Energy Sources in Stars and the Origin of the Elements (15.2-15.3)

What makes a star shine?

Energy Sources in Stars (§ 15.2)

What makes a star shine?

Sun's Energy Output

$L_{sun} = 4 \times 10^{26}$ W (J s^{-1})

$t_{sun} = 4.5 \times 10^{9}$ yr (1.4×10^{17} s) - oldest rocks - radioactive dating

L_{sun} ~ constant over geological timescale (fossil evidence)

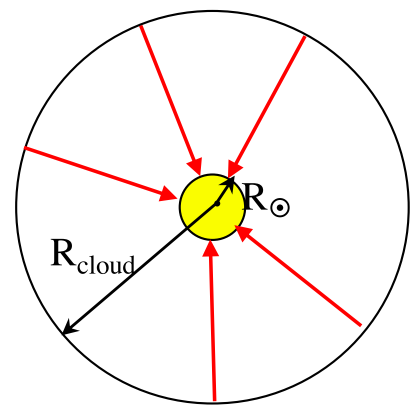
Therefore total energy output $E_{tot} = L_{sun} \times t_{sun} = 6 \times 10^{43}$ J

How might the Sun have generated this energy?

Energy Sources in Stars

Energy Source #1:Gravitational contraction

We believe Sun collapsed from a large gas cloud ($R \sim \infty$) to its present size



No change in Mass, $M_{\odot}$

Potential energy released is:

$\Delta E_{grav} = -(E_{now} - E_{initial})$ take $E_{initial} = 0$

P.E. is obtained from integrating grav. potential over all points in the sphere

$$\text{P.E.} = -\int_0^{R_{sun}} G \frac{\left(\frac{4}{3}\pi\rho r^3\right)\left(4\pi r^2 \rho dr\right)}{r}$$

$$= -\left(\frac{16}{15}\right)\pi^2\rho^2 G R^5 \quad \text{for constant } \rho$$

With $M = \frac{4}{3}\pi R^3 \rho$, we get

$$\text{P.E.} = -\frac{3GM_{sun}^2}{5R_{sun}}$$

Energy Sources in Stars

Energy Source #1:Gravitational contraction

$$P.E. = -\frac{3Gm_{sun}^2}{5R_{sun}}$$

However, from Virial Theorem (E_{th} = -1/2 E_{grav}), only 1/2 of the gravitational energy is available for energy release, the other half heats the star.

So, $\Delta E_{grav} = -(E_{now} - E_{initial})$

$= 3/10(GM_{sun}^2)(1/R_{sun} - 1/\infty)$

$\sim 10^{41}$ J

$\sim 1/600\ E_{tot}$ needed over Sun's lifetime

By this process, Sun could only radiate for

$t = \Delta E_{grav}/L_{sun} \sim 10^7$ years

$t = \Delta E_{grav}/L_{sun}$ is Kelvin-Helmholtz timescale

Conclusion: Gravitational Contraction not the main energy source in Sun

Energy Sources in Stars

Energy Source #2:Chemical Reactions

e.g. How much energy would be released if Sun was completely ionized and then all gas recombined into neutral atoms? (binding energy of the H atom 13.6 ev: 1 ev = 1.6×10^{-19} J $\rightarrow$ 13.6 ev ~ 2×10^{-18} J)

Assume Sun is 100% H (X = 1)

Total # H atoms $N_H = M_{sun}/m_H \sim 10^{57}$

$\therefore E_{tot}$ ionization ~ $10^{57} \times (2 \times 10^{-18})$ J ~ 2×10^{39} J

This is only ~ 1/30,000 E_{total} of Sun's energy output

Sun's energy output ~ burning 7000 kg coal each hour on every sq. metre Sun's surface!

Conclusion: Chemical Reactions cannot be source Sun's energy

Energy Sources in Stars

Energy Source #3:Nuclear Energy

Fission?

$Z = 0.02$ (2% Sun consists of heavy elements).

Fissionable isotopes like ^{235}U are rare (isotopes are nucleii with the same # protons but different # neutrons) e.g.

$n + U^{235} \rightarrow Ba^{141} + Kr^{92} + 3n$ (~3×10^6 more energy than burning coal)

Fusion?

e.g. 4 protons get converted into 2p + 2n i.e. $4H \rightarrow 1\ ^4He$

H is very abundant in the Sun ($X = 0.73$)

4 H nuclei: mass = $4 \times m_p = 6.693 \times 10^{-27}$ kg

1 ^4He nucleus: mass = 6.645×10^{-27} kg (binding energy included)

Difference is 0.048×10^{-27} kg = 0.7% of original mass

Where does this mass go?

Energy Sources in Stars

Energy Source #3:Nuclear Energy

The "lost" mass is converted into energy according to Einstein's equation for the rest energy of matter: $E = mc^2$ (E energy, m mass, c speed light)

Example:

Suppose that Sun was originally 100% H and only 10% of that was available for fusion. Thus…

M_H available = 0.1 M_{sun}

ΔM_H destroyed through fusion = 0.1 M_{sun} x 0.007

total nuclear energy available $E_{tot} = (7 \times 10^{-4}\ M_{sun})c^2 \sim 1.3 \times 10^{44}$ J

Sun's total lifetime energy output = 6×10^{43} J

$E_{tot} \sim 2$ x the total energy Sun has emitted!!

→ Sun could shine for at least another 5×10^9 yr at its current luminosity

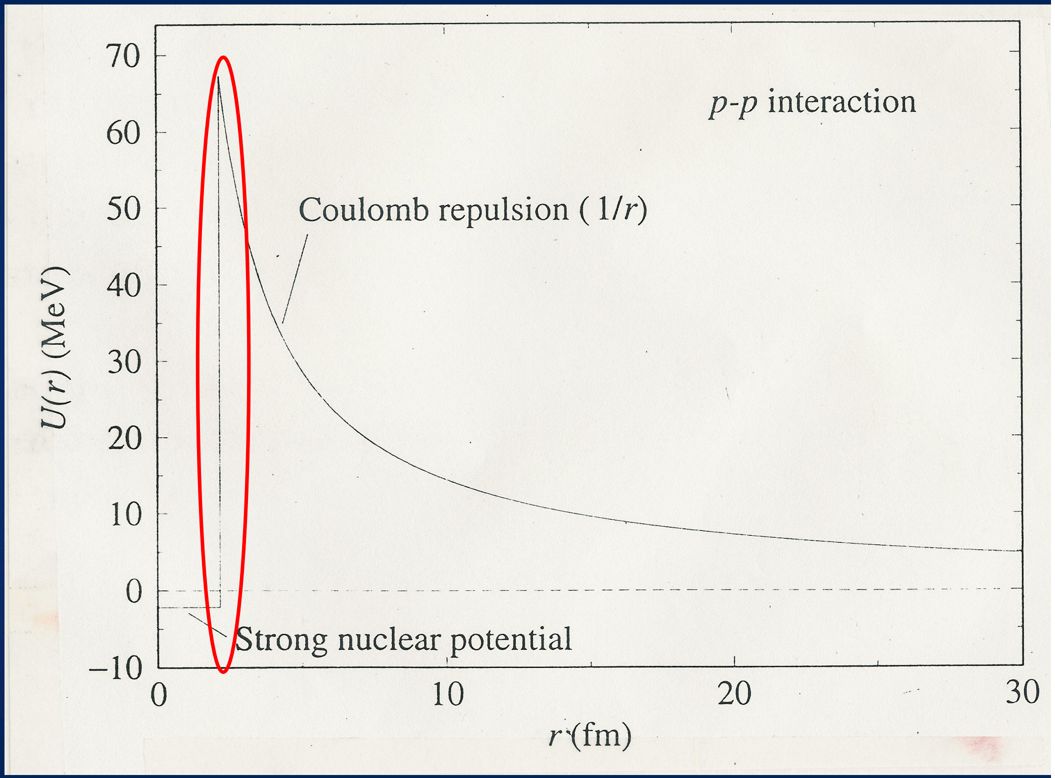
Hence $t_{nuclear}$ for Sun ~ 10^{10} yrs

Can Fusion Occur in Sun's Core?

Protons must overcome mutual electrostatic repulsion: Coulomb Barrier

Classical Approach:

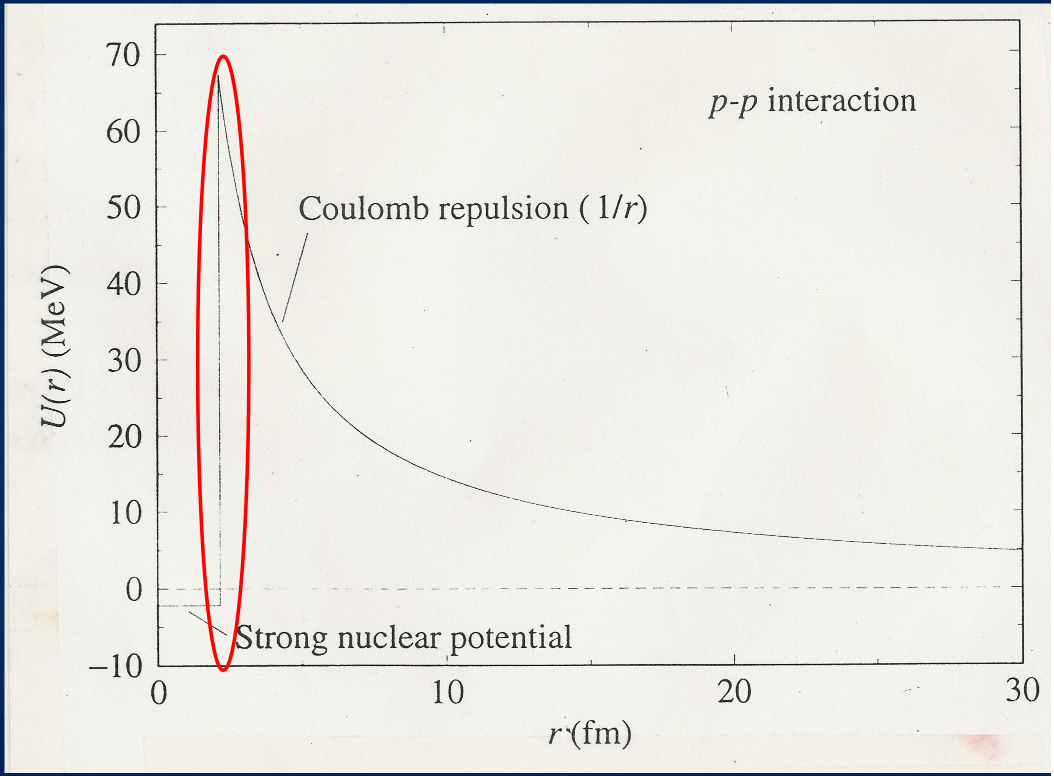
- E_{kin} proton > $E_{coulomb}$
- $E_{coulomb}$ normally written as $k[e_1e_2]/r$ with e's the charge on the protons and k Coulomb's Constant – but if SI units used the charge is defined so that k = 1 and is dimensionless – then e^2/r has units of energy with the value of e (SI) for a proton then $4x10^{-10}$. Hence for a separation between protons of 10^{-13} cm (10^{-15} m) the energy is $(4x10^{-10})^2/10^{-13}$ = $1.6x10^{-6}$ ergs = $1.6x10^{-13}$ Joules



Can Fusion Occur in Sun's Core?

Classical Approach:

- E_{kin} proton > $E_{coulomb}$
- $1/2\ m_p v^2 > e^2/r$ (cgs form)
- LHS is thermal gas energy, thus, $3/2\ kT > e^2/r$
- At $T_{central} \sim 1.6 \times 10^7$ K → $E_{kin} \sim 3.3 \times 10^{-16}$ J
- For successful fusion, r ~ 10^{-15} m (1 fm) →
  - $E_{coulomb} \sim 1.6 \times 10^{-13}$ J
  - $E_{kin} \sim 10^{-3}\ E_{coulomb}$



Classically, would need T~10^{10} to overcome Coulomb barrier

So, no Fusion??

Can Fusion Occur in Sun's Core?

Can we be helped by considering the distribution of energies? - not all protons will have just $E_{th} = 3/2\ kT$

Proton velocities are distributed according to the Maxwellian equation: $P(v) = 4\pi(m/2\pi kT)^{3/2}\ v^2\ e^{-mv^2/2kT}$

At the high energy tail of the Maxwellian distribution, the relative number of protons, with $E > 10^3 \times E_{th}$ is:

$N(E_{coulomb} \sim 10^3 \langle E_{thermal} \rangle)/N(\langle E_{thermal} \rangle) \sim e^{-\Delta E/kT} \sim e^{-1000} = 10^{-430}$!!

Number protons in Sun = $M_{sun}/M_H \sim 10^{57}$ so 1 in 10^{430} of these will have enough energy to overcome Coulomb barrier.

So again, no nuclear reactions??

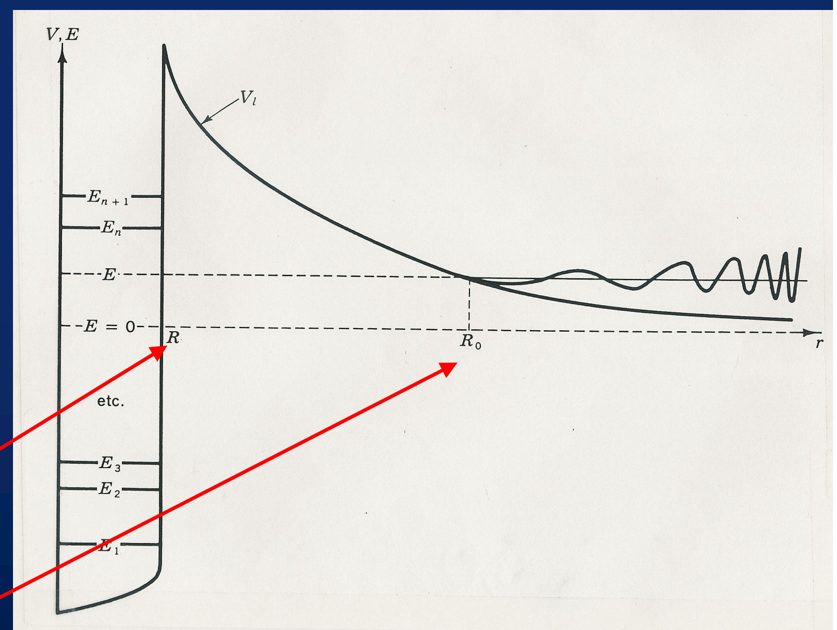
Can Fusion Occur in Sun's Core?

Quantum Approach:

- Heisenberg Uncertainty Principle - $\Delta p \Delta x > h/2\pi$
- If Δp small, then Δx may be large enough that protons have non-negligible probability of being located within 1 fm of another proton inside Coulomb barrier

Called Quantum Tunneling

- Particles have wavelength, ($\lambda = h/p$), associated with them (like photons) called de Broglie wavelength
- Proved in many experiments - for example diffraction electrons (wave phenomenon)
- Eg. λ free electron (3×10^6 m/s) = 0.242 nm (size atom)
- λ person (70 k gm) jogging at 3 m/s = 3×10^{-36} m (negligible - person won't diffract!)
- Increased wavelength reflects loss momentum.



Classically, the proton is reflected at R_0.

Quantum mechanically, there exists a finite probability for a proton to reach interaction radius, R.

Can Fusion Occur in Sun's Core?

Even with tunneling, are stars hot enough for nuclear reactions to proceed?

$\lambda = h/p$ is the wavelength associated with a massive particle (p = momentum)

In terms momentum, the kinetic energy of a proton is:

$$1/2\, m_p v^2 = p^2/2m_p$$

Assuming proton must be within 1 de Broglie λ of its target to tunnel set distance, r, of closest approach to λ, (where barrier height = original K.E. incoming particle) gives:

$$e^2/r = e^2/\lambda = p^2/2m_p = (h/\lambda)^2/2m_p$$

Solving for λ ($=h^2/2m_p e^2$) and substituting $r = \lambda$ into $3/2\, kT = e^2/r$, we get the QM estimate of the temperature required for a nuclear reaction to occur:

$T_{QM} = 4/3(e^4 m_p/kh^2)$ - putting in numbers

$T_{QM} = 10^7$ K which is comparable to T_{core}. Therefore, fusion is feasible at centre of Sun

Proton-Proton Chain

Basic particles involved in nuclear reactions

Particle	Symbol	Rest mass (kg)	Charge (e-)	Spin
photon	γ	0	0	1
neutrino	ν	>0	0	1/2
anti-neutrino	ν-	>0	0	1/2
electron	e-	9×10^{-31}	-1	1/2
positron	e+	9×10^{-31}	+1	1/2
proton (^1_1H)	p+	1.6×10^{-27}	+1	1/2
neutron	n	1.6×10^{-27}	0	1/2
deuteron	^2_1H	3.2×10^{-27}	+1	

Conservation Laws in nuclear reactions:

(1) mass-energy

(2) charge

(3) difference between number particles and anti-particles conserved - ie particle cannot be created from anti-particle or vice-versa but a pair can be formed or destroyed without violating this rule or #p + #n conserved

Proton-Proton Chain

Types of nuclear reactions

Beta Decay: $n \rightarrow p^+ + e^- + \nu^-$ proceeds spontaneously - also for neutron inside nucleus

eg $[Z\text{-}1, A] \rightarrow [Z, A] + e^- + \nu^-$ ($Z = \#\ p^+$, $N = \#\ n$, $A = Z+N$)

${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \nu^-$

Inverse Beta Decay: $p^+ + e^- \rightarrow n + \nu$

eg $[{}^{26}_{13}Al + e^+ \rightarrow {}^{26}_{12}Mg + \nu]$

(p^+, γ) process: ${}^{A}Z + p^+ \rightarrow {}^{A+1}(Z+1) + \gamma$

eg $[{}^{12}C + p^+ \rightarrow {}^{13}N + \gamma]$

(α, γ) process: α particle (${}^4He$) added to nucleus to make heavier particle $[{}^{A}Z + {}^{4}He \rightarrow {}^{A+4}(Z+2) + \gamma]$

eg. $[{}^{8}Be + {}^{4}He \rightarrow {}^{12}C + \gamma]$

Proton-Proton Chain

Since the Sun's mass consists mostly of H and He, we anticipate nuclear reactions involving these two elements

eg $p^+ + p^+ \rightarrow {}^2He$

$p^+ + {}^4He \rightarrow {}^5Li$

${}^4He + {}^4He \rightarrow {}^8Be$

But there is a problem here - what is it?

Proton-Proton Chain

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eg $p^+ + p^+ \rightarrow {}^2He$

$p^+ + {}^4He \rightarrow {}^5Li$

${}^4He + {}^4He \rightarrow {}^8Be$

But there is a problem here - what is it?

All these reactions produce unstable particles

eg $p^+ + p^+ \rightarrow {}^2He \rightarrow p^+ + p^+$

$p^+ + {}^4He \rightarrow {}^5Li \rightarrow p^+ + {}^4He$

${}^4He + {}^4He \rightarrow {}^8Be \rightarrow {}^4He + {}^4He$

So among lightest elements there are no two-particle exothermic reactions producing stable particles. We have to look to more peculiar reactions or those involving rarer particles

Proton-Proton Chain

Using considerations of:
- minimizing Coulomb barriers,
- crossections,
- and making sure conservation laws are obeyed

the nuclear reaction chain at right produces the energy observed in the Sun.

Hans Bethe 1967

The proton-proton chain PPI Branch

$$\text{STEP 1:}\quad p^+ + p^+ \rightarrow {}^2_1H + e^+ + \upsilon$$

Slowest step

Deuterium ($p^+ + n$) positron neutrino

proton → neutron via "inverse Beta decay"

$$p^+ \rightarrow n + e^+ + \upsilon_e$$

Weak nuclear force

$$\text{STEP 2:}\quad p^+ + {}^2_1H \rightarrow {}^3_2He + \gamma$$

Low-mass isotope of He ($2p+ + n$)

Gamma ray photon

Repeat steps 1 & 2 to produce 2 ${}^3_2He$ nuclei

$$\text{STEP 3:}\quad {}^3_2He + {}^3_2He \rightarrow {}^4_2He + p^+ + p^+$$

NET RESULT

Total of ~~6~~ 4 protons → 1 He nucleus + ~~2~~ protons + $2e^+$ $m_{e+} \sim 5 \times 10^{-4}\, m_{p+}$ + neutrinos + energy

Very low mass $m_{\nu e} \leq 10^{-5}\, m_{p+}$

Proton-Proton Chain

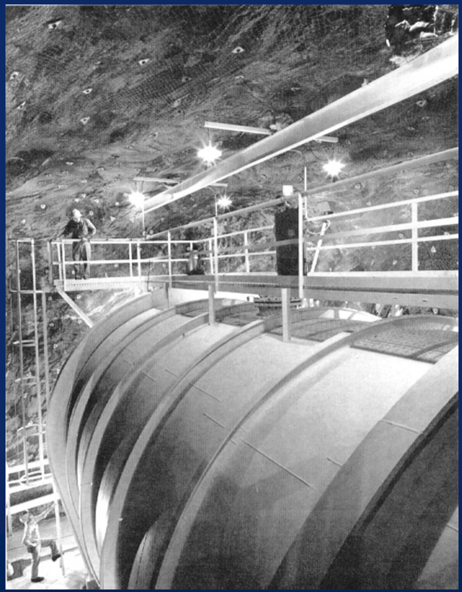
Summary of proton-proton chain:

- $6p^+ \rightarrow {}^4He + 2p^+ + 2e^+ + 2\nu + 2\gamma$ or $4p^+ \rightarrow {}^4He + 2e^+ + 2\nu + 2\gamma$
- Mass difference between $4p^+$ and 1 ^{4}He = 26.7 Mev / 6.242 x 10^{18} ev/J = 4.2 x 10^{-12} J
- ~3% of this energy (0.8 Mev) is carried off by neutrinos and does not contribute to the Sun's luminosity
- 2 e^+ immediately annihilate with 2 e^- and add 4 x 0.511 Mev (= 2.04 Mev)
- So, the total energy available for the Sun's luminosity per ^{4}He formed is: (26.7 - 0.8 +2.04) Mev = 27.9 Mev = 4.3 x 10^{-12} J
- # ^{4}He formed/sec = L_{sun}/4.3 x 10^{-12}J = 3.9 x10^{26} J/s / 4.3x10^{-12} J = 9.0 x 10^{37} ^{4}He/s
- Increase of ^{4}He mass/time = dm_{He}/dt = 9.0 x 10^{37} ^{4}He/s x 6.68 x 10^{-27} kg/^{4}He = 6.0 x 10^{11} kg ^{4}He /s

After 10^{10} years (3 x 10^{17} sec), M(^{4}He) = 1.8 x 10^{29} kg = 10% mass Sun

Check on Proton-Proton Chain

- # ^{4}He formed/sec = 9.0 x 10^{37} ^{4}He/s and each ^{4}He is accompanied by production of 2 ν → 2.0 x 10^{38} ν /s
- Flux neutrinos at Earth is 2 x $10^{38}/4\pi R_{ES}^2$ = 7 x 10^{14} ν m^{-2} s^{-1}
- ν can be capture at Earth by $\nu + {}^{37}Cl \rightarrow {}^{37}Ar + e$



- Prediction was 5.8/day – detected 2.1/day
- Concern that T Sun was thus lower than model
- Answer was 3 flavors of ν change one to other inside Sun – new physics – finally resolved in SNO which was not sensitive to ν types – detected expected # of ν.

Ray Davis 2002

Art McDonald 2015

Proton-Proton Chain

The previous nuclear reaction chain (PPI) is not the only way to convert H into He.

eg last step:

$^3He + {}^3He \rightarrow {}^4He + 2p^+$

can proceed differently if there is appreciable ^4He present

A second branch (PPII)

In solar centre, 69% of ^3_2He combines with another ^3_2He, but 31% can fuse with ^4_2He

$$^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$$

$$^7_4Be + e^- \rightarrow {}^7_3Li + \upsilon_e$$

$$^7_3Li + p^+ \rightarrow {}^4_2He + {}^4_2He$$

A third branch (PPIII)

In the Sun's core, a ^7_4Be nucleus can capture a p^+ instead of an e^- about 0.3% of the time.

$$^7_4Be + p^+ \rightarrow {}^8_5B + \gamma$$

Very unstable $\longrightarrow$ $^8_5B \rightarrow {}^8_4Be + e^+ + \upsilon_e$

$$^8_4Be \rightarrow {}^4_2He + {}^4_2He$$

PPI + PPII + PPIII operate simultaneously

Proton-Proton Chain: Summary

$$p^+ + p^+ \rightarrow {}^2_1H + e^+ + \nu_e$$

$${}^2_1H + p^+ \rightarrow {}^3_2He + \gamma$$

69%

31%

$${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2p^+$$

(PPI)

$${}^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$$

99.7%

0.3%

$${}^7_4Be + e^- \rightarrow {}^7_3Li + \nu_e$$

$${}^7_3Li + p^+ \rightarrow 2\,{}^4_2He$$

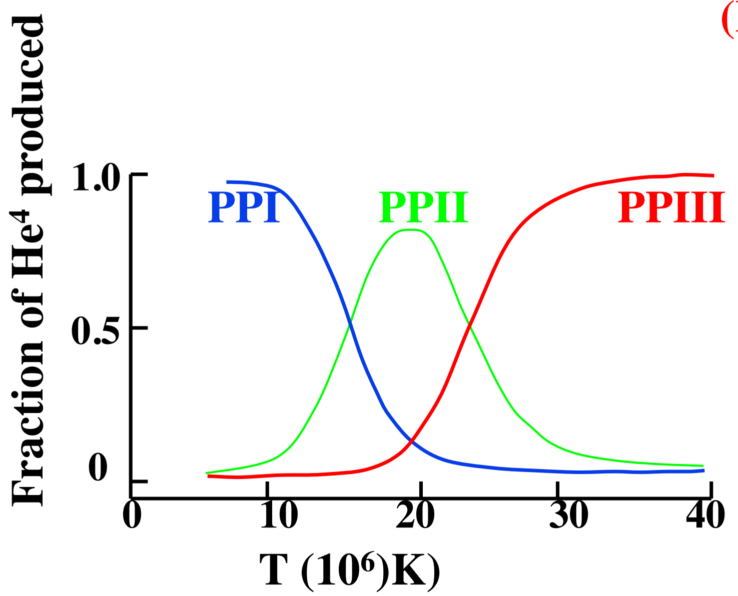
(PPII)

$${}^7_4Be + p^+ \rightarrow {}^8_5B + \gamma$$

$${}^8_5B \rightarrow {}^8_4Be + e^+ + \nu_e$$

$${}^8_4Be \rightarrow 2\,{}^4_2He$$

(PPIII)



Alternate Fusion Reactions H $\rightarrow$ ^{4}He

CNO Cycle (or bi-cycle)

- First step in PP chain has very low reaction rate (8×10^9 years - weak interaction).
- ^{12}C can act as a catalyst in fusion reaction.
- Net result:
  $^{12}C + 4p^+ \rightarrow {}^{12}C + {}^4He + 2e^+ + 2\nu + 3\gamma$
- Note: ^{12}C neither created nor destroyed. Also isotopes of N & O are temporarily produced.
- CNO cycle dominates over PP chain if $T_{core} > 1.8 \times 10^7$ (slightly hotter than Sun)

THE CNO BI-CYCLE

($T < 10^8$ K)

$C^{12} + H^1 \longrightarrow N^{13} + \gamma$	10^6 yr
$N^{13} \longrightarrow C^{13} + e^+ + \nu$	14 min
$C^{13} + H^1 \longrightarrow N^{14} + \gamma$	3×10^5 yr
$N^{14} + H^1 \longrightarrow O^{15} + \gamma$	3×10^8 yr
$O^{15} \longrightarrow N^{15} + e^+ + \nu$	82 s
$N^{15} + H^1 \longrightarrow C^{12} + He^4$	10^4 yr

$\boxed{\text{or } (\sim 4 \times 10^{-4})}$ **X as frequently**

$$\left.\begin{array}{l} N^{15} + H^1 \longrightarrow O^{16} + \gamma \\ O^{16} + H^1 \longrightarrow F^{17} + \gamma \\ F^{17} \longrightarrow O^{17} + e^+ + \nu \\ O^{17} + H^1 \longrightarrow N^{14} + He^4 \end{array}\right\}$$

Once for every 2500 instances of Cycle 1

Helium Burning

No stable nuclei with A=8 therefore He burning proceeds in 2 steps

1. $^{4}He + ^{4}He \leftrightarrow ^{8}Be$ – builds up small concentration ^{8}Be
2. $^{8}Be + ^{4}He \rightarrow ^{12}C$ – possible at $T=10^{8}$ K

Net effect is $^{3}He4 \rightarrow ^{12}C + \gamma$

When sufficient ^{12}C is produced we can have
$^{12}C + ^{4}He \rightarrow ^{16}O$

Carbon Burning and Beyond

$^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$ or $^{20}Ne + {}^{4}He$ or $^{23}Na + p$

Can be followed by

$^{20}Ne + \gamma \rightarrow {}^{16}O + {}^{4}He$ or $^{20}Ne + {}^{4}He \rightarrow {}^{24}Mg$

Oxygen burning

$^{16}O + {}^{16}O \rightarrow {}^{32}S + \gamma$ or $^{28}Si + {}^{4}He$ or $^{31}P + p$ (many other channels)

Si burning does not occur directly but in a complex way by photodisintegrations (alpha, γ) and alpha capture $\rightarrow$ core ^{56}Fe as it is most tightly bound of all the nuclei

eg $^{28}Si + {}^{4}He \rightarrow {}^{32}S$, $^{32}S + {}^{4}He \rightarrow {}^{36}Ar$, $^{36}Ar + {}^{4}He \rightarrow {}^{40}Ca$,

$^{40}Ca + {}^{4}He \rightarrow {}^{44}Ti$, $^{44}Ti + {}^{4}He \rightarrow {}^{48}Cr$, $^{48}Cr + {}^{4}He \rightarrow {}^{52}Fe$

$^{52}Fe + {}^{4}He \rightarrow {}^{56}Ni$ (these kinds of reactions produce many of the heavy elements up to Fe that we see in the Universe)

Summary Other Fusion Reactions

Fusion of	into	at $T_{central}$	Ref.
^{4}He triple α process	Be, C	$\sim 10^8$ K	p. 368
^{12}C	O, Ne, Na, Mg	6×10^8 K	p. 409
^{16}O	Mg, Si, S, etc.	$\sim 10^9$ K	p. 409

We believe Universe began with only H, He, (Li, Be)

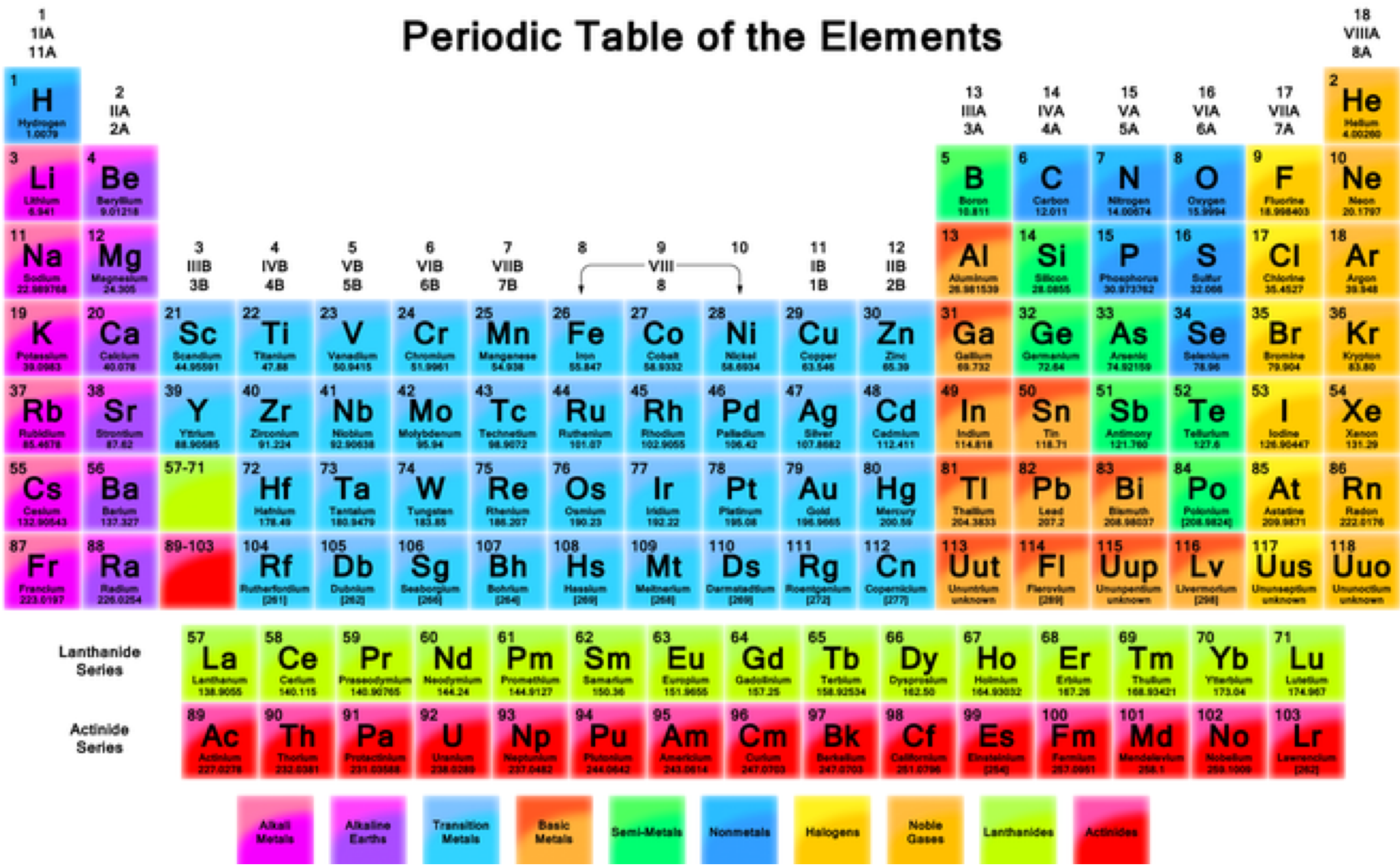
All other elements created in core of stars (stellar nucleosynthesis)

We are all made of "STARSTUFF"

Elements beyond iron: s-process

Where do the elements beyond Fe come from (eg gold = $^{197}Au_{79}$?)

Coulomb barriers too high for expected stellar central temperatures for reactions like $^{56}Fe + ^{56}Fe \rightarrow$?



Elements beyond iron: s-process

Where do the elements beyond Fe come from (eg gold = $^{197}Au_{79}$?)
Coulomb barriers too high for expected stellar central temperatures for reactions like $^{56}Fe + ^{56}Fe \rightarrow$?

Solved by understanding of the s-process (slow neutron capture) where a heavy nucleus captures a neutron to make a heavier isotope. If the isotope is stable, it can capture another neutron until the nucleus becomes unstable at which point it can β-decay to produce an element of the next highest atomic number. These are NOT charged particle reactions so problem of Coulomb barriers is not an issue. Fe is the seed of the s-process and a source of neutrons is required. The stars wherein this process occurs are luminous giant stars.

Elements beyond iron: s-process

Neutron sources: $^{13}C + {^4He} \rightarrow {^{16}O} + n$ or $^{22}Ne + {^4He} \rightarrow {^{25}Mg} + n$

s-process example: $^{56}Fe + n \rightarrow {^{57}Fe}$, capture 2 more n $\rightarrow$ ^{59}Fe, Fe 56-58 are all stable but ^{59}Fe unstable (half life 44.5 days).

If flux n not too high, and don't capture another n within 44.5 days, ^{59}Fe can decay to ^{59}Co by β-decay ($n \rightarrow p + e^- + \nu$) and we now have an element one element up in periodic table.

The process can continue to form higher and higher mass elements and thus we can populate the high mass end of the periodic table.

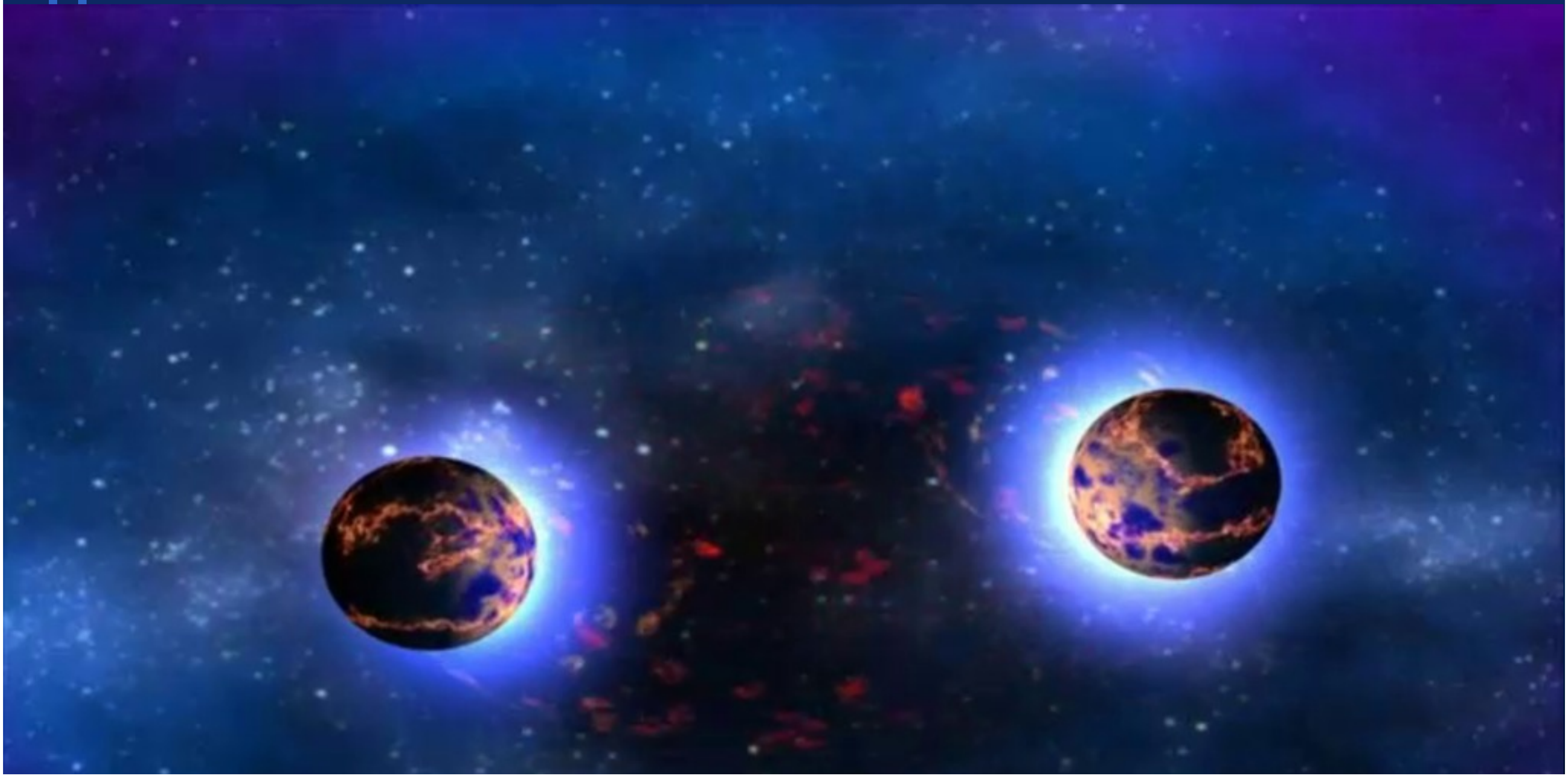
Elements beyond iron: r-process

In contrast to the scenario just described, imagine that the flux of neutrons is high and the time between neutron captures is small compared to the half life of the isotopes under consideration, super-rich isotopes can then be formed. When the neutron flux stops, these super-neutron-rich isotopes will undergo a series of β-decays until a stable isotope is reached. As you might imagine, this scenario can take place during a supernova explosion (more about this soon).

For example, here is a scenario to make GOLD

$^{195}Y_{70} + n \rightarrow {}^{196}Y_{70}$, ^{196}Y is unstable and decays $^{196}Y \rightarrow {}^{196}Lu_{71} + e^- + \nu$
Lu (Lutecium) then captures a n to make ^{197}Lu which is unstable and it decays to $^{197}Hf_{72}$. After multiple n captures and decays we end up with $^{197}Pl_{78}$ which then β-decays $^{197}Pl_{78} \rightarrow {}^{197}Au_{79} + e^- + v$. $^{197}Au_{79}$ is the one stable isotope of GOLD.

Elements beyond iron: r-process: Collision Neutron Stars as Neutron Source



Origin of the Elements