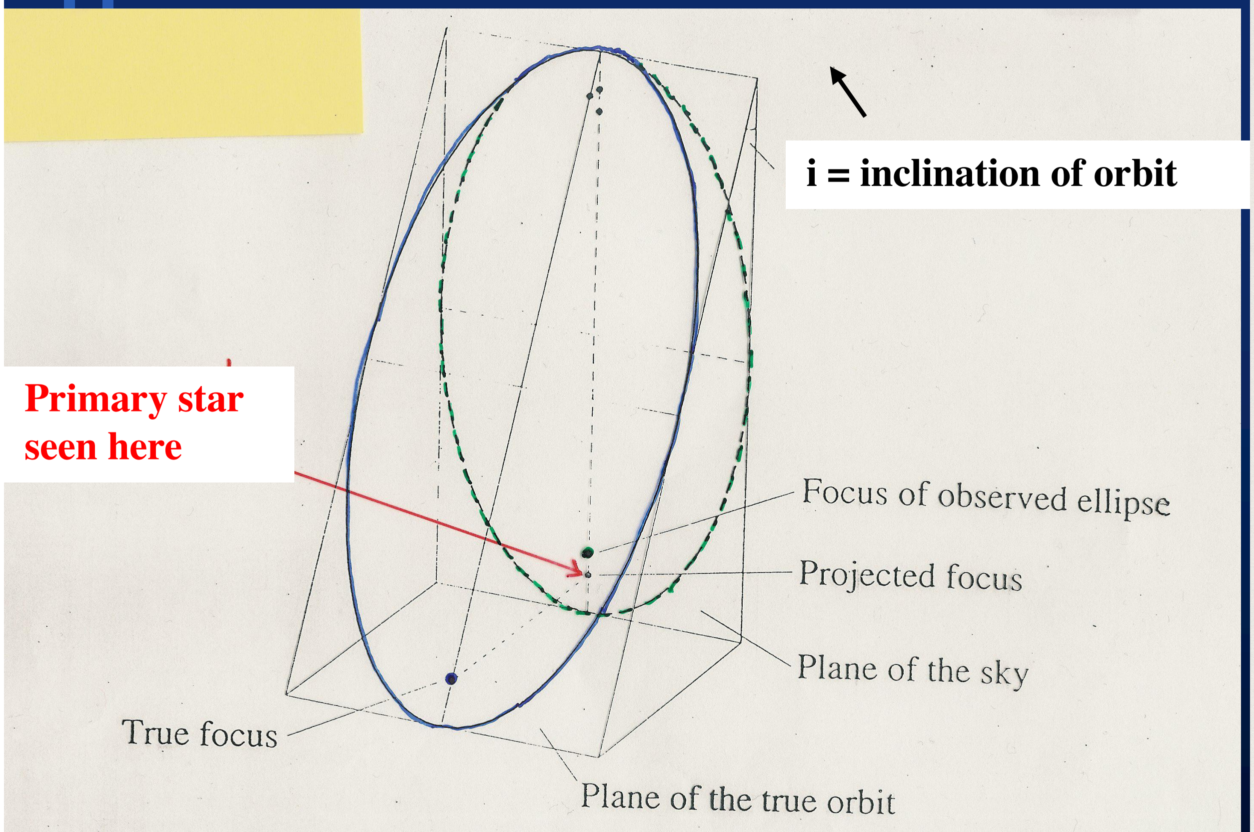


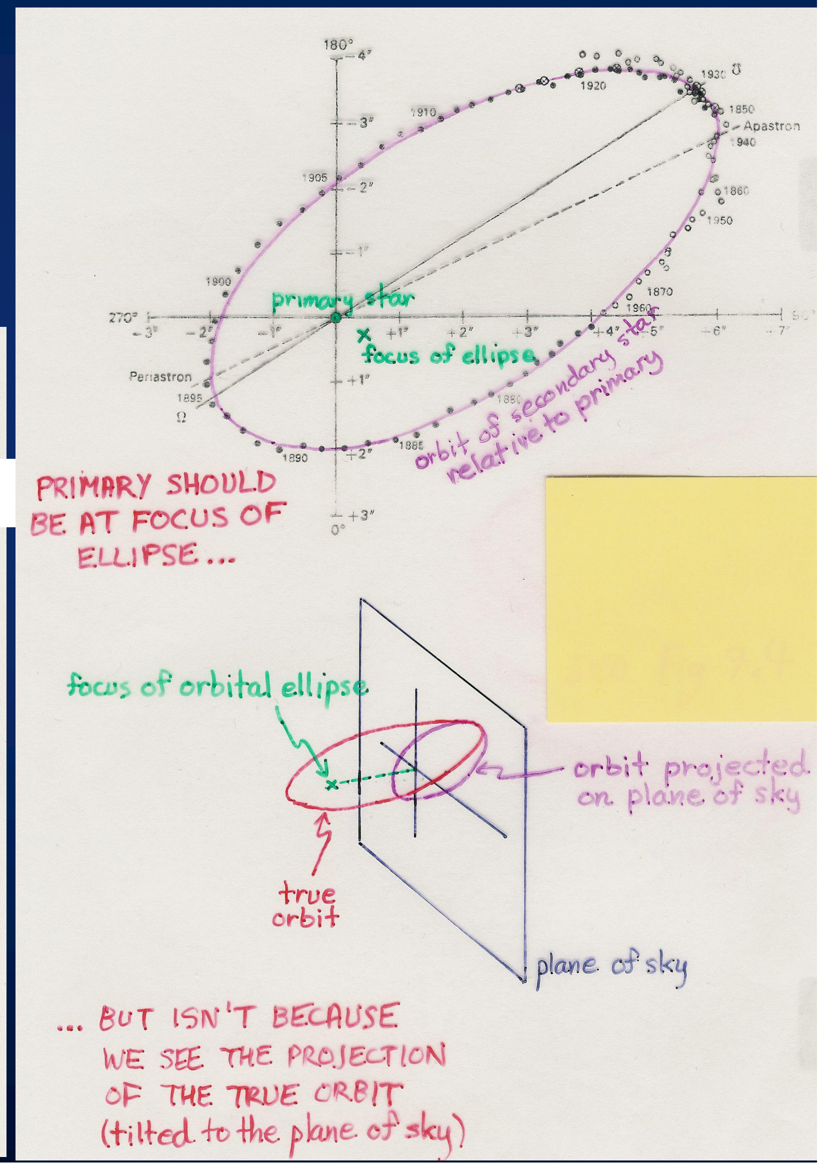


Masses from Visual Binaries

Assumed orbit in plane of the sky – generally not true

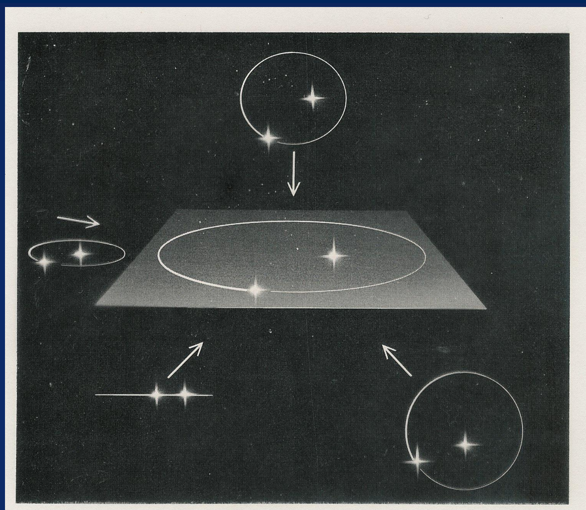
Inclination, i, can be deduced by fact that CM will not be at focus of projected ellipse

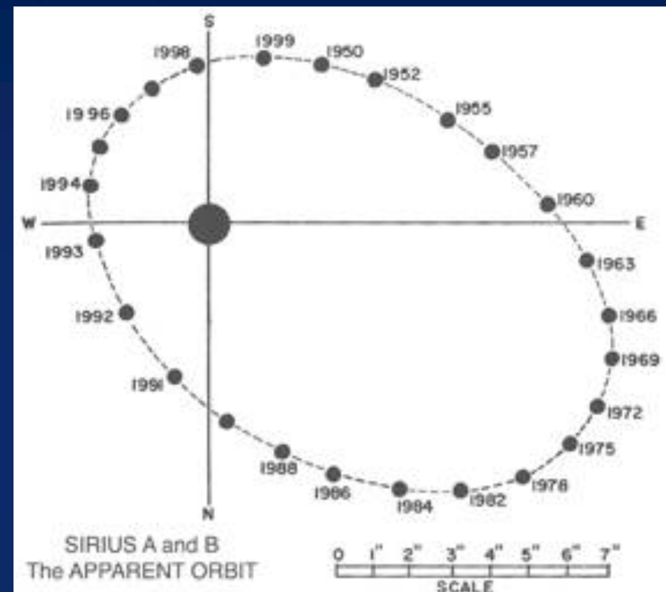




Masses from Visual Binaries

Mass calculation if orbit inclined by angle i (most common situation)





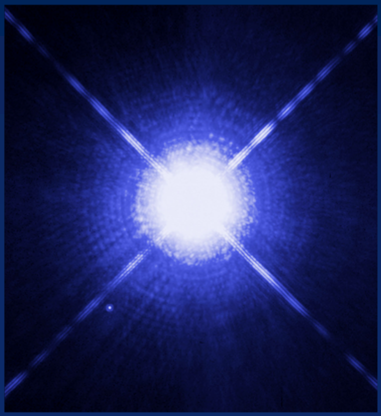
Does not affect mass ratio: $m_1/m_2 = a_2 \cos i / a_1 \cos i = a_2'/a_1'$. a_2' and a_1' are the projected semi-major axes on sky.

But $a = a'/\cos i$, so apparent solution to Kepler's 3rd Law would be $p^2 = [1/(m_1 + m_2)](a'/\cos i)^3 \rightarrow m_1 + m_2 = [a'^3/p^2]/(\cos i)^3$

Inclination, i, can be deduced by fact that CM will not be at focus of projected ellipse - best usual case for mass determination.

Stellar Distances for Visual Binary Stars

How can we use binary stars to get stellar distances



Hint: Kepler's third law

$p^2 = 4\pi^2 a^3/G(M1 + M2)$

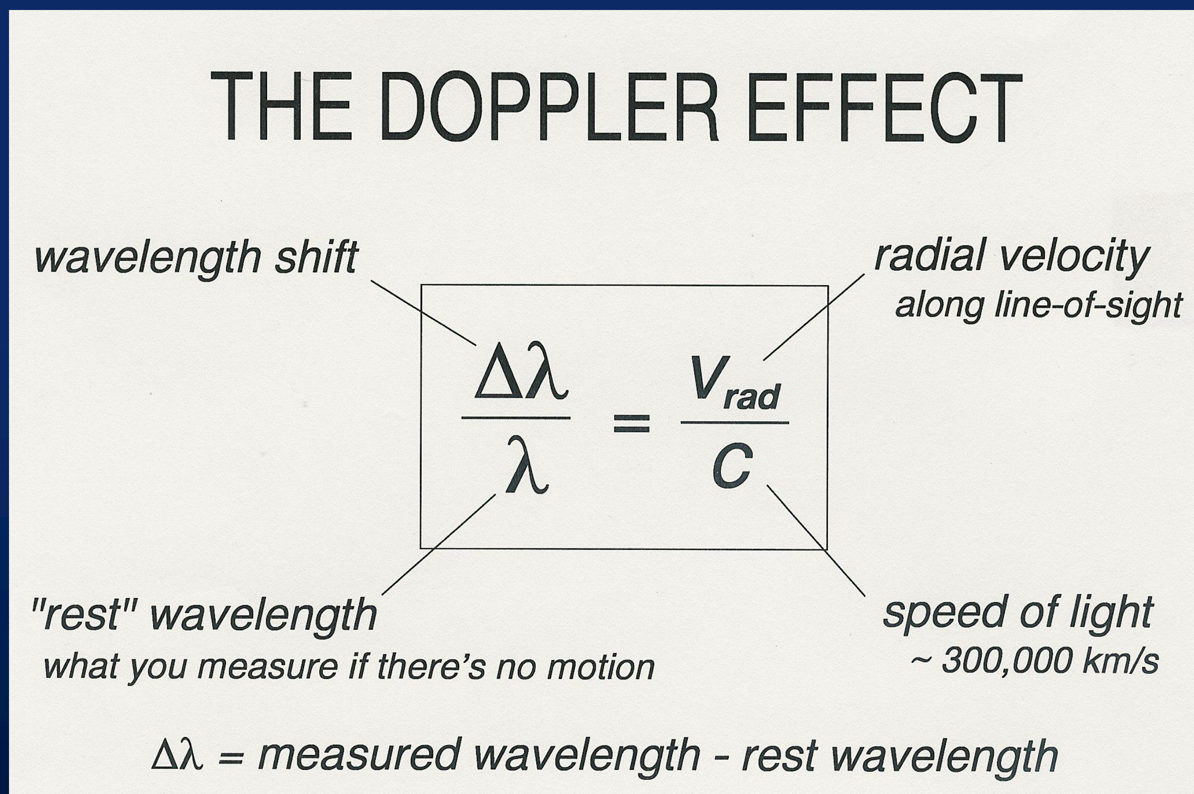
Measure period – knowing mass of stars gives *a* then from angular separation and *a* get distance

eg Sirius A and B – Mass A=2.02 M_{sun} Mass B=0.98 M_{sun}
P = 50.1 yrs gives *a* (semi major axis)= 2.84×10^{12} m
Observed angular semi major axis = 7.50"

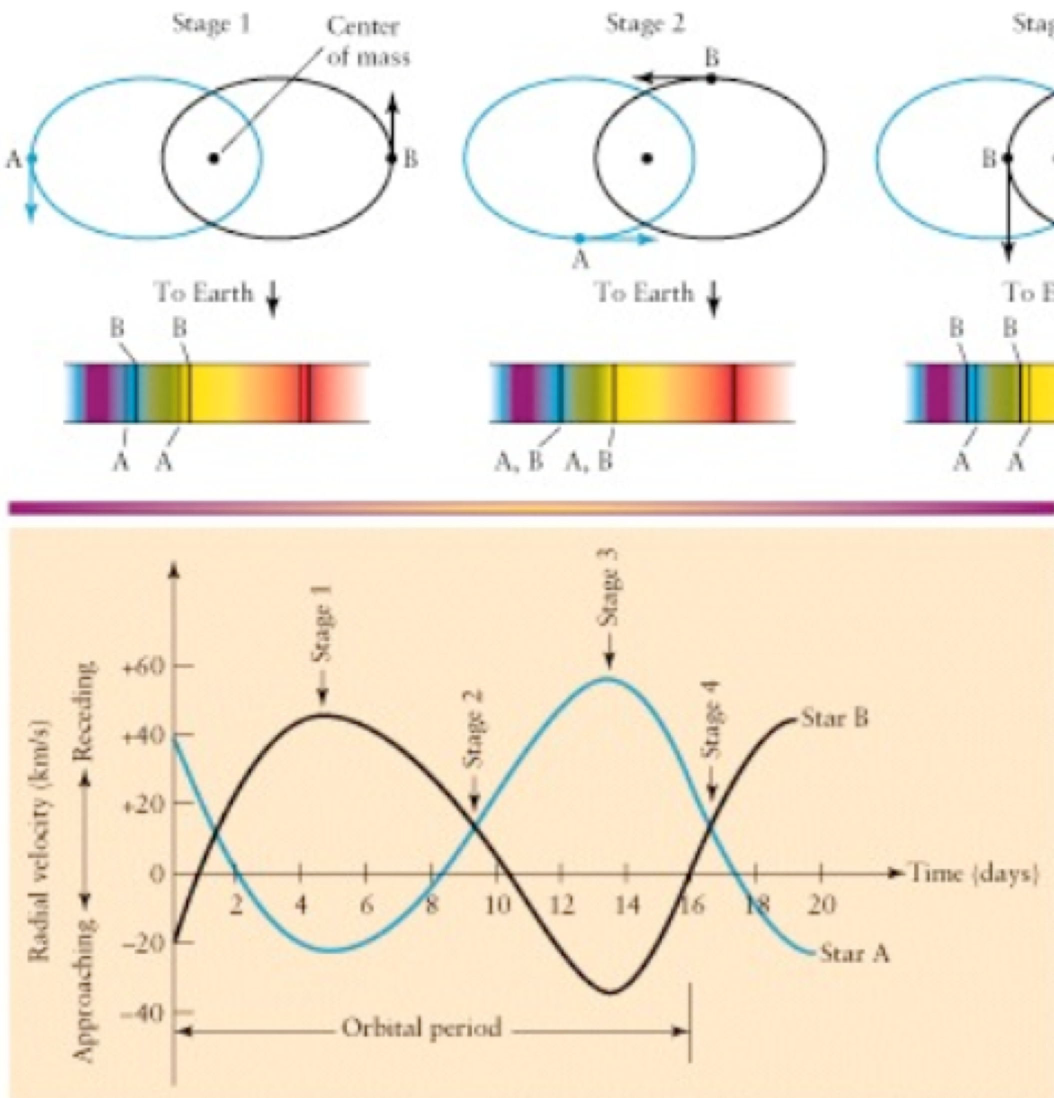
Distance (from small angle formula) = 8.7 light years

Spectroscopic Binaries

SB: - two stars unresolved but binarity revealed by periodic shifts in wavelengths of spectral features due to Doppler Effect



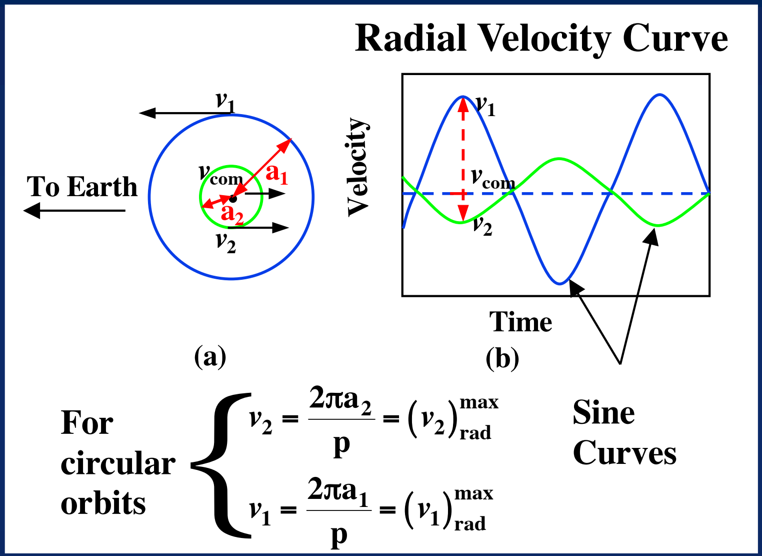
Spectroscopic Binaries



Double-lined spectroscopic binary showing the orbits and resultant composite spectra. COM of system has velocity ~15km/s.

Spectroscopic Binaries

Case (a) Spectra of both stars are present (double-line SB) and orbital plane is along line-of-sight (as below – eclipsing spectroscopic binary).



NB: if orbits are inclined - still sine curves, but amplitude is changed by sin i - need other info to get i, v

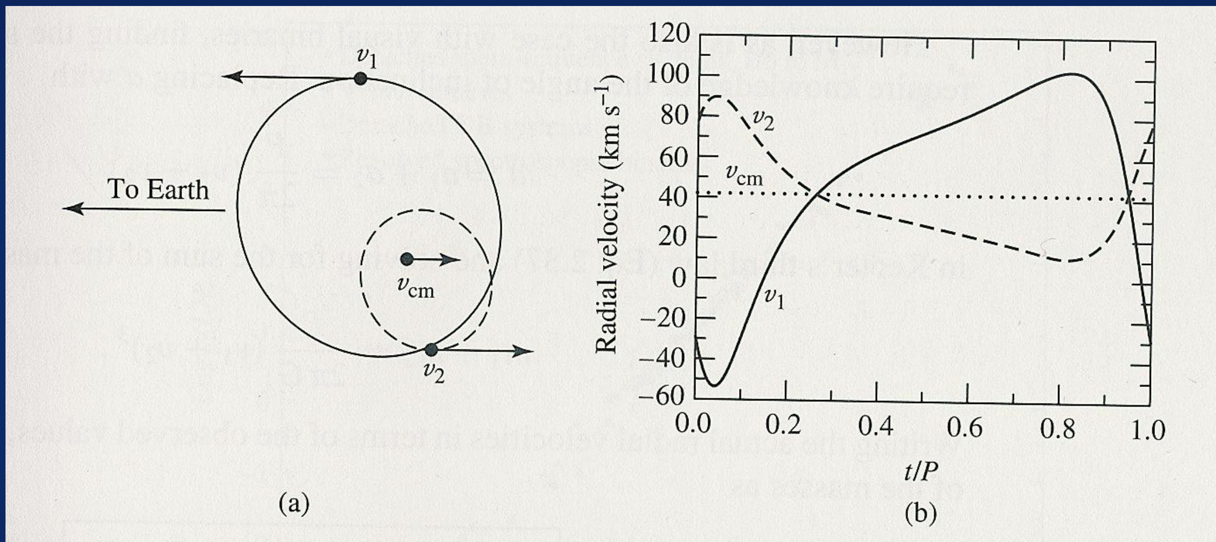
$(v_2)_{max} = 2\pi a_2/p$; $(v_1)_{max} = 2\pi a_1/p$ (for circular orbits)

$(v_2)_{max}/(v_1)_{max} = a_2/a1 = m_1/m_2$ ($m_1 a_1 = m_2 a_2$ - Radial Velocity curve gives mass ratio)

$a = a_1 + a_2 = (p/2\pi)(v_1 + v_2)$; then from $p^2 = 4\pi^2 a^3/G(m_1 + m_2)$ - Kepler's Third Law - we derive $m_1 + m_2 = p(v_1 + v_2)^3/2\pi G$ - have $m_1 + m_2$ and ratio m_1/m_2 - can solve for individual masses.

Spectroscopic Binaries

When $e \neq 0$, velocity curves are skewed. Exact shapes of curves depend both on e and the orientation of the orbit with respect to observer.



These velocity curves are for 2 stars with masses = 1 and 2 solar masses, p = 30 days, radial velocity of the centre of mass = 42 km/s, e = 0.4, and plane of orbit lying along the line of sight to observer.

Usually, curves like these have to be modeled to choose the best-fit parameters.

Spectroscopic Binaries

Case (b) Orbits inclined by angle i;

$v_{rad} = v \sin i$ (v_{rad} is observed)

No effect on mass ratio:

$m_1/m_2 = v_2/v_1 = (v_{2rad}/\sin i)/(v_{1rad}/\sin i) = v_2/v_1$

So we can again get the ratio of the stellar masses without knowing i

Again, from Kepler's 3rd Law:

$m_1 + m_2 = p(v_1 + v_2)^3/2\pi G$

Thus, we have that $m_1 + m_2 = p(v_{1rad} + v_{2rad})^3/2\pi G \sin^3 i$ ….(eq.1)

Spectroscopic Binaries

Case (c) Only 1 star's spectrum visible (single-line SB) - currently great interest as companion very faint or dark like neutron star, white dwarf or PLANET.

$\rightarrow$ v_{2rad} not measured, but we know $m_1/m_2 = v_{2rad}/v_{1rad}$

so $v_{1rad} + v_{2rad} = v_{1rad}[1 + v_{2rad}/v_{1rad}] = v_{1rad}[1+m_1/m_2]$,

Sub into (eq.1) $(m_1 + m_2) = p(v_{1rad} + v_{2rad})^3/2\pi G \sin^3 i$)

$= p(v_{1rad})^3(1+m_1/m_2)^3/2\pi G \sin^3 i$

or $m_2^3 \sin^3 i/(m_1 + m_2)^2 = pv_{1rad}^3/2\pi G$ …..(eq. 2)

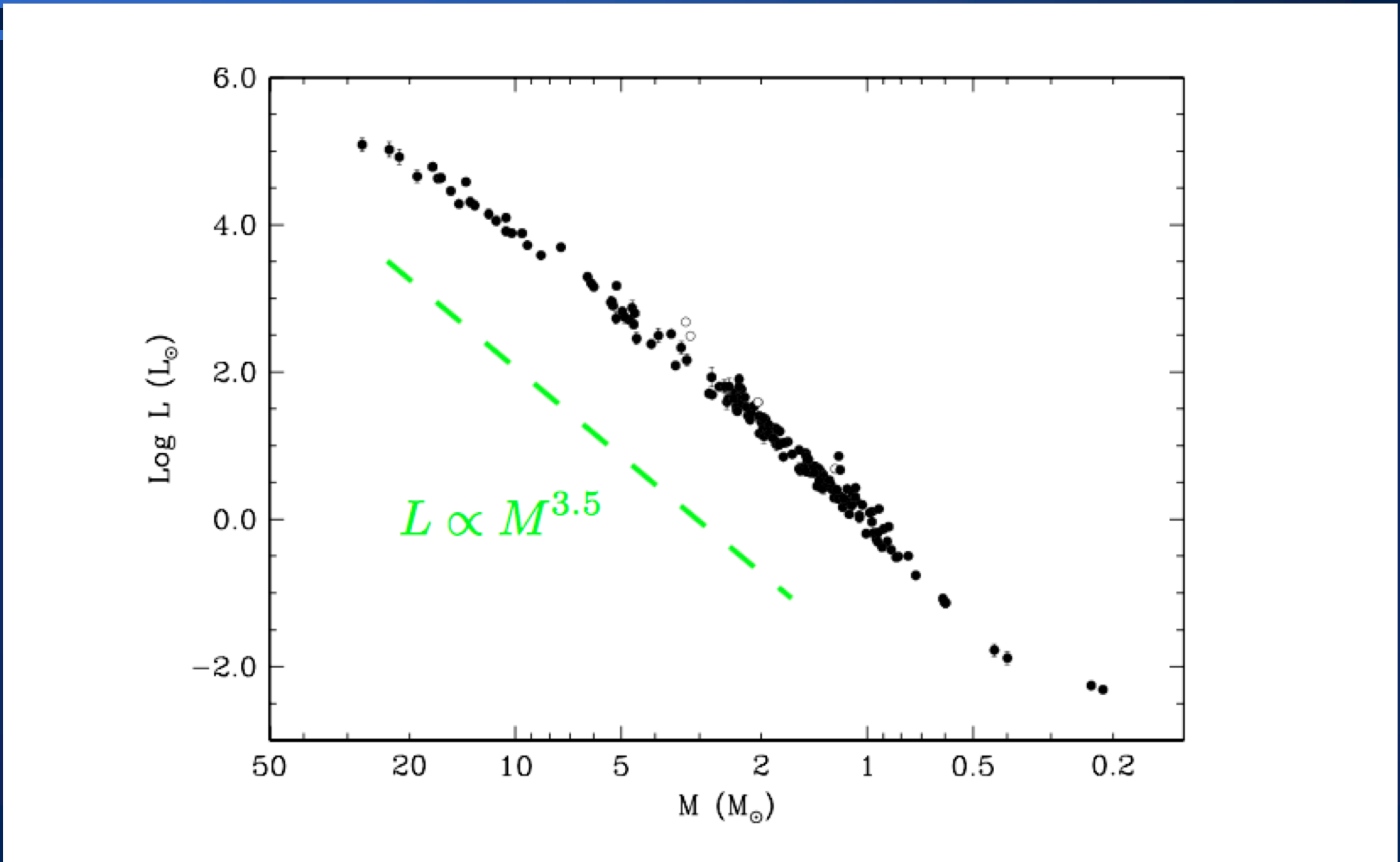
$pv_{1rad}^3/2\pi G$ is called the mass function. It depends only on observed quantities. The mass function does not give the mass ratio (as only 1 spectrum seen) but it is useful for statistical studies if an estimated mass of at least 1 component known (colour, spectra).

Eg assume mass m_1, get minimum mass m_2 assuming $i = 90^\circ$.

If $m_1 >> m_2$ (eg planet) mass function simplifies to $m_2^3 \sin^3 i/m_1^2$ (get minimum mass planet)

If $m_1 << m_2$ (unseen star is a black hole) mass function becomes $m_2 \sin^3 i$, and mass function gives lower limit on mass unseen object

Mass-Luminosity Relation 13.6



Masses were obtained from observations of many binaries.
Luminosities were obtained from parallax measurements.
Note the relatively narrow mass range.

Fig. 13.11 is similar

State of Stellar Matter

With masses and radii known, the densities and state of matter of stars can be determined.

Sun:

M_{sun} = 2 x 10^{30} kg = 2 x 10^{33} g
R_{sun} = 7 x 10^{5} km = 7 x 10^{10} cm

Mean density $\rho_{sun} = M_{sun}/(4/3\pi R_{sun}^3) = 1.4$ g/cm^3 (a bit more than ρ water)

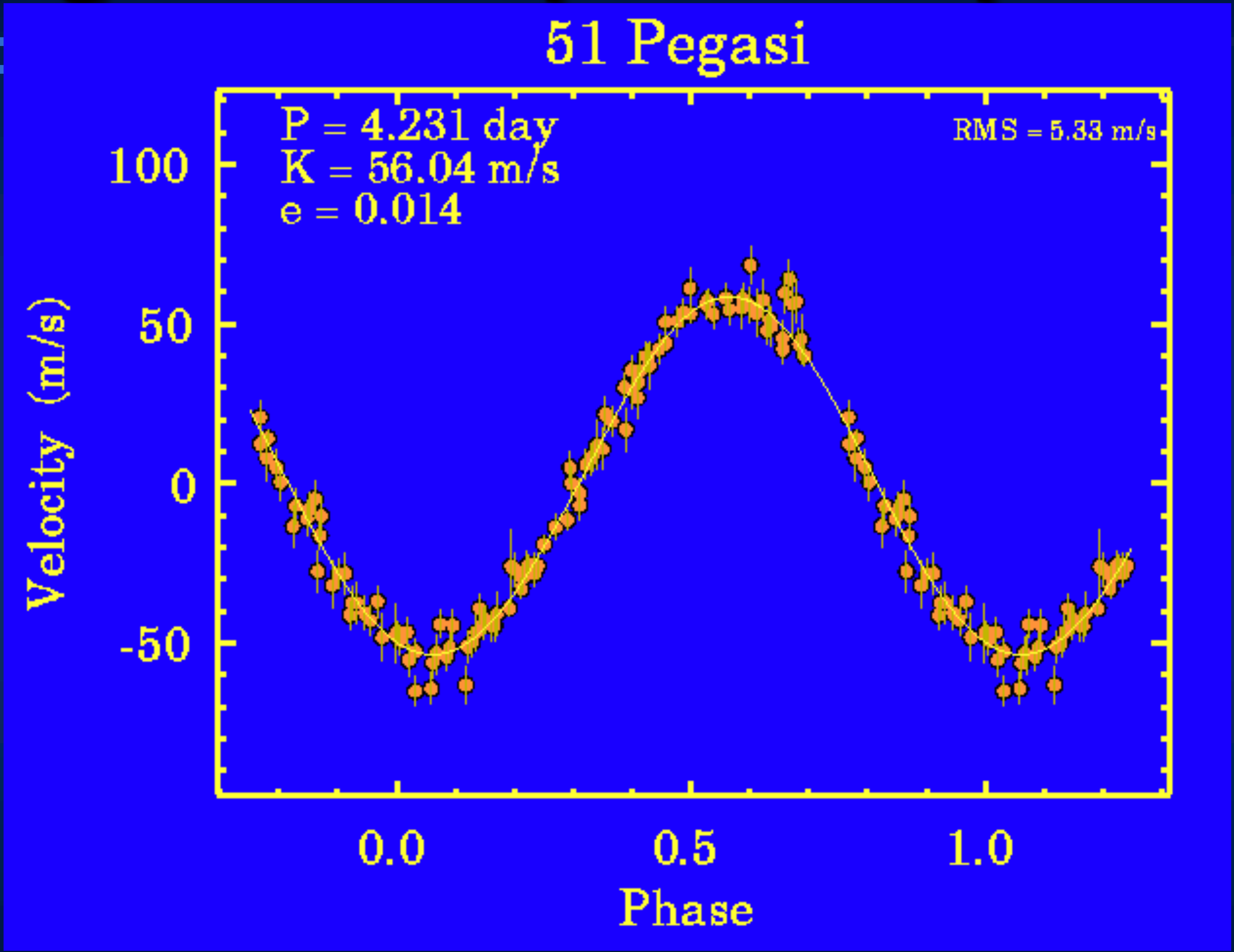
Giant Stars:

M_{star} ~ few x M_{sun} in some cases
R_{star} ~ 500 R_{sun}

ρ_{giant} typically $< 10^{-7}$ g/cm^3!

3000K $< T_{eff} <$ 50,000 K $\rightarrow$ stars are gaseous

Single-Lined Spectroscopic Binary



What can you say about unseen mass? 51 Peg like Sun