

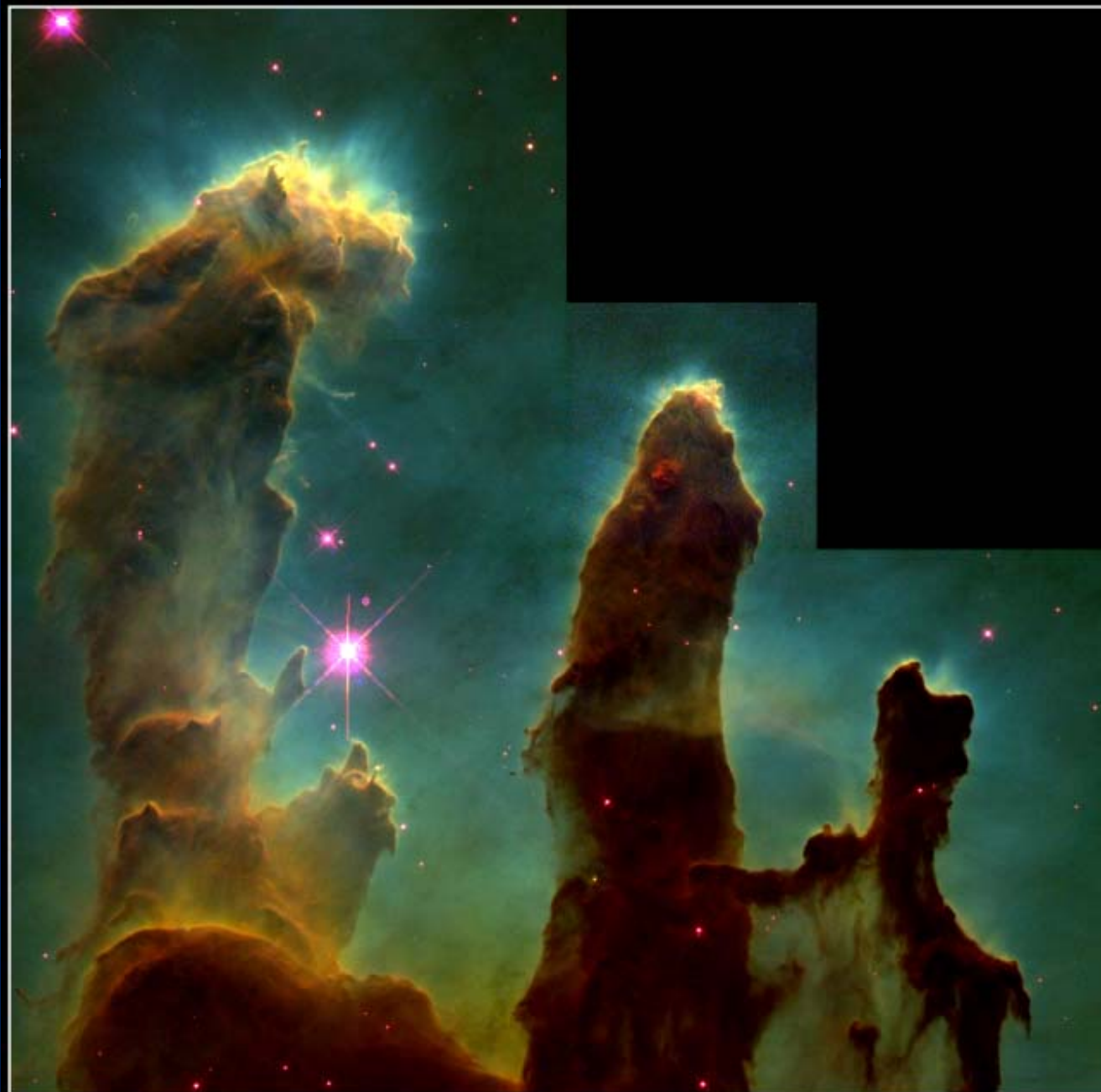


Stellar Evolution: Parts of Chapter 17, 18

Stars lose energy - hence they must evolve.

There are different time scales on which stars (Sun) evolve:

- (1) Nuclear time scale $t_{\text{nuclear}} = (0.007 \times 0.1 \times Mc^2)/L \sim 10^{10} \text{ yr}$
- (2) Thermal time scale $t_{\text{thermal}} = (0.5GM^2/R)/L \sim 2 \times 10^7 \text{ yr}$
- (3) Dynamical time scale $t_{\text{dynamical}} = \text{remove pressure - free fall time from surface to centre} = (2R^3/GM)^{1/2} \sim 0.6 \text{ hour!}$



Gaseous Pillars · M16

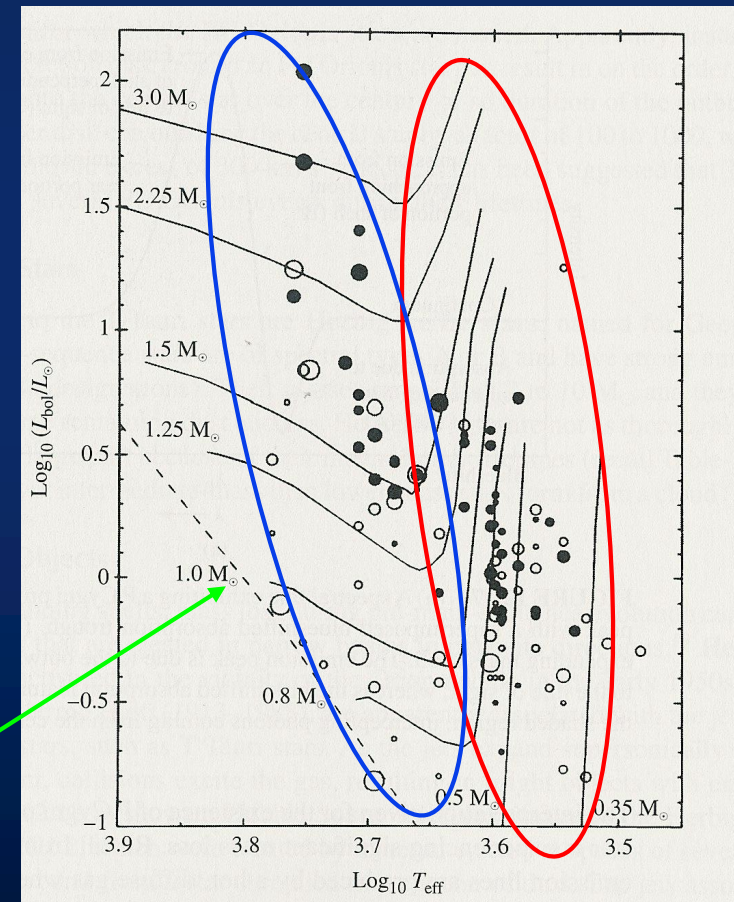
HST · WFPC2

PRC95-44a · ST ScI OPO · November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA



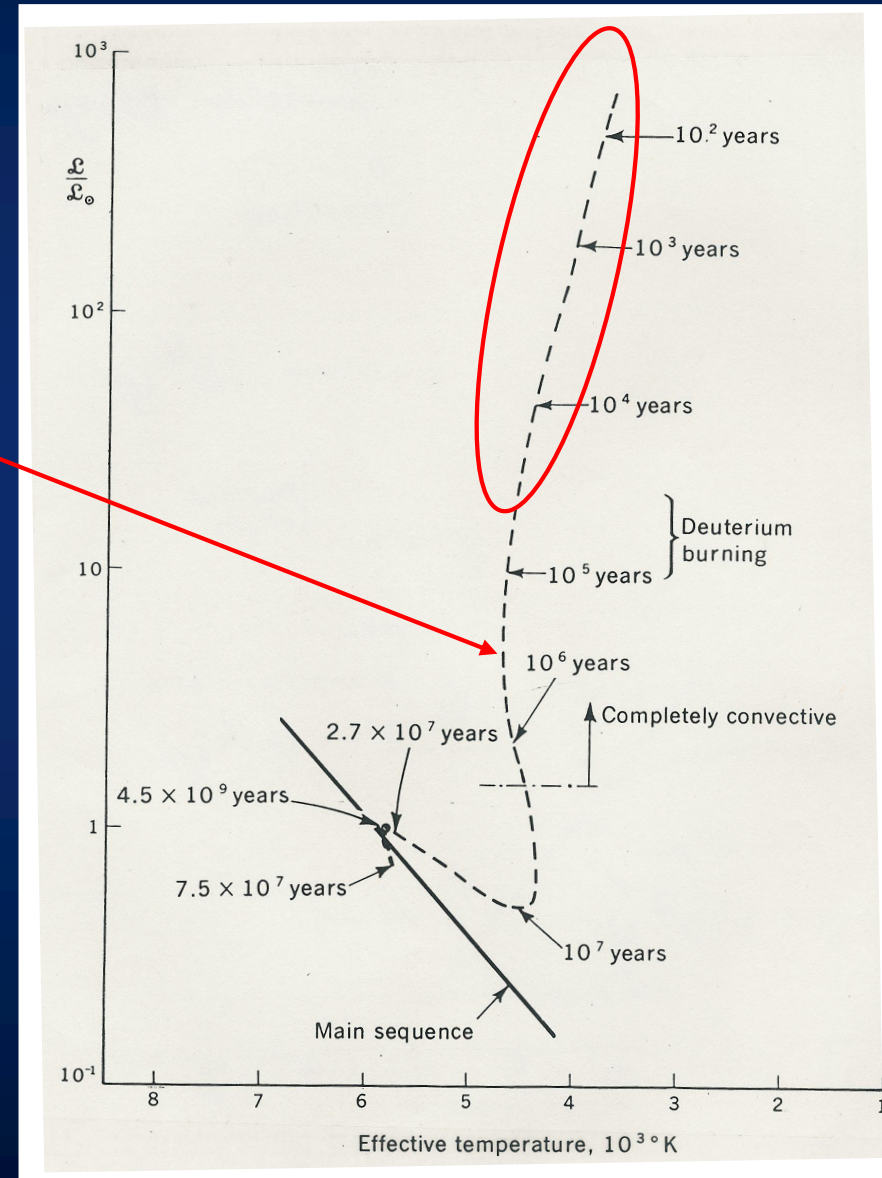
Stellar Evolution (Formation)

- In the collapsing cloud, energy is released by gravitational contraction. The star shrinks and begins to heat up.
- Initially, radiation passes right through star so that T_{eff} stays nearly constant. Therefore, as R drops so does L ($L = 4\pi R^2 \sigma T_{\text{eff}}^4$).
- Contraction is originally on the dynamical timescale. Eventually density increases and so does opacity. A larger fraction of energy is transformed into heat. Pressure increases, raising the resistance to the collapse so the contraction slows. T_{eff} rises with little change in R . Therefore, star moves to higher L .
- Eventually, the star's core is hot enough for nuclear reactions. Star reaches hydrostatic equilibrium, **on main sequence**.

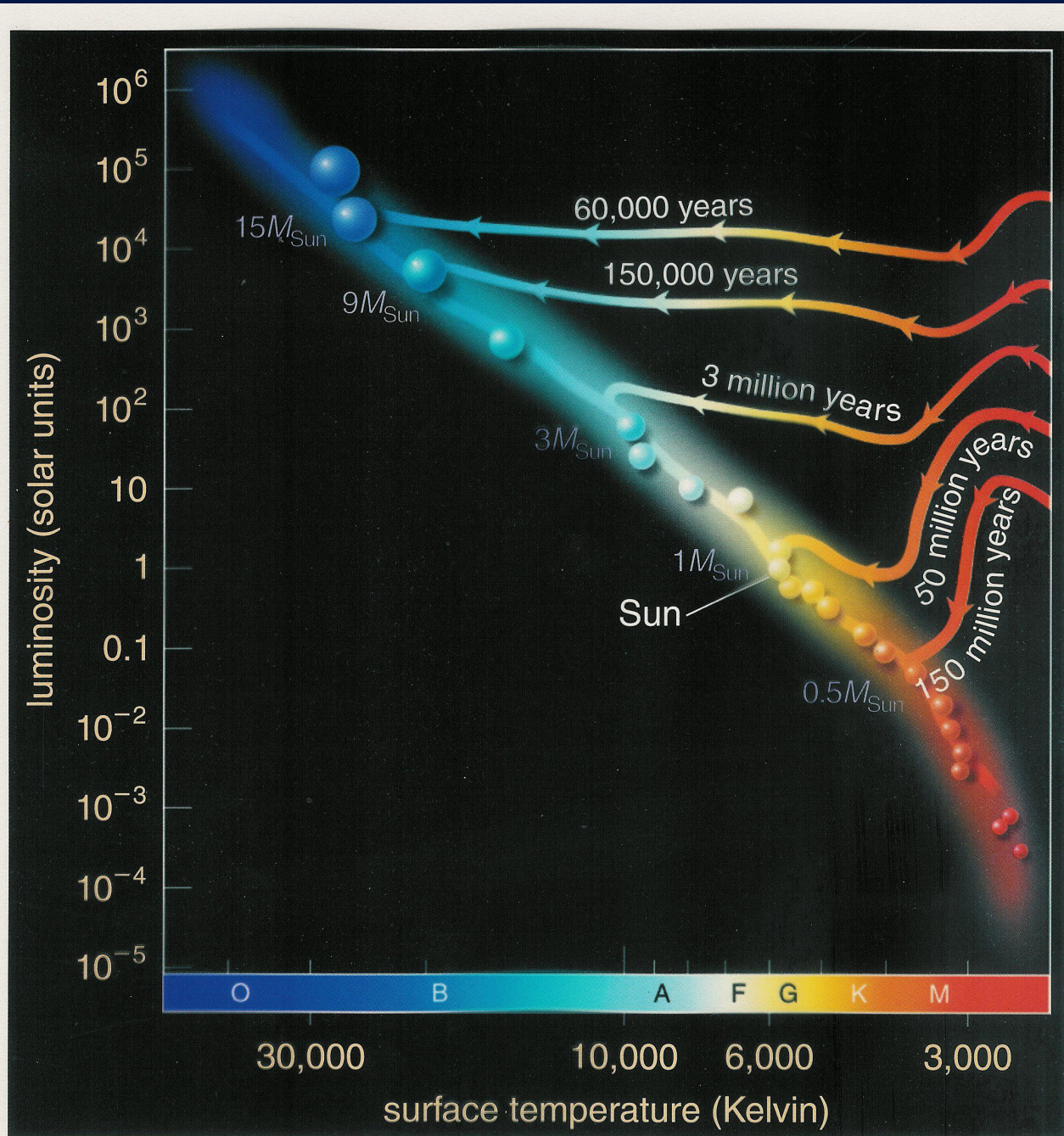


★ Stellar Evolution (Pre Main Seq.)

- For first few $\times 10^4$ years, the star contracts from the interstellar medium and radiates away gravitational energy.
- By 10^6 years, the first nuclear reactions involving light elements start:
eg $p + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma$
- ${}^2\text{H}$ (from Big Bang) has very low abundance so the energy released from this reaction just slows the contraction rate a bit.
- Other reactions:
 ${}^6\text{Li} + p \rightarrow {}^3\text{He} + {}^4\text{He}$
 ${}^7\text{Li} + p \rightarrow 2 {}^4\text{He}$
 ${}^{11}\text{B} + p \rightarrow 3 {}^4\text{He}$
- All abundances are low, so the contraction rate is minimally affected.



★ Stellar Evolution (Pre Main Seq.)



Young Stars and Planet Formation





Stellar Evolution (Main Sequence)

- Life on Main Sequence (§ 17.2)
- Core H burning and hydrostatic equilibrium is the longest lived stage in a star's life. **HOW LONG DOES IT LAST?**
- Mass - Luminosity relation $L \propto M^{3.5}$
- Energy produced by fusion $E = \Delta M c^2$
 $\Delta M = \text{mass destroyed} = 0.007 \times \text{mass H fused (inner 10\% of M)}$

so $E \propto M$
- During main sequence stage, $L \sim \text{constant}$, thus
 $L = \text{energy from fusion} / \text{MS lifetime} = E/t_{\text{ms}}$
or $t_{\text{ms}} = E/L = M/M^{3.5} \rightarrow t_{\text{ms}} = M^{-2.5}$
- Thus, massive stars have shorter main sequence lifetimes than small mass stars.
- In terms of Sun's lifetime: $t_{\text{ms}}/t_{\text{ms sun}} = (M/M_{\text{sun}})^{-2.5}$



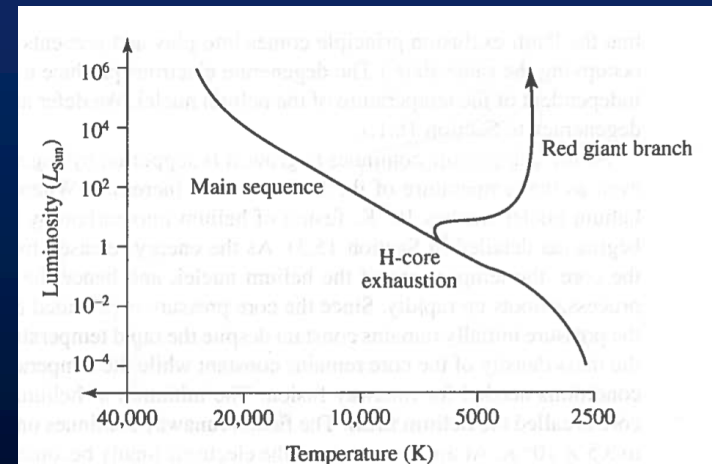
Stellar Evolution (Main Sequence)

M/M_{sun}	L/L_{sun}	t/t_{sun}	t (yr)
0.1	3×10^{-4}	300	3×10^{12} (> age universe!!)
1	1	1	10^{10}
5	~ 300	0.02	2×10^8
10	~ 3000	0.003	3×10^7
30	1.5×10^5	2×10^{-4}	2×10^6
100	10^7	10^{-5}	10^5 (100,000 yrs!)

- Low mass stars live for very long times
- High mass stars are short-lived

Evolution Off Main Sequence

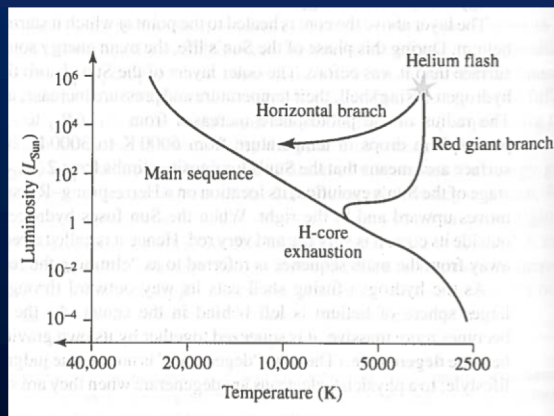
1. On MS composition changes slowly μ increases from 0.6 to 1.32
2. Since $P \propto (1/\mu) \rho T$ and $1/\mu$ decreases either one or both of ρ and T increases at centre in order to maintain Hydrostatic Equilibrium
3. Increases Energy Generation so L increases: $0.7 L_{\text{sun}}$ 4.6 Gyr ago, L_{sun} now, $2.2 L_{\text{sun}}$ in 6 Gyr causes runaway greenhouse on Earth - like Venus.
4. The way to increase ρ or T is to have the core contract $E_{\text{grav}} \rightarrow E_{\text{thermal}}$
5. Shell above core heats and starts H-burning $\text{H} \rightarrow \text{He}$
6. Outer atmosphere absorbs this and work is done on it and it expands
7. Radius expands by factor ~ 100 , photospheric T drops $6000\text{K} \rightarrow 3000\text{K}$, inner planets destroyed, Earth way too hot for life
8. Since $L \propto R^2 T^4$, L increases by factor 1000
9. T goes down and L goes up \rightarrow star moves to upper right in HR Diagram,
8. Star now in **Red Giant** phase of evolution





Red Giant Evolution

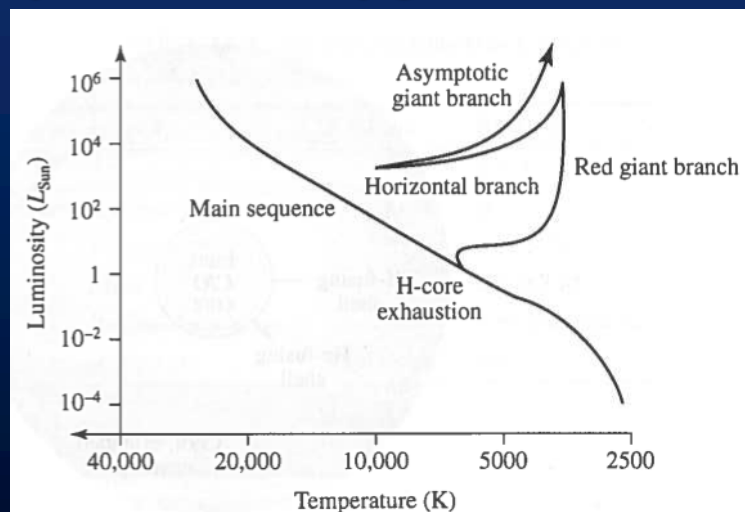
- Core Sun depleted of H, initially no more nuclear energy to heat the gas. Gravity will win over gas pressure and the core will contract and heat up.
- Gravitational potential energy of the core will be converted to heat as it shrinks. Hydrogen burns in a shell surrounding the core.
- HELIUM CORE BECOMES DEGENERATE
- At tip RGB helium begins to burn explosively as it is degenerate (helium flash)





Horizontal and Asymptotic Giant Branch Evolution (AGB)

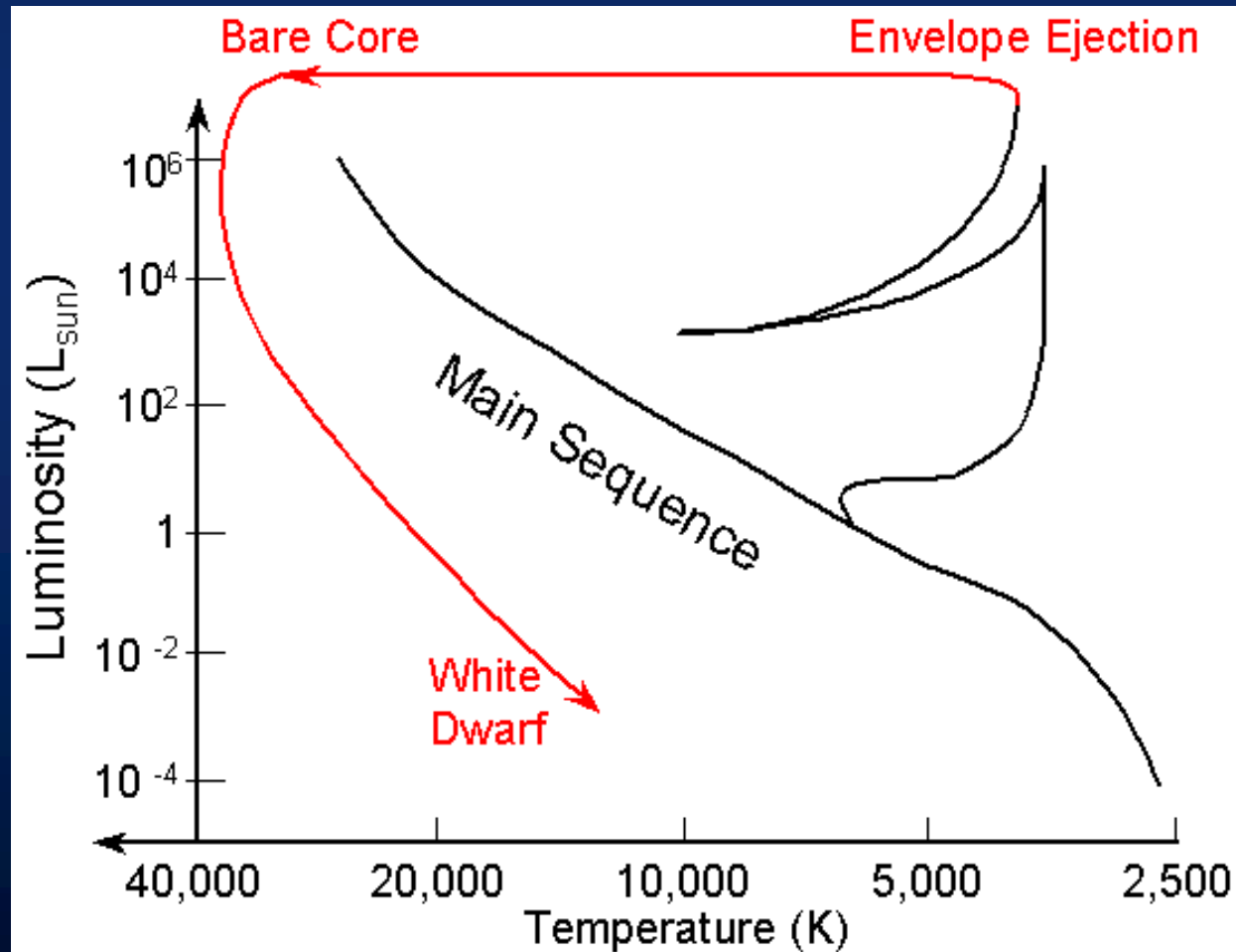
- After He flash star evolves to Horizontal Branch at higher T_{eff} and lower Luminosity (H-burning shell major energy source and it is pushed outward to cooler temperatures when He core ignites)
- Lifetime on HB $\sim 10^8$ years (He eventually exhausted)
- Star evolves up AGB – 3 energy sources (contracting core and 2 burning shells (see Fig 17.5))
- Very large and luminous – unstable – pulsates – ejects outer atmosphere – only carbon-oxygen core remains





Post AGB Evolution

- Eventually, the surface gas will have enough kinetic energy to escape core's gravity (to form a planetary nebula), leaving behind a “bare core” of C and O → WHITE DWARF.



Stellar Evolution (Death of the Sun)

Planetary nebulae





Stellar Evolution (Death of the Sun)

- When nuclear fuel is exhausted, the core will contract again. Low mass stars never get hot enough for nuclear reactions beyond formation C, O.
- Core will shrink and electrons in core will be forced together.
- Effects of Uncertainty Principle come into play when matter is compressed $\Delta x \Delta p \sim \hbar$ so there is zero-point energy (even at $T = 0$ K)



Stellar Evolution (Death of the Sun)

➤ $E_{\text{deg}} = p^2/2m = \hbar^2/2m(\Delta x)^2$ which increases as size decreases

$$P_{\text{deg}} = E_{\text{deg}}/V$$

➤ So for N particles in volume V , $P_{\text{deg}} = (N/V)E_{\text{deg}}$ where particle density $n = N/V$, $\Delta x = (V/N)^{1/3}$

➤ Hence $P_{\text{deg}} = (N/V)(p^2/2m) =$

$$(N/V)(\hbar^2/2m) (N/V)^{2/3} =$$

$$(\hbar^2/2m)n^{5/3} \text{ so } P_{\text{deg}} \propto \rho^{5/3}$$



Stellar Evolution (Death of the Sun)

- The pressure of the degenerate electrons can provide a pressure force **even at 0 K**.
- $P_{\text{deg}} \propto \rho^{5/3}$ (independent of T). NB P_{deg} not $\propto \rho T$ - perfect gas law breaks down.
- Recall that $P \propto M^2/R^4$ and $\rho \propto M/R^3$. With $P \propto \rho^{5/3}$ we get:
$$M^2/R^4 \propto (M/R^3)^{5/3}$$
$$\rightarrow R \propto M^{-1/3} \text{ or}$$
$$M \propto 1/V \text{ (Note how counter-intuitive this relation is).}$$



Stellar Evolution (Death of the Sun)

White Dwarfs (Sirius B for example)

$$\rightarrow \rho_{\text{wd}} = (M_{\text{wd}}/M_{\text{sun}})/(R_{\text{wd}}/R_{\text{sun}})^3 \rho_{\text{sun}} =$$
$$(0.96)/(0.0084)^3 \rho_{\text{sun}} =$$

$$1.6 \times 10^6 \rho_{\text{sun}} = 2.2 \times 10^9 \text{ kg m}^{-3}$$

(10 tons per teaspoon!)

$$\rightarrow 10^5 \text{ K} > T_{\text{eff}} > 3000 \text{ K}$$

$$\rightarrow P_{\text{c wd}} = (M_{\text{wd}}/M_{\text{sun}})^2/(R_{\text{wd}}/R_{\text{sun}})^4 P_{\text{c sun}} =$$
$$(0.96)^2/(0.0084)^4 P_{\text{c sun}} = 2 \times 10^8 P_{\text{c sun}} =$$

$$5 \times 10^{24} \text{ Pa (Earth } 10^5 \text{ Pa} = 1 \text{ Atmosphere)}$$

★ Upper Mass Limit to WD (Chandrasekhar Limit)

If WD mass increases, since $M \propto 1/V$ radius WD decreases

Degenerate electrons must move faster to provide enough pressure to support the star: $v \rightarrow c$ (speed light) ie electrons highly relativistic

When non-relativistic $\Delta x \Delta p = \hbar$ thus $\Delta v = \Delta p / m_e$
 $= \hbar n_e^{1/3} / m_e$

If electrons relativistic $E_{\text{rel}} = pc$, $E_{\text{rel}} = (\Delta p)c =$
 $\hbar n_e^{1/3} c$

★ Upper Mass Limit to WD (Chandrasekhar Limit)

$$\text{Now } P = E/\text{vol} = \hbar n_e^{1/3} c / (\Delta x)^3 = \hbar n_e^{1/3} c / (n_e)^{-1} = \hbar c n_e^{4/3}$$

Compare this with non-relativistic $P \propto n_e^{5/3}$

$$\text{Recall } P = GM^2/R^4 = \hbar c (\rho/m_H)^{4/3} = (\hbar c/m_H^{4/3})(M^{4/3})/R^4$$

Note that R^4 cancels so we just solve for

$$M = (\hbar c/G)^{3/2} m_H^{-2}$$

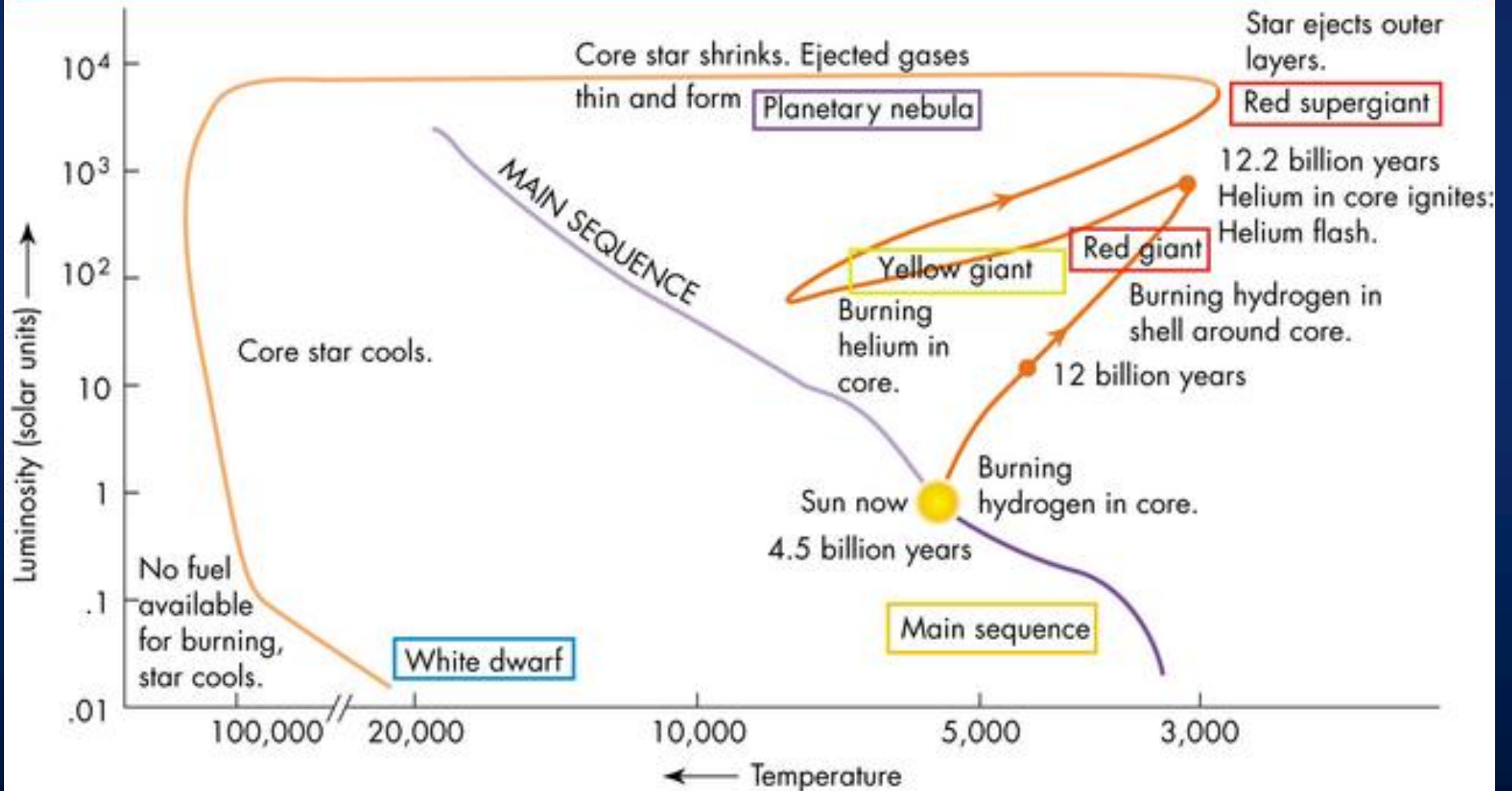
$$\begin{aligned} \text{Put in numbers } (\hbar &= 1.054 \times 10^{-34} \text{ J s}, \\ c &= 2.998 \times 10^8 \text{ ms}^{-1}, G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \\ m_H &= 1.673 \times 10^{-27} \text{ Kg} \end{aligned}$$

Which gives 1.8 Solar Masses, if $M > 1.8$ collapses
→ Supernova?

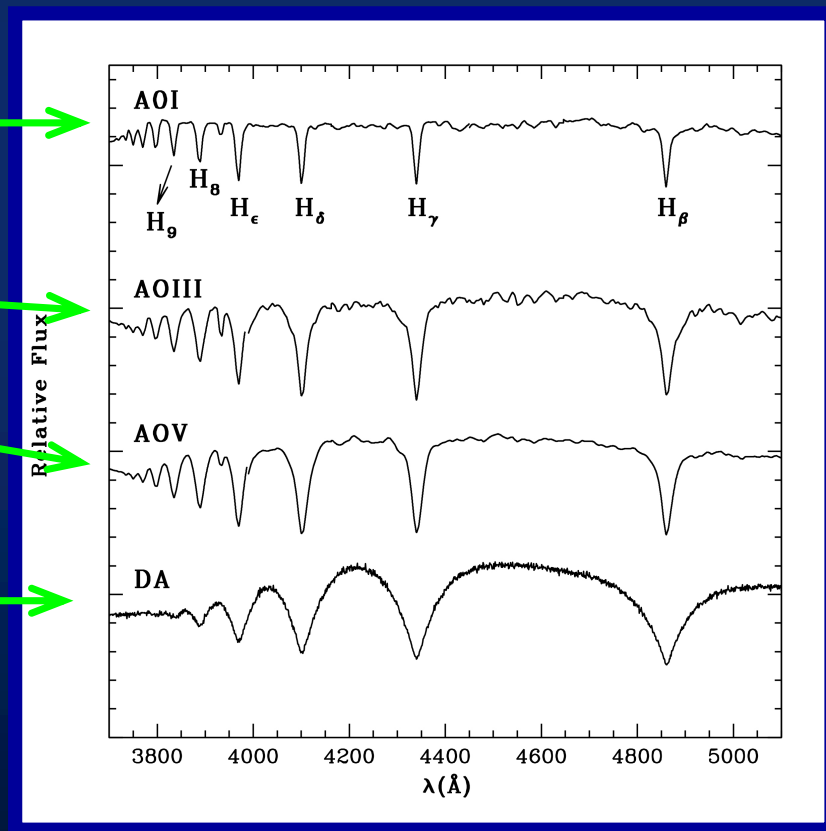
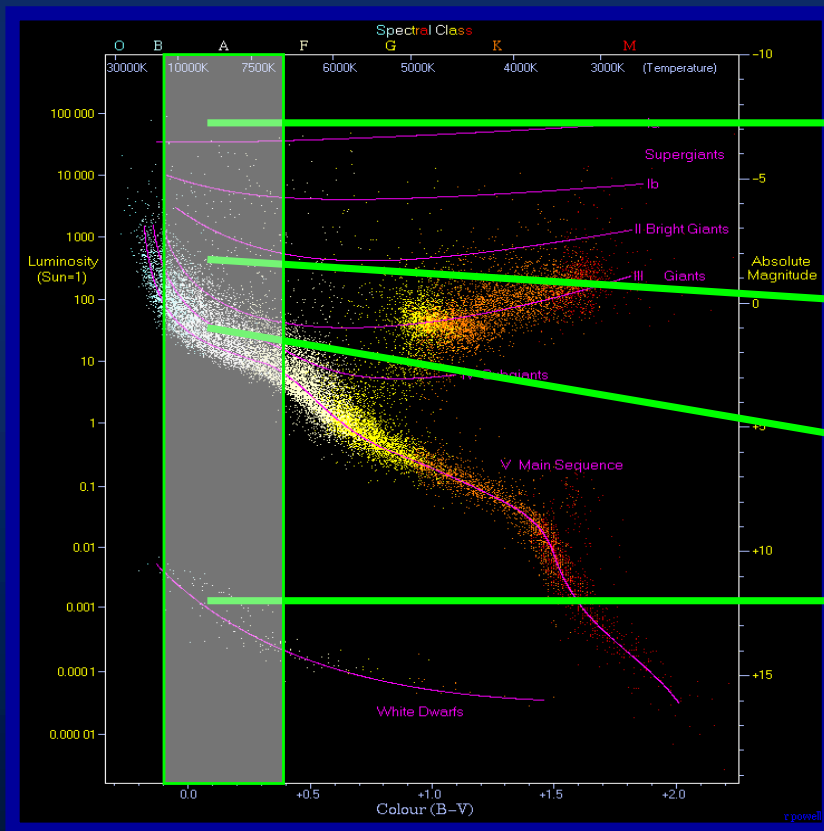


Stellar Evolution Summary

	~9 billion yrs	~1 billion yrs	~100 million yrs	~10,000 yrs	
Time spent as	Main sequence	Red giant	Yellow giant	Planetary nebula	White dwarf
Sun's age	4.5 billion yrs (now)	12.2 billion yrs	12.3 billion yrs	12.3305 billion yrs	12.3306 billion yrs

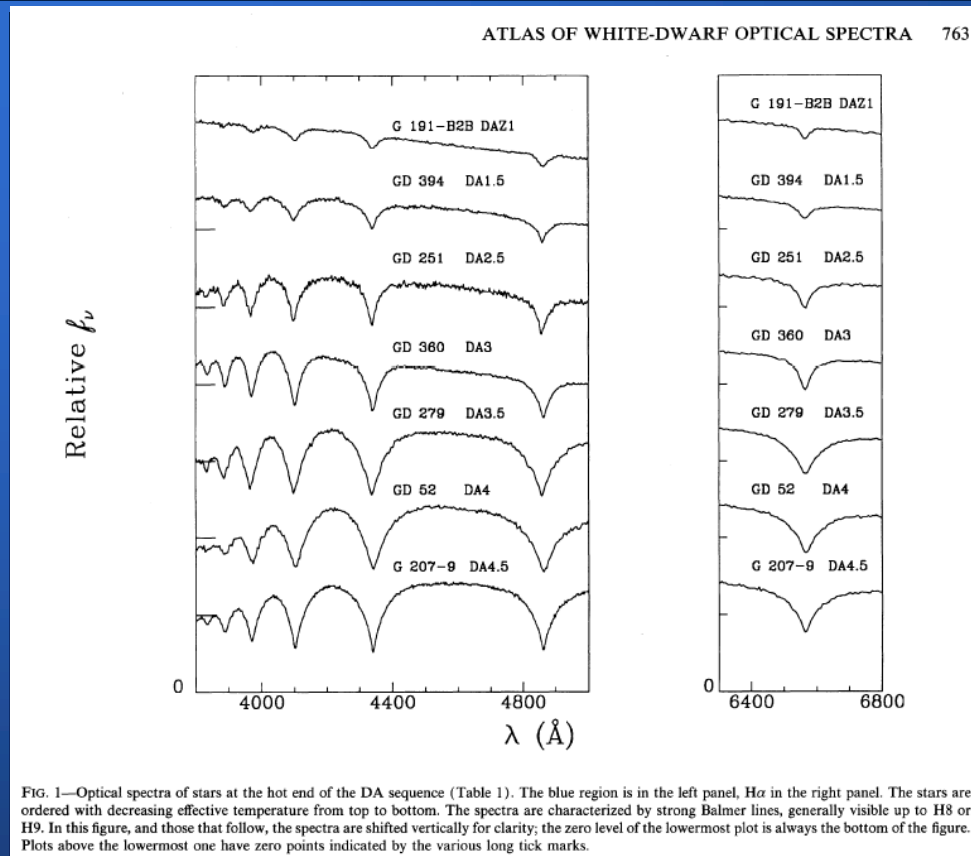


Spectrum of White Dwarfs

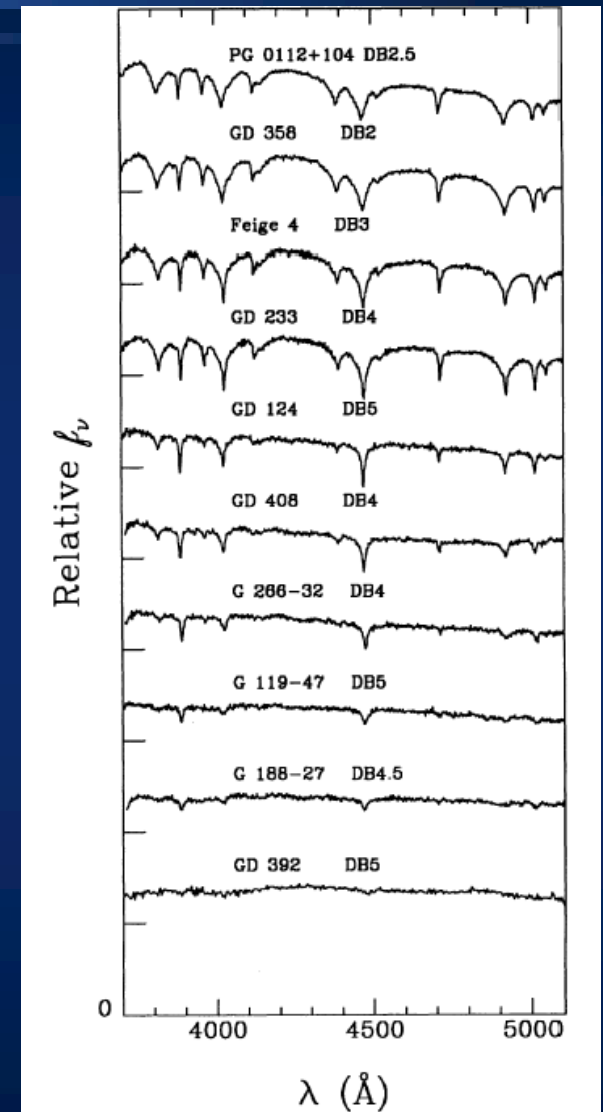


DA Spectra

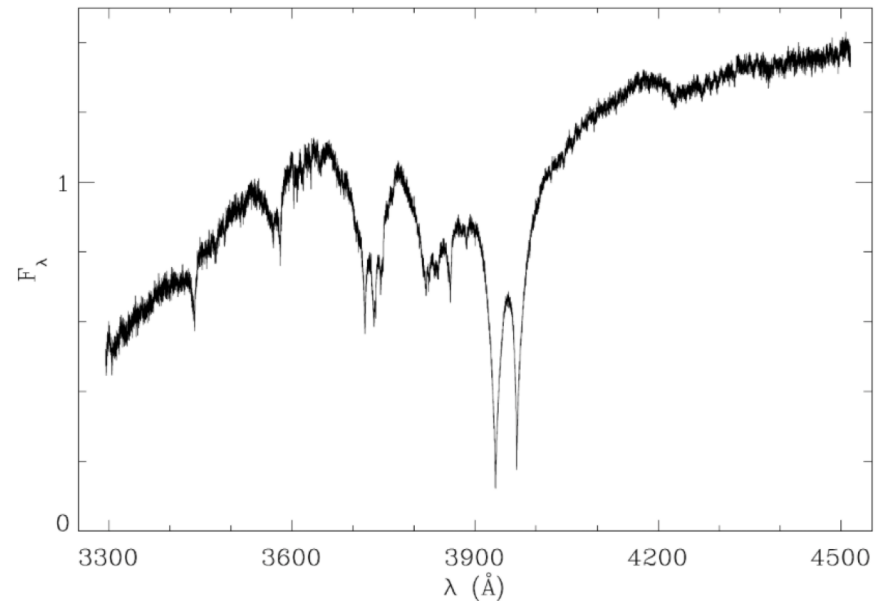
DB Spectra



Rapid settling of elements
heavier than H and He in high
gravity



DZ White Dwarf Spectra

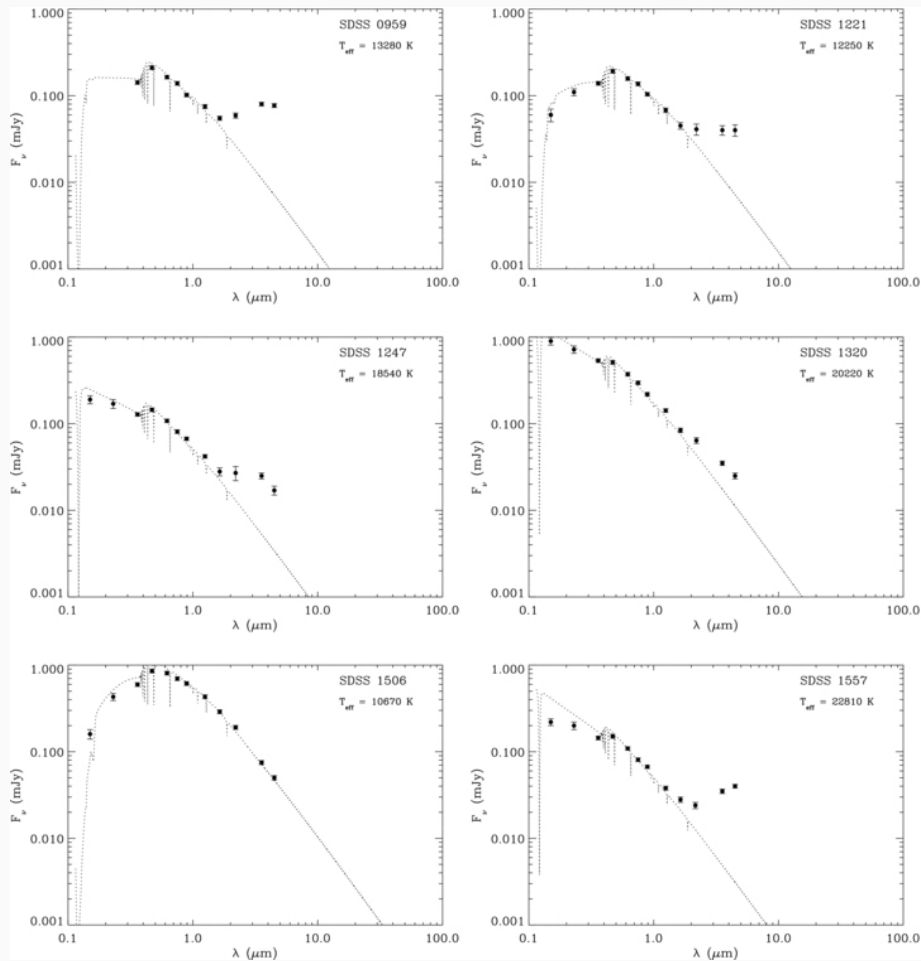


Many DZ WDs exhibit IR excesses suggesting dust surrounds the WD and it is being heated by the UV flux from the star.

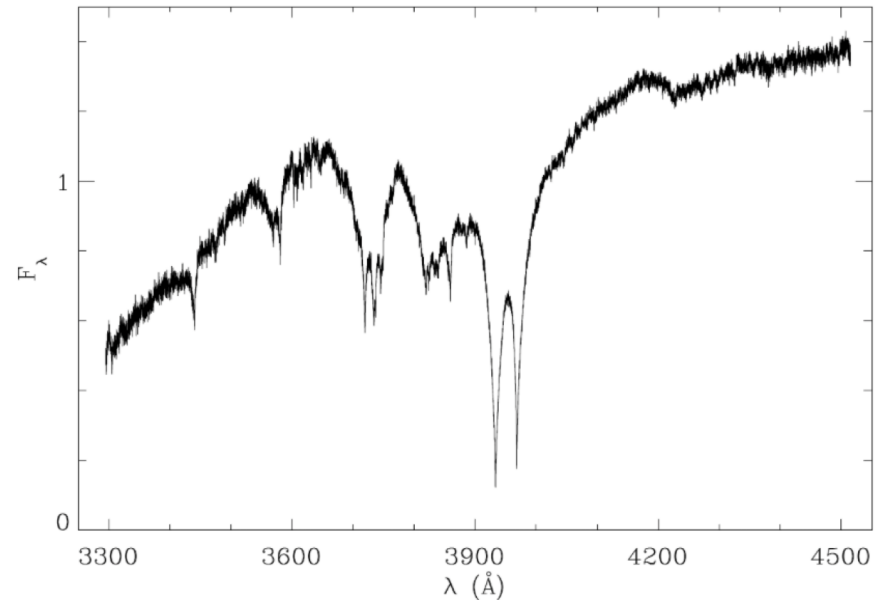
Implications????

DZ White Dwarf Spectra

Figure 2



Ultraviolet through infrared spectral energy distributions of the DA-type, infrared excess candidates. Stellar atmosphere models are plotted as dotted lines, using parameters derived from model fits to hydrogen Balmer lines in the SDSS or WHT spectra, and matched to the observed *g*-band fluxes. [Table 1](#) photometry is shown as data points with error bars.



Many DZ WDs exhibit IR excesses suggesting dust surrounds the WD and it is being heated by the UV flux from the star.

Implications????



Stellar Evolution : Star Clusters

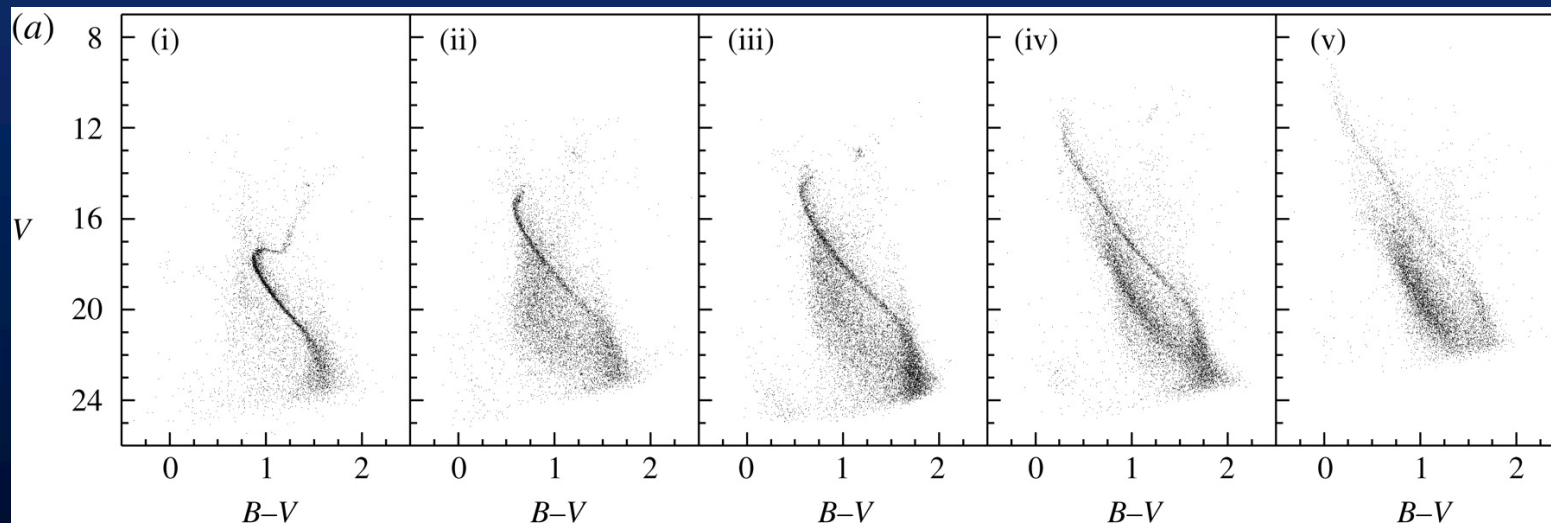
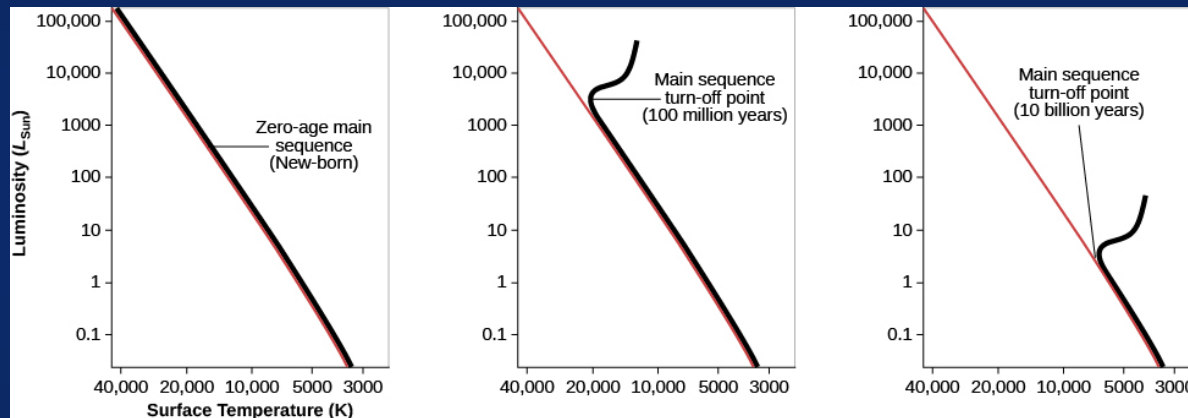
Star clusters are:

- Coeval (all stars same age)
- Same metal abundance (formed from same gas cloud)
- Have a range in masses in them (mass function)



Stellar Evolution (Star Clusters)

- Many stars can condense from the same giant molecular cloud → star cluster.
- Massive stars reach main sequence first and evolve from it earlier also
($t_{\text{ms}} \propto M^{-2.5}$)
- As time advances, high mass tip of main sequence shortens and evolved stars appear in upper right of H-R Diagram
- Position of main sequence turnoff → age of cluster



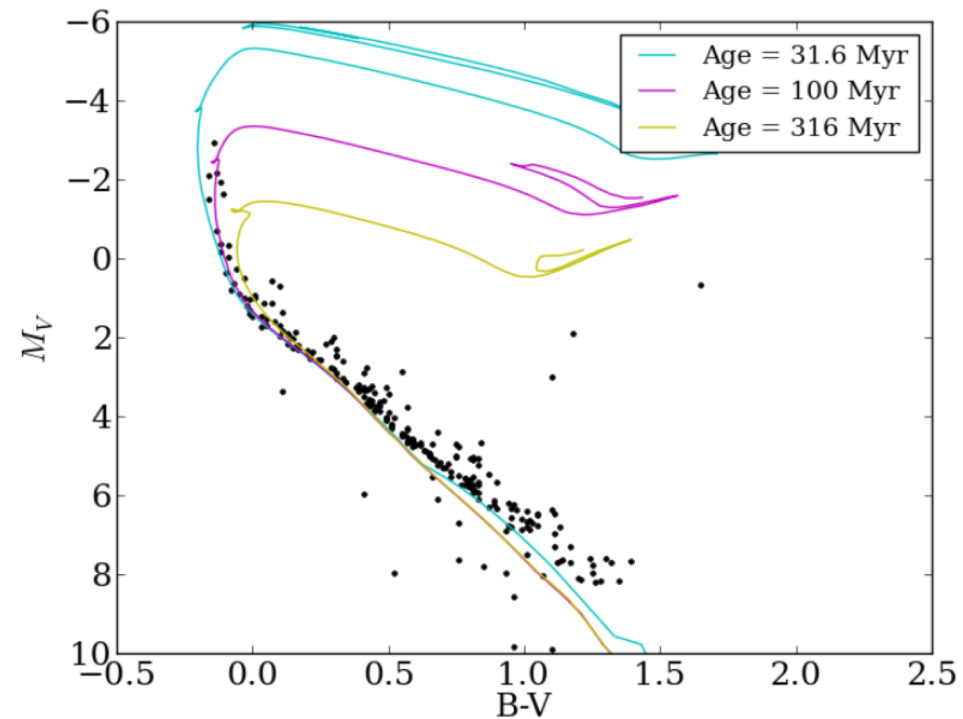
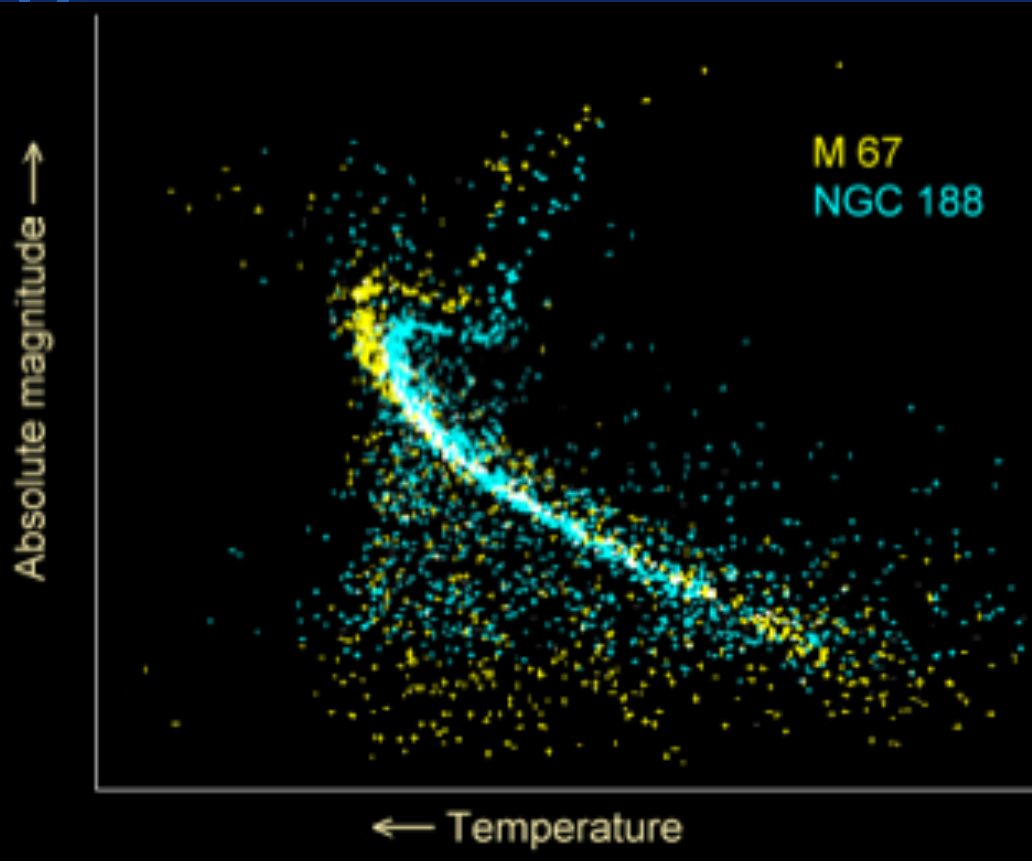
Stellar Evolution (Star Clusters)

Two star clusters different ages

M67 4 Gyrs

NGC 188 7 Gyrs

Pleiades 10^8 years





White Dwarfs in Clusters

Open Clusters have a wide range of ages (10 Myrs to 9 Gyrs, the age of the disk)

Use white dwarfs as chronometers

Use cooling models to derive the ages of globular clusters

Forensics: Diagnose the long dead population of massive stars

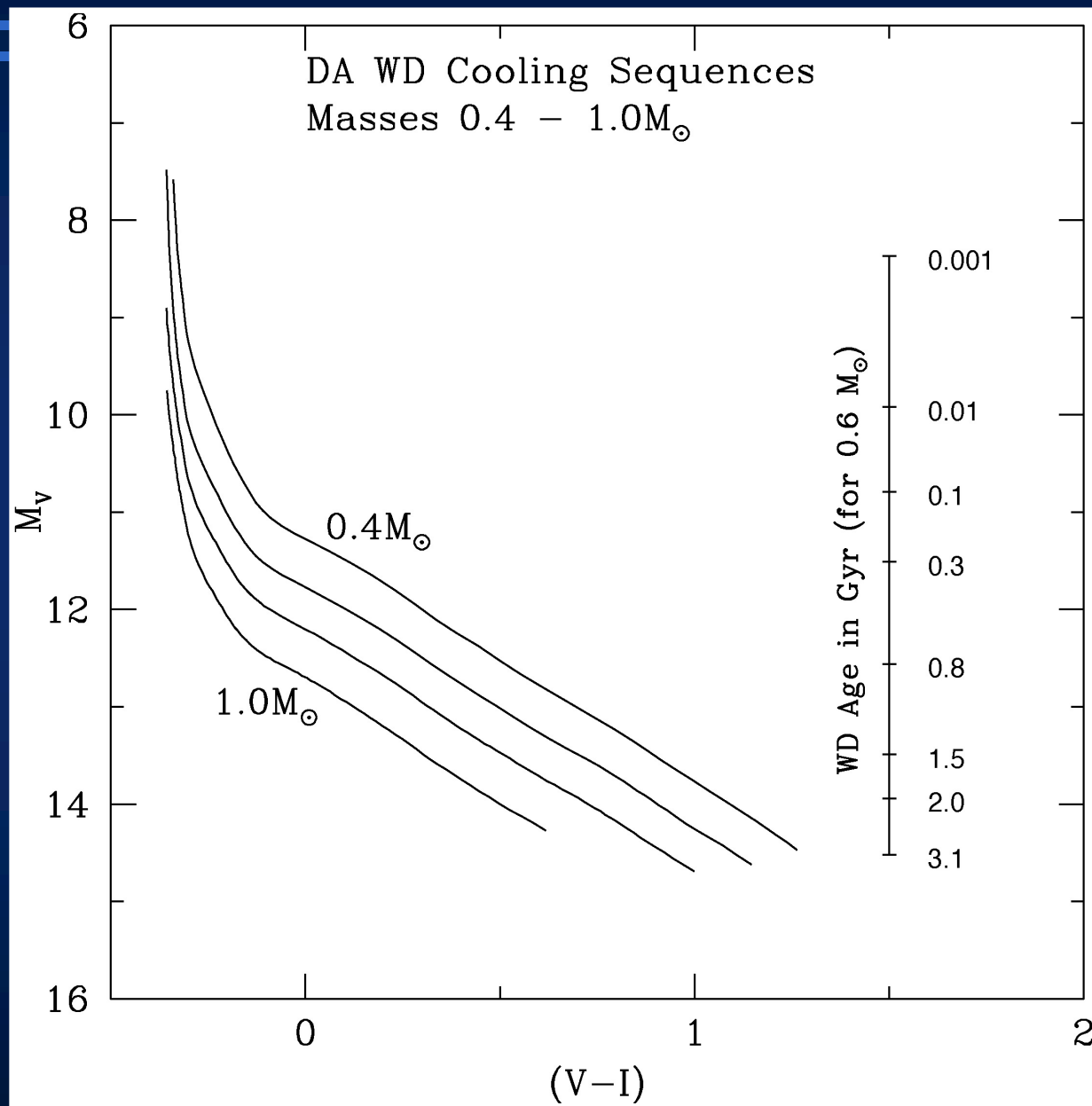


Cooling of White Dwarfs

- $L = d/dt(3/2kT * M/A_{mp})$ [Thermal energy * # ions]
- Simplify by $L = 4\pi R^2 \sigma T^4$
- $T^4 = C dT/dt$
- Integrate to get $T \propto t^{-1/3}$ (rate cooling $\sim 1.6 \mu \text{ K/year}$)
- More formally $T \propto t^{-2/5}$ Mestel Cooling (1952)
- Cool WDs are old



WDs Are Good Clocks





White Dwarf Cooling

0.5 Solar Mass WD, $T=2 \times 10^7$ K, 100% C^{12}

Take $\langle L \rangle = 10^{-4}$ Solar Luminosities

Total # ions = $0.5 \times 2 \times 10^{30} / 12 m_H$

= 5×10^{55} C ions, initially with energy

$E = 3/2 kT$ with $T = 2 \times 10^7$ K = 4×10^{-16} J

$E_{\text{total}} = 4 \times 10^{-16} \times 5 \times 10^{55} = 2 \times 10^{40}$ J

If $\langle L \rangle = 10^{-4}$ Solar = 4×10^{22} J/S (Watt)

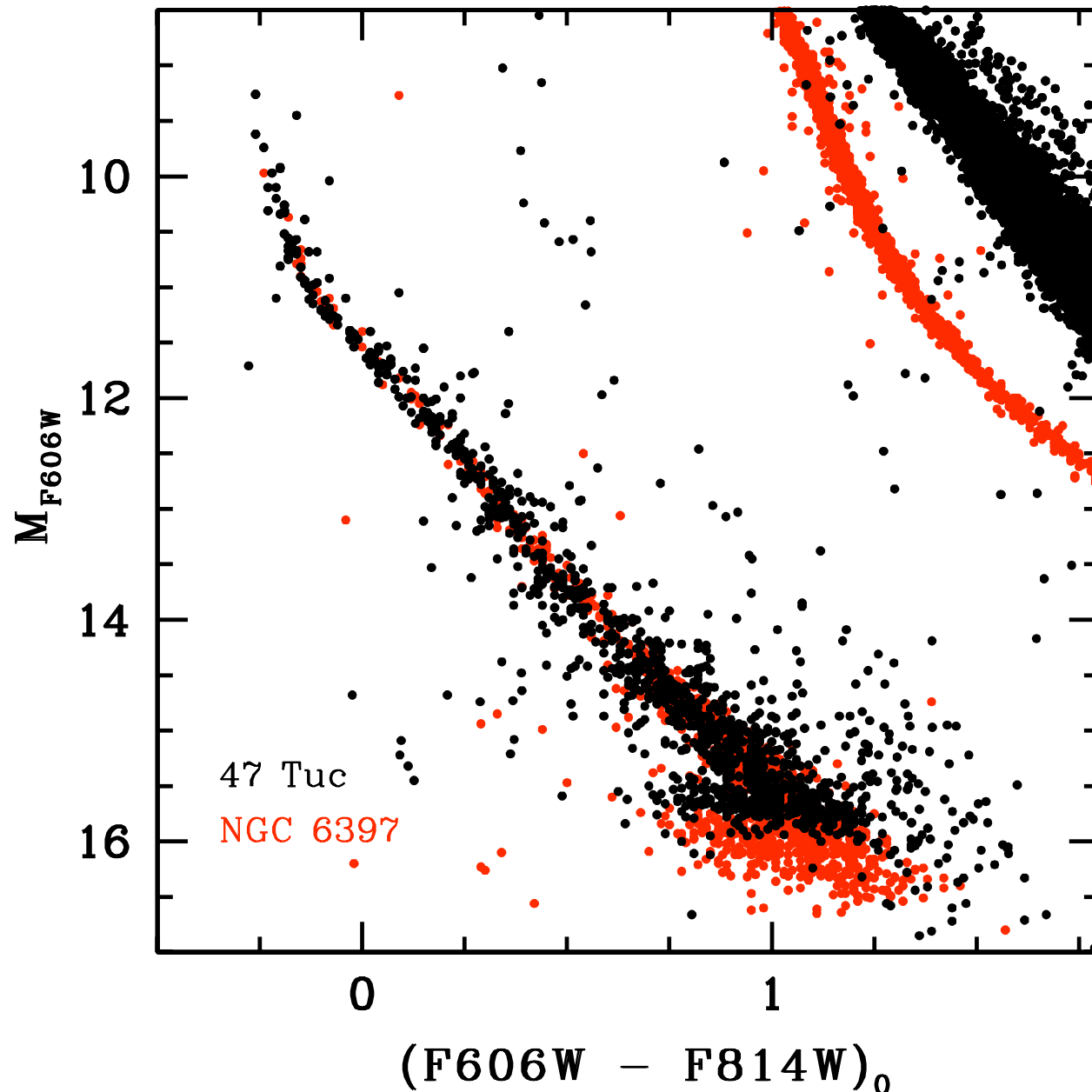
WD lasts for 2×10^{40} J / 4×10^{22} J/S = 5×10^{17} S

~17 Billion Years **OLDER WDs ARE FAINTER**

★ White Dwarf Cooling 2 Clusters

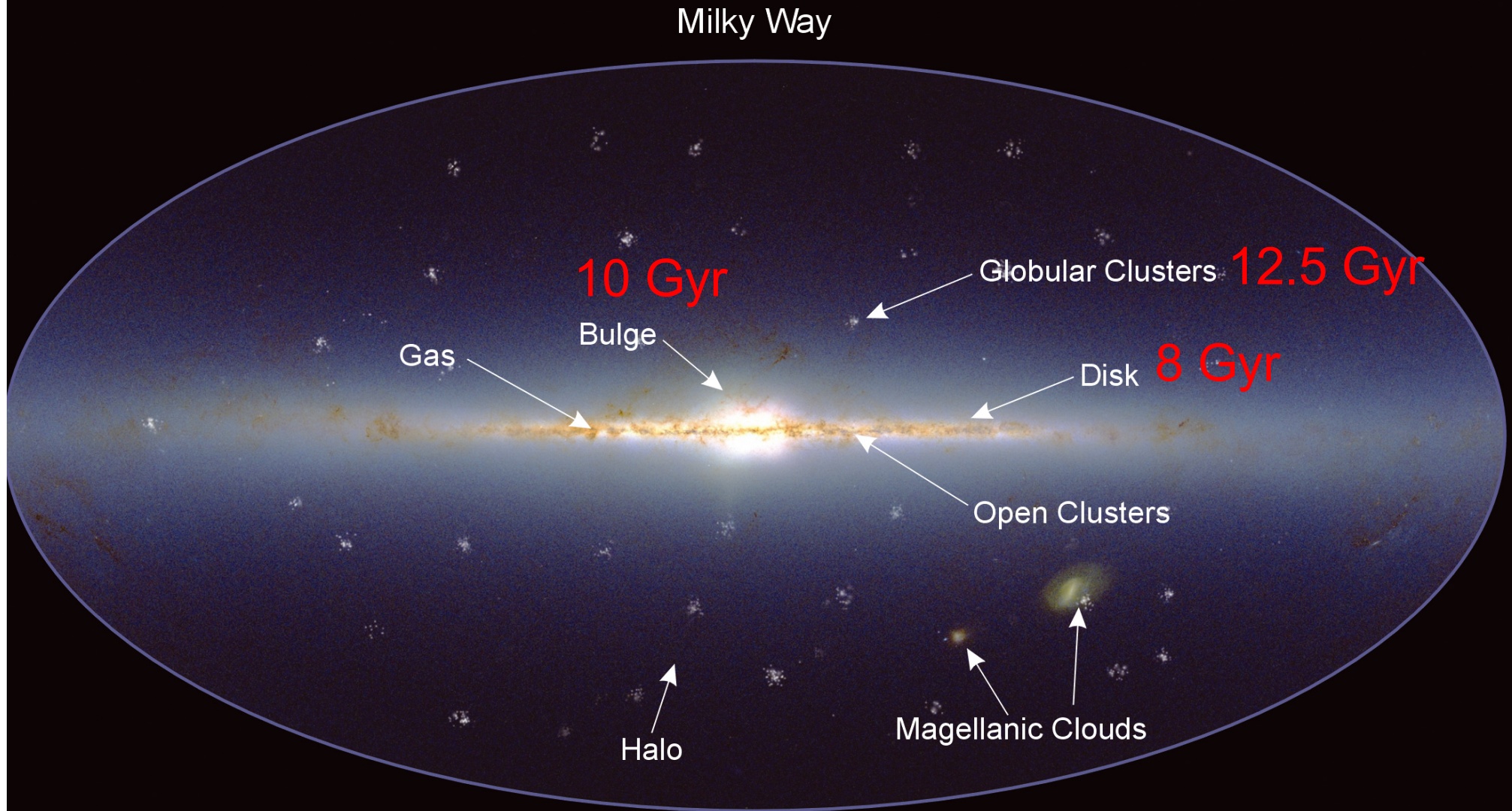
Note:

NGC 6397 fainter
(cooler) truncation
--> OLDER



Richer et al 2013

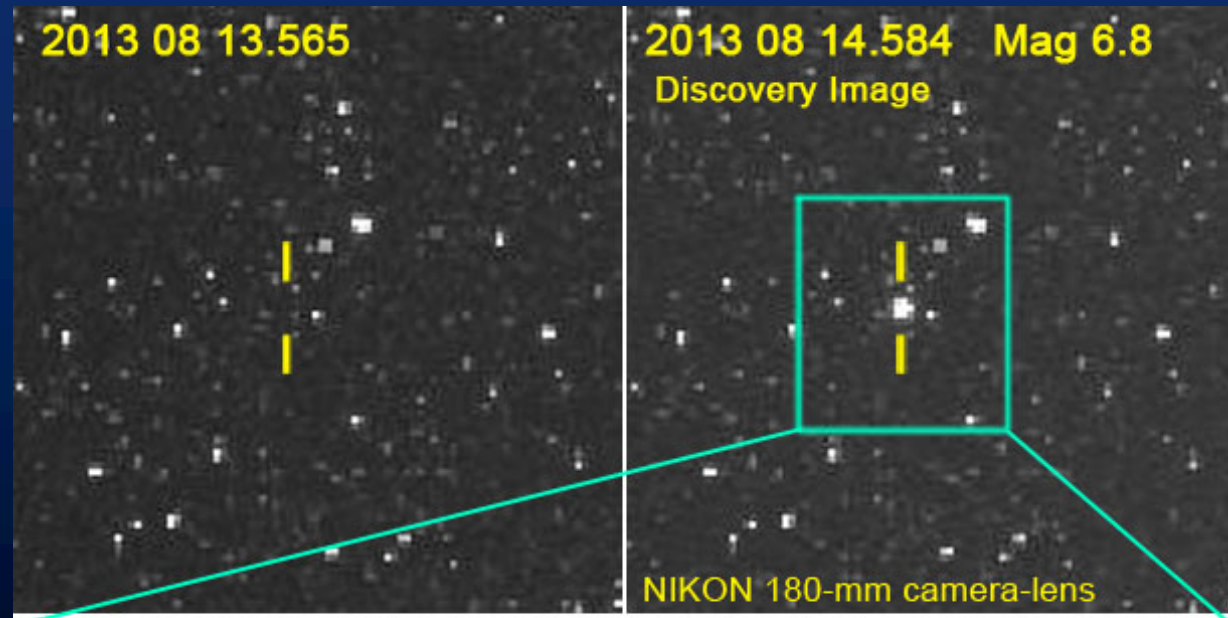
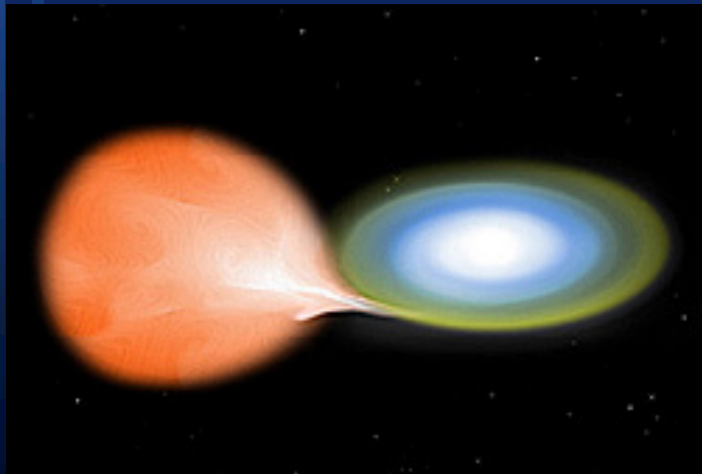
Chronology Formation Galaxy from WD Cooling





Classical Nova

Discuss supernovae shortly, but first consider less energetic events. Binary system: WD and another star. Star evolves (or separation is small) mass (mainly H) can be transferred to the WD, can also form a disk - mass heated and nuclear reactions – flares up - NOVA – fades on timescale ~ 1 month, often repeats.

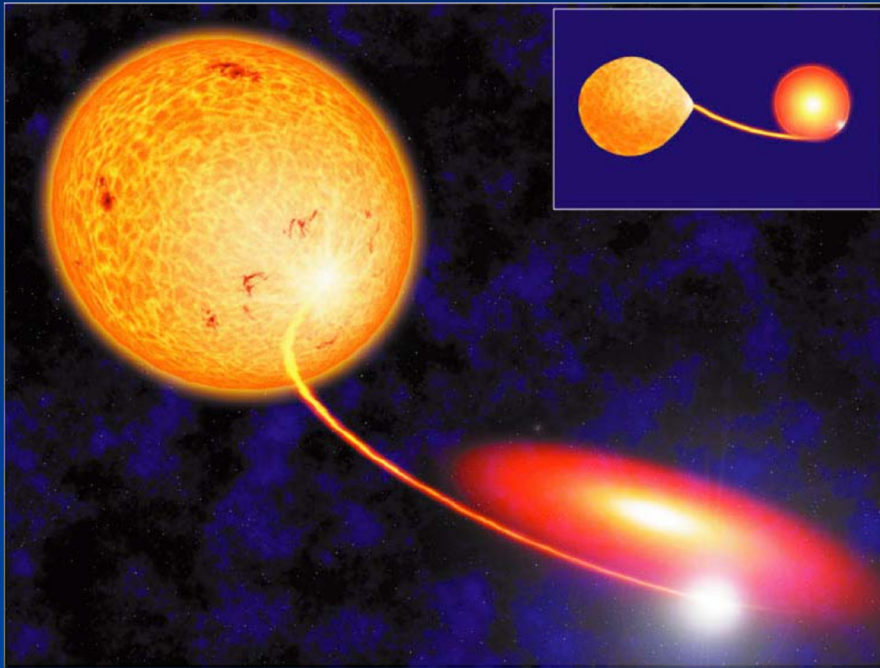




Type I Supernovae

Thermonuclear Bombs in Space!

Explosions of White Dwarfs in Binary Systems



**WD Accretion From Main
Sequence Companion**

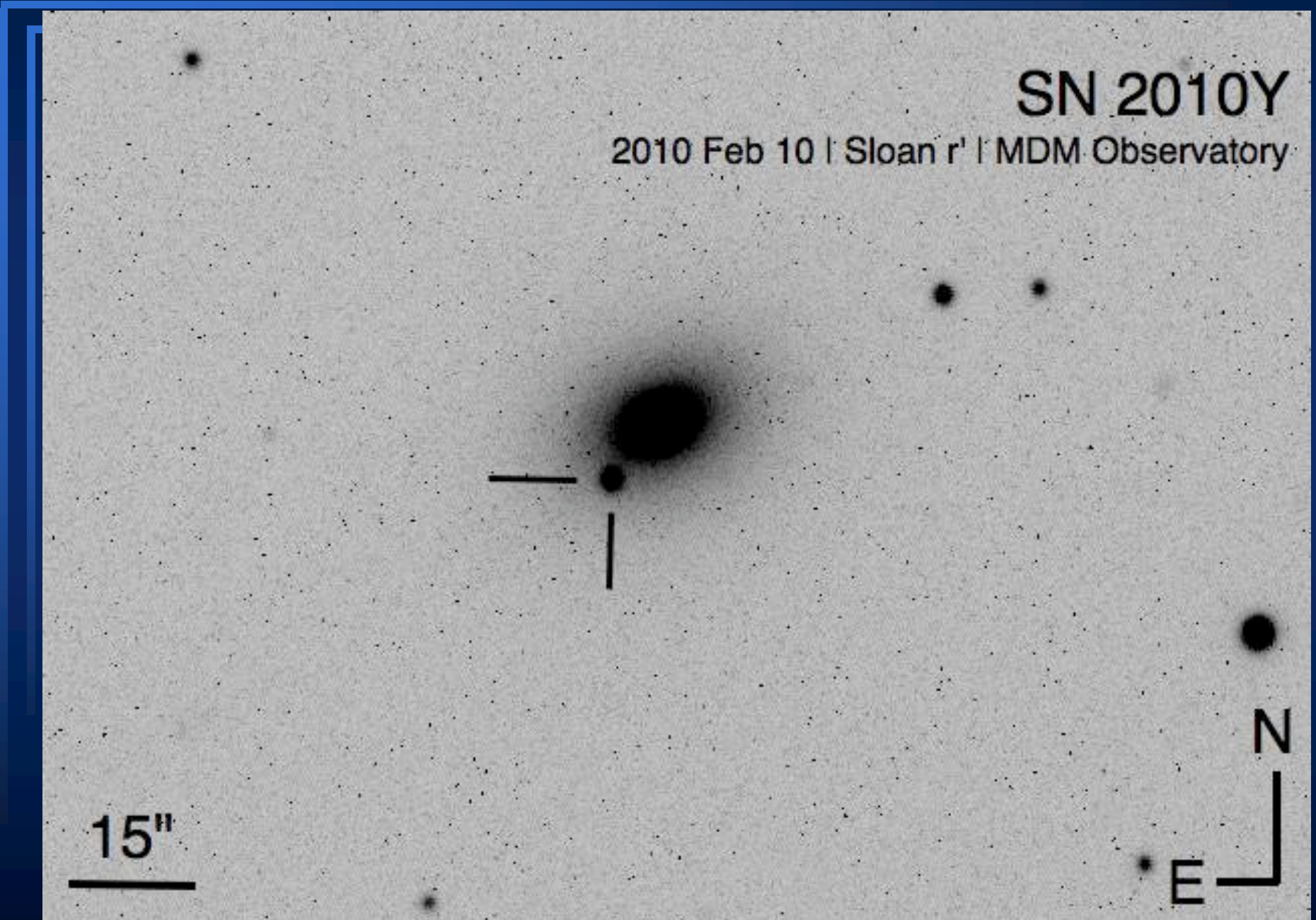


Merger of 2 White Dwarfs

If the total mass of the WD system exceeds $\sim 1.4 M_{\text{sun}}$ (the Chandrasekhar mass), it goes supernova



Type I SN in Elliptical Galaxy





Standard Candles

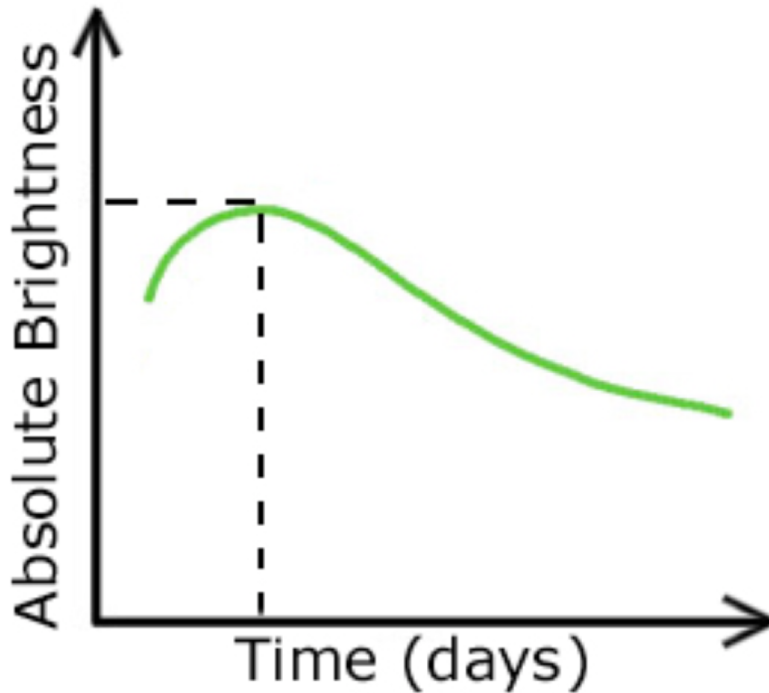
A **Standard Candle** is a theoretical astronomical object of known intrinsic luminosity L , like a 100 Watt light bulb in space





Standard Candles

Type Ia V Band Light Curve



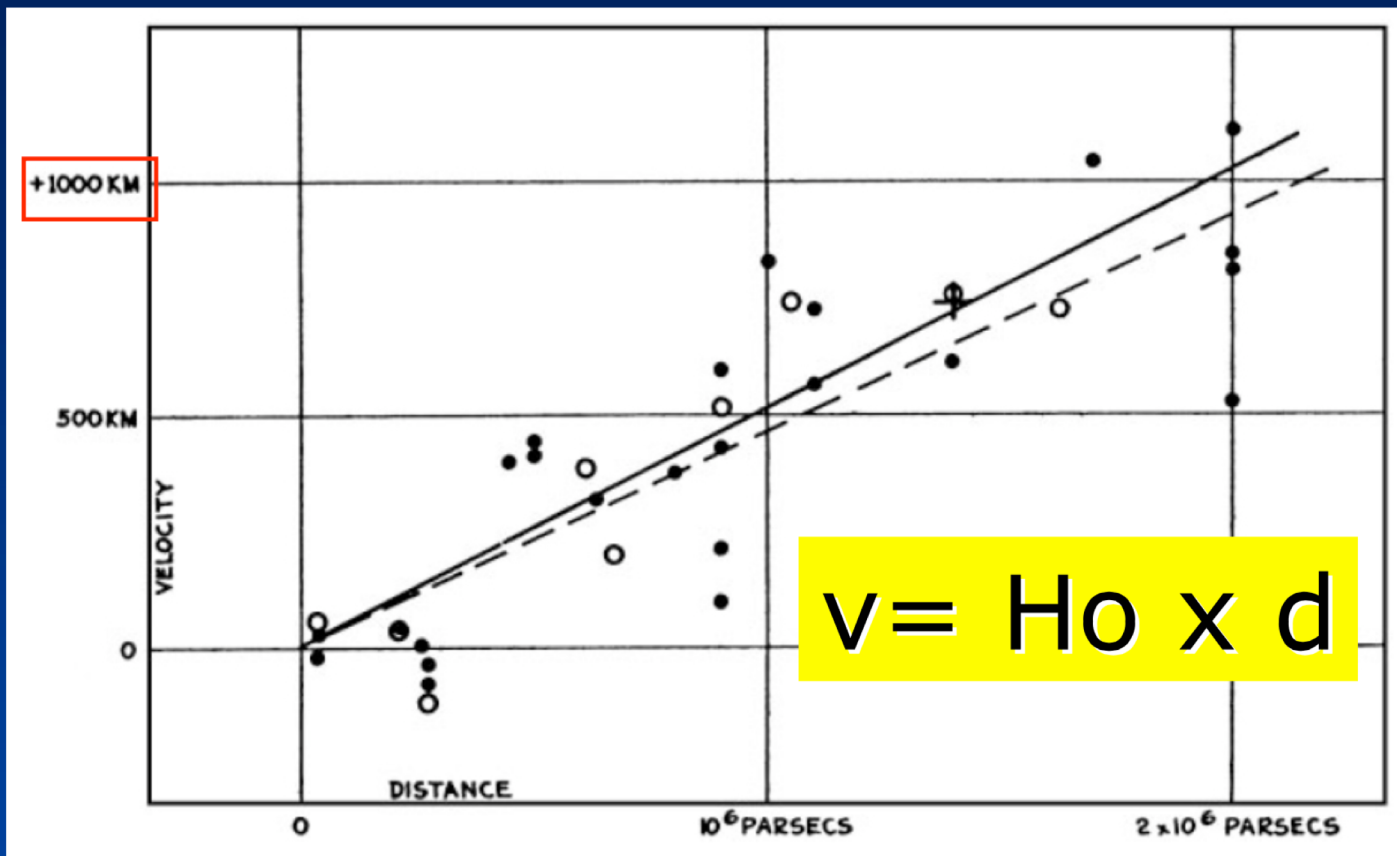
$$F = \frac{L}{4\pi d^2}$$

- The peak absolute brightness (or luminosity L) of a Type Ia supernova is *roughly* constant from event to event
- If we measure the apparent brightness (or flux F), we can infer the distance d if we somehow know L

Original Hubble Diagram

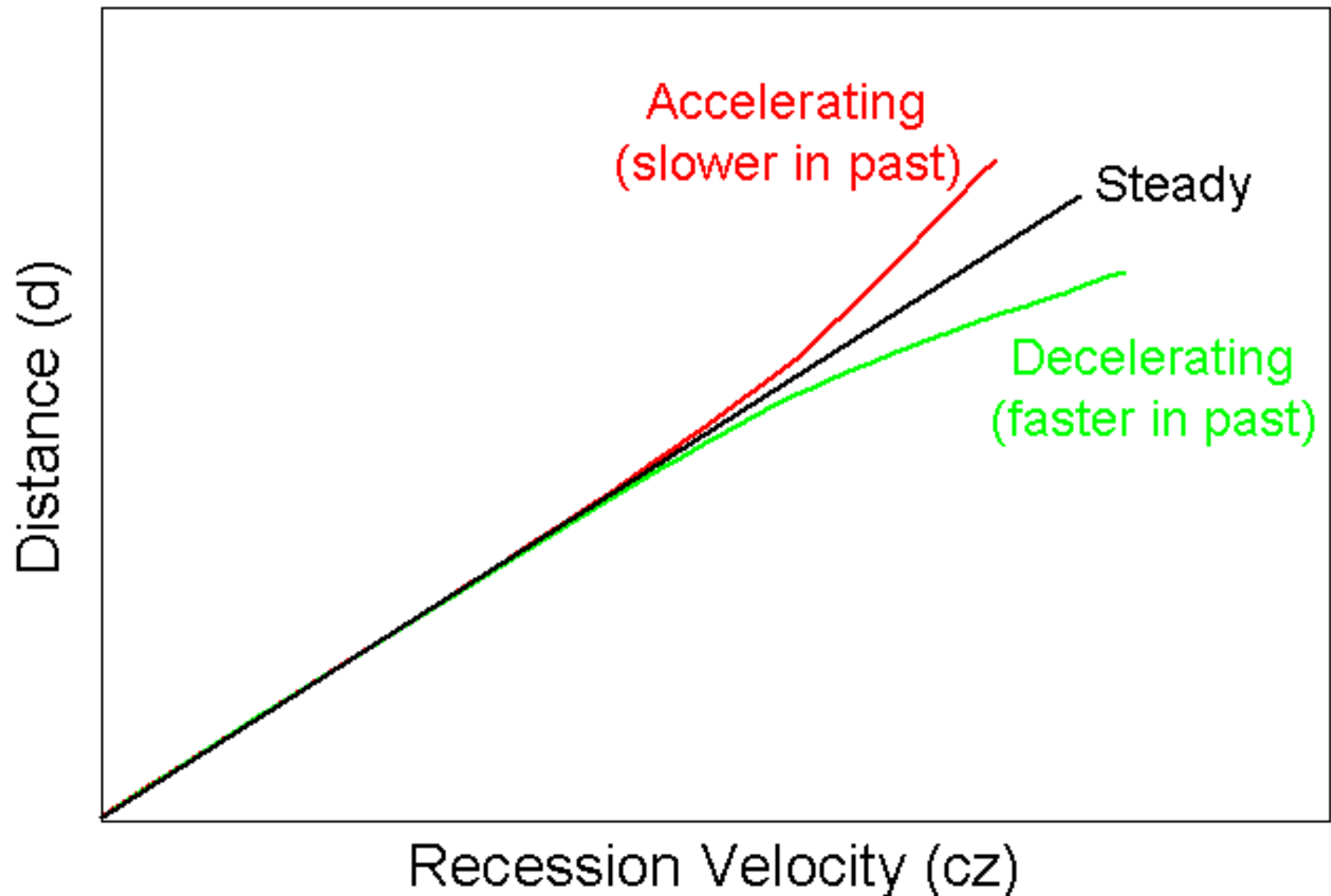
Hubble's Diagram (1929)

velocity (v)

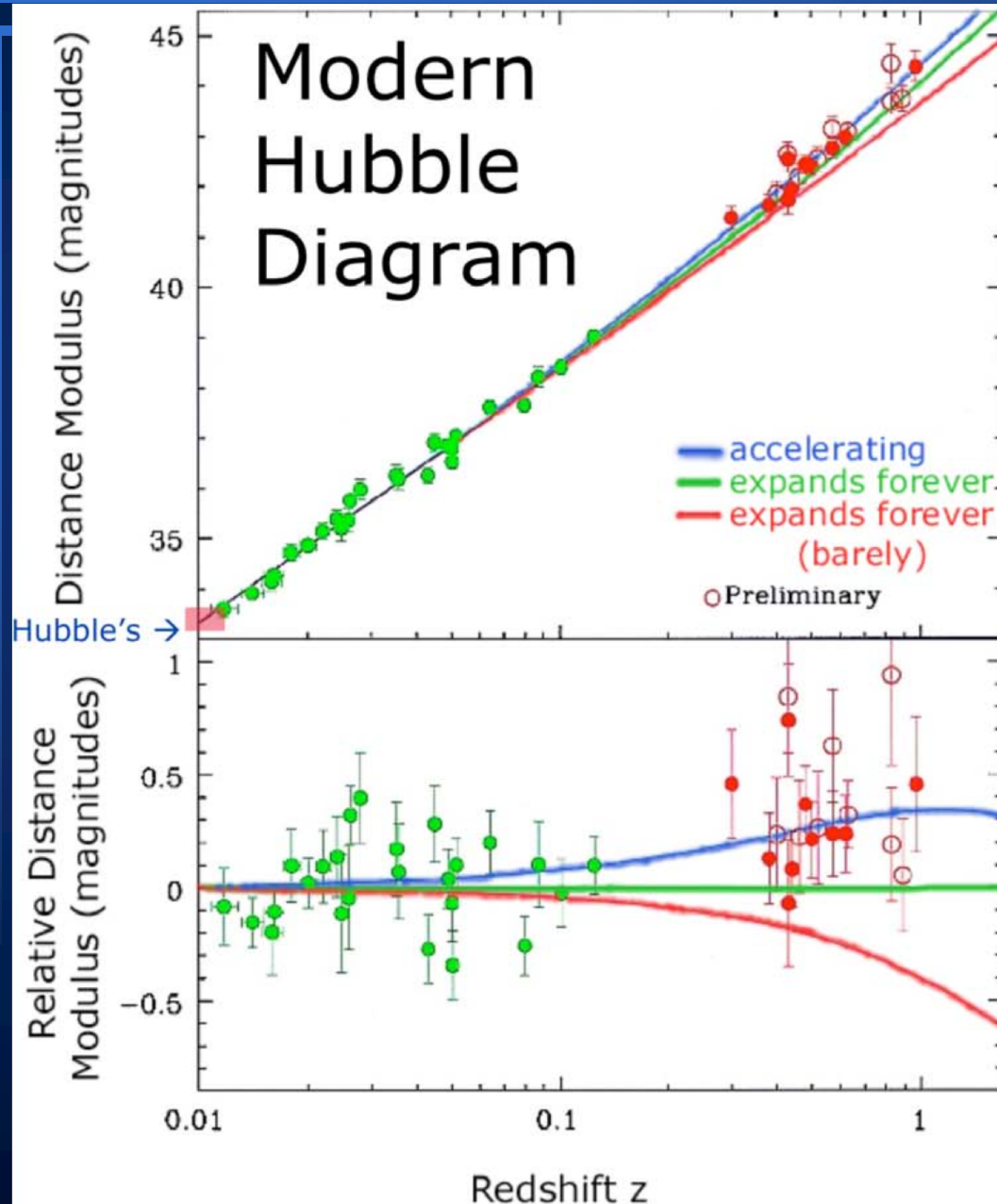


distance (d)

Modern Hubble Diagram



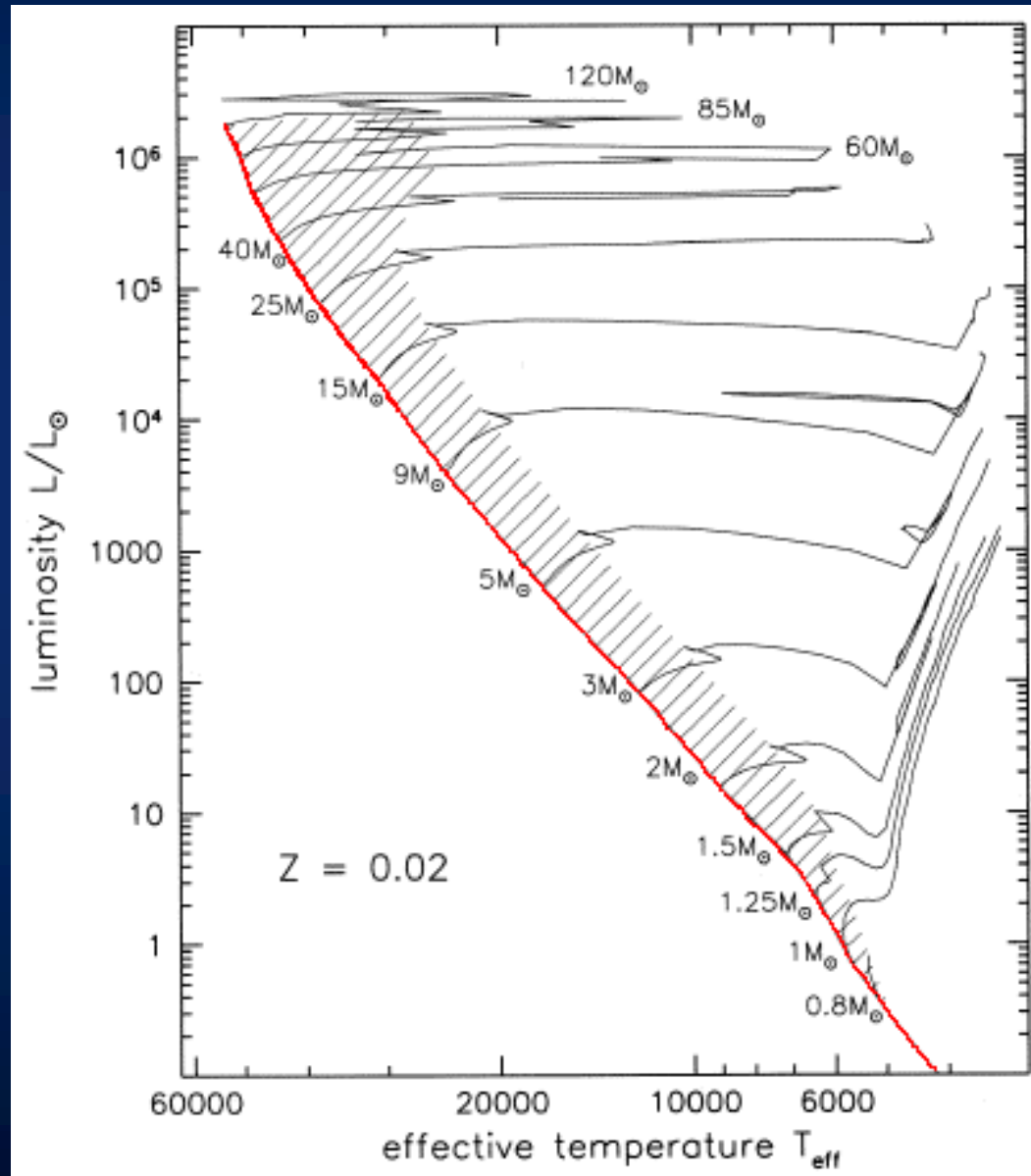
Modern Hubble Diagram



Nobel Prize Physics
2011

Shared by 3 astronomers
from 2 different teams

Evolution off Main Sequence High Mass Stars & Type II Supernova





Stellar Evolution (Death Massive Star)

- Gravity strong enough to squeeze C-O core until T rises high enough to fuse C.
- $C \rightarrow Mg, Na, Ne$ and O, and briefly stabilizes star until C used up.
- Again, core collapses. E_{grav} turned into more heat \rightarrow core hot enough fuse Ne.
- $Ne \rightarrow O, Mg$ and cycle repeats with $O \rightarrow Si, S, P$ then $Si \rightarrow Fe$ then $Fe \rightarrow ??$
- Fe most strongly bound nucleus. Its formation consumes energy. By this time, $\rho > 10^9 \text{ g/cm}^3$ (10^{12} kg/m^3), $T > 3 \times 10^9 \text{ K}$.
- Now get $^{56}\text{Fe} + \gamma \rightarrow 13 \text{ } ^4\text{He} + 4n$ - what kind of nuclear reaction is this? (γ here $\sim 100 \text{ MeV}$ and further drains thermal energy of core).
- Star collapses rapidly, T continues to rise, star almost in free-fall
 $^4\text{He} \rightarrow 2p + 2n$ eventually $p + e^- \rightarrow n + \nu$
- Two possible outcomes - n degeneracy stops collapse (**neutron star**) or complete gravitational collapse (**black hole**).



Stellar Evolution (Death Massive Star)

The final 1.4 second:

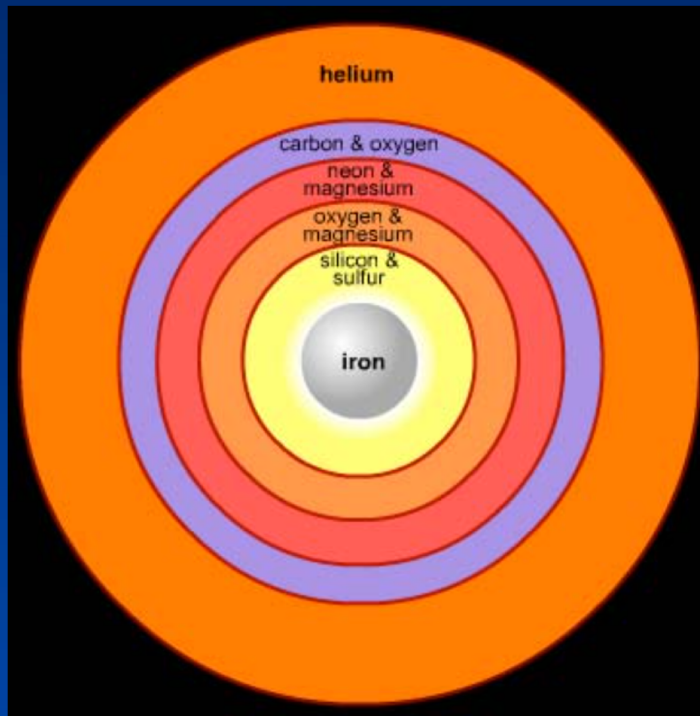
- Sudden core collapse. T soars to $> 5 \times 10^9$ K, ρ reaches 4×10^{17} kg/m³ (thimble = mass of a mountain!), intense flood of neutrinos emitted.
- Pressure forces electrons to combine with protons to make neutrons - core composed entirely of neutrons.
- Superdense matter in core stops contracting abruptly - but layers above still plunging inward at 10-15% speed of light. This material hits core and “bounces”.
- Shock wave rebounds throughout star, ripping away outer envelope and most of star's mass ejected ($>90\%$).

SUPERNOVA

- In next second a SN can release as much energy (mostly neutrinos) as Sun does in 10^{10} years.

Late-Time Structure Massive Star

Gravitational Core Collapse of Massive Stars



- For stars with $M > 8 M_{\text{sun}}$ main sequence nuclear fusion results in an onion-like structure w/ an Iron core
- Star can't get any more energy from fusing Iron

Once the pressure support from fusion DEMO disappears, the star's core collapses, leading to a supernova as the outer layers fall in and rebound

Supernova 1987a in LMC





Supernova in M51





Neutron Stars

Structure similar to WDs – both supported by degeneracy pressure but in case neutron stars the neutron is the degenerate particle of interest – both particles are Fermions and subject to uncertainty principle

Recall for WDs $P_{\text{deg}} = (\hbar^2/2m)n^{5/3}$ where m is mass of electron and n is particle density (N/R^3) so
$$P_{\text{deg}} = (\hbar^2/2m)(N/R^3)^{5/3}$$



Neutron Stars

Also recall that $P \propto M^2/R^4$ so that

$M^2/R^4 \propto (1/2m)N^{5/3}/R^5$ - now for a neutron star m here is not mass electron but mass neutron m_n

So rearranging $M^2R \propto (1/2m)N^{5/3}$

If we now divide this by a similar relation for WDs and assume (reasonably) that M and hence N is the same for both stars we have



Neutron Stars

$R_{\text{ns}}/R_{\text{wd}} \propto m_e/m_n$ or generally

$R \propto 1/\text{mass degenerate particle}$

with the mass of the electron $1/1839$ mass of neutron, we expect neutron star radii in the range $6000/1839 = 3$ km (somewhat too small but it gives an idea of the kind of object we are considering)



Neutron Stars / Pulsars

- Masses $\sim 1.4 M_{\text{sun}}$ - presence made known as **PULSARS**
- Size $\sim 10 \text{ km}$
- Radio pulsar period 0.714 s →
- Distribution of periods ↓

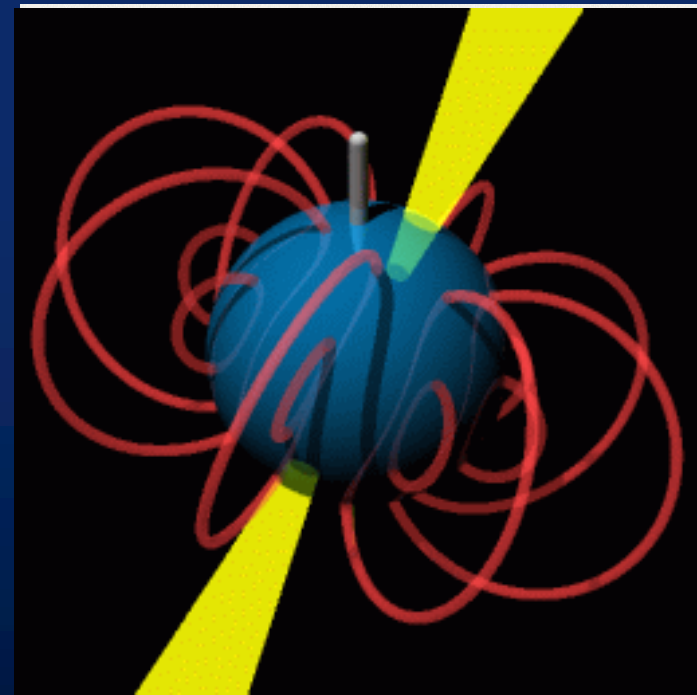
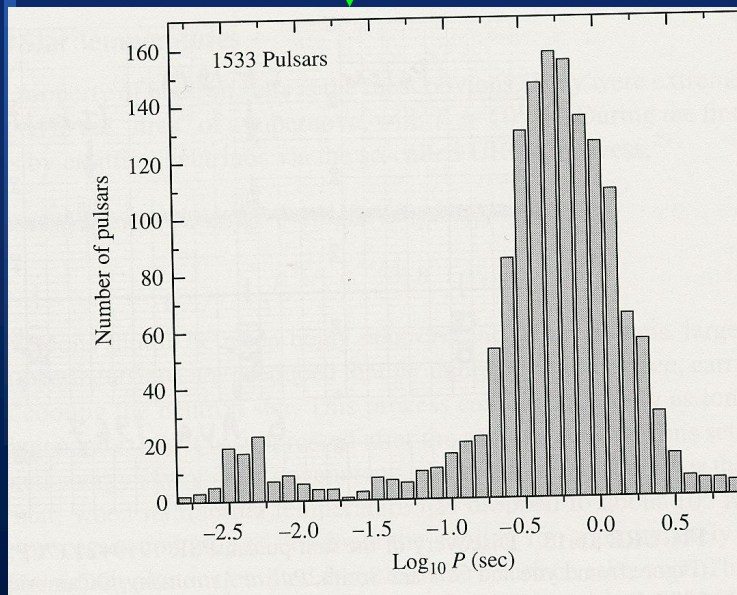
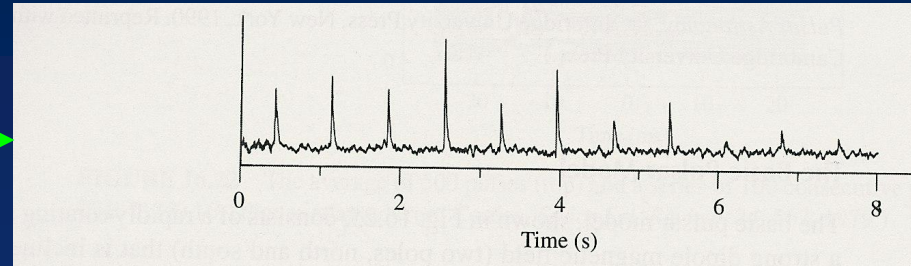
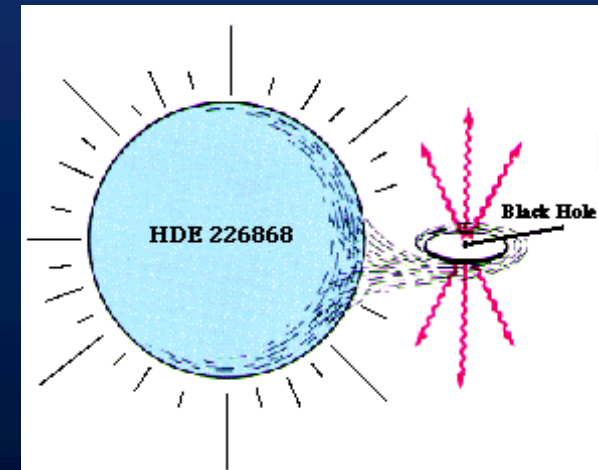
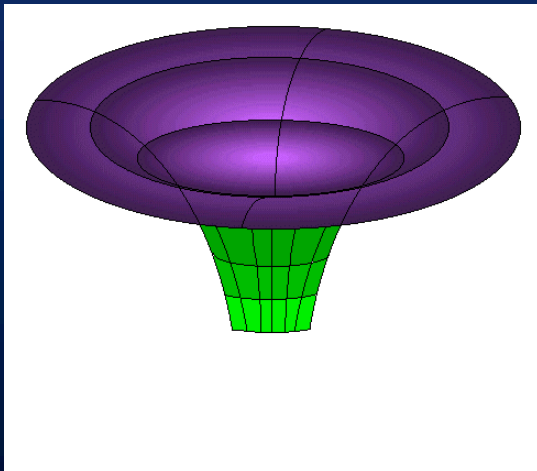


FIGURE 16.25 A basic pulsar model.



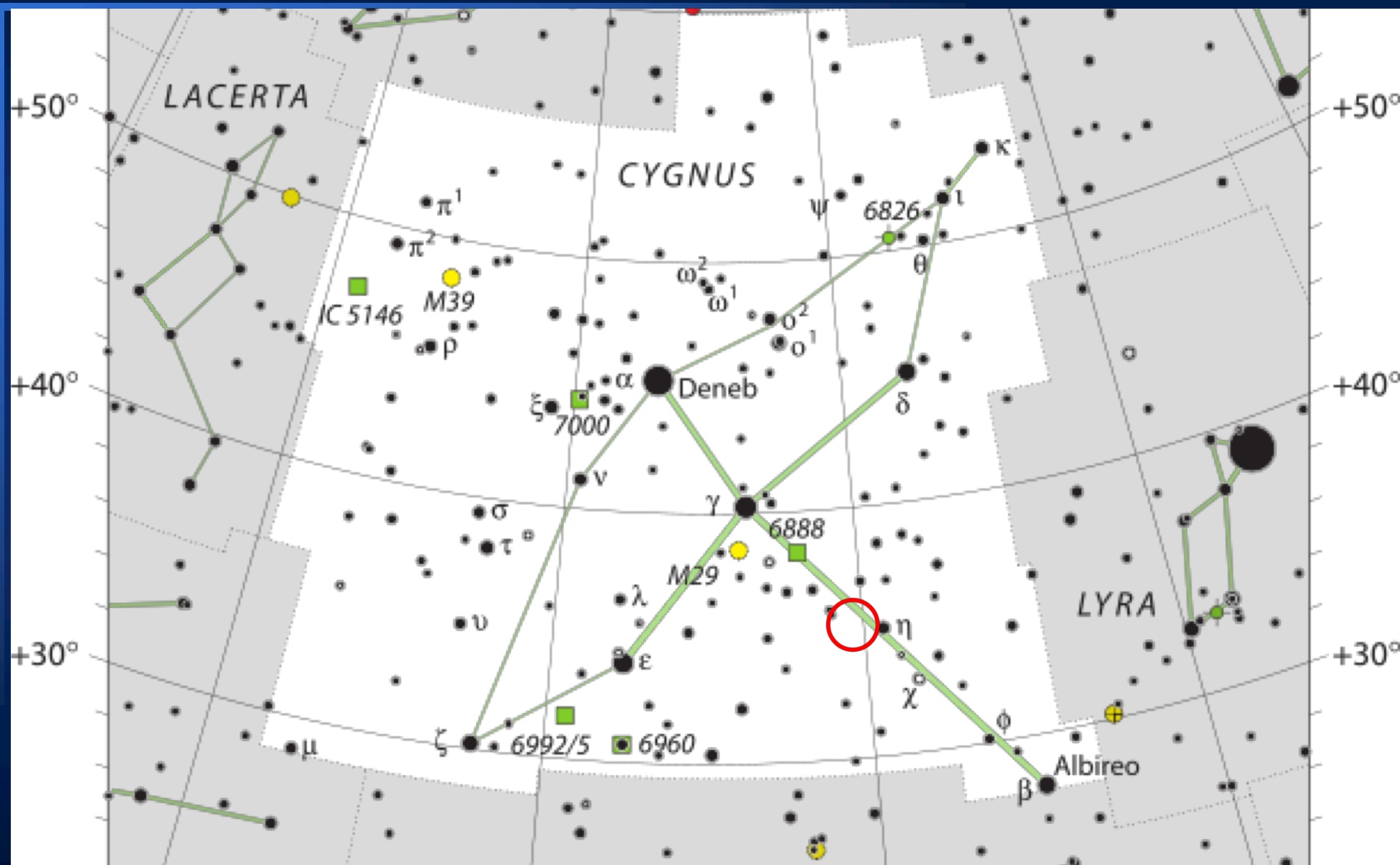
Black Holes

- Escape velocity > speed light $V_{\text{esc}} = (2GM/R)^{1/2} > c$
→ $R < 2GM/c^2$ or $R < 1.5 \times 10^{-27} M$ (M in Kgm, R in m)
- R is called Schwarzschild Radius and defines the event horizon.
- For example with $M = 10 M_{\text{sun}}$ $R < 30 \text{ km}$.
- For Earth to be a black hole $R < 1 \text{ cm}$!
- Presence of black holes is suggested in binary systems - high mass dark component and x-ray source.
O9I $\sim 15 M_{\text{Sun}}$



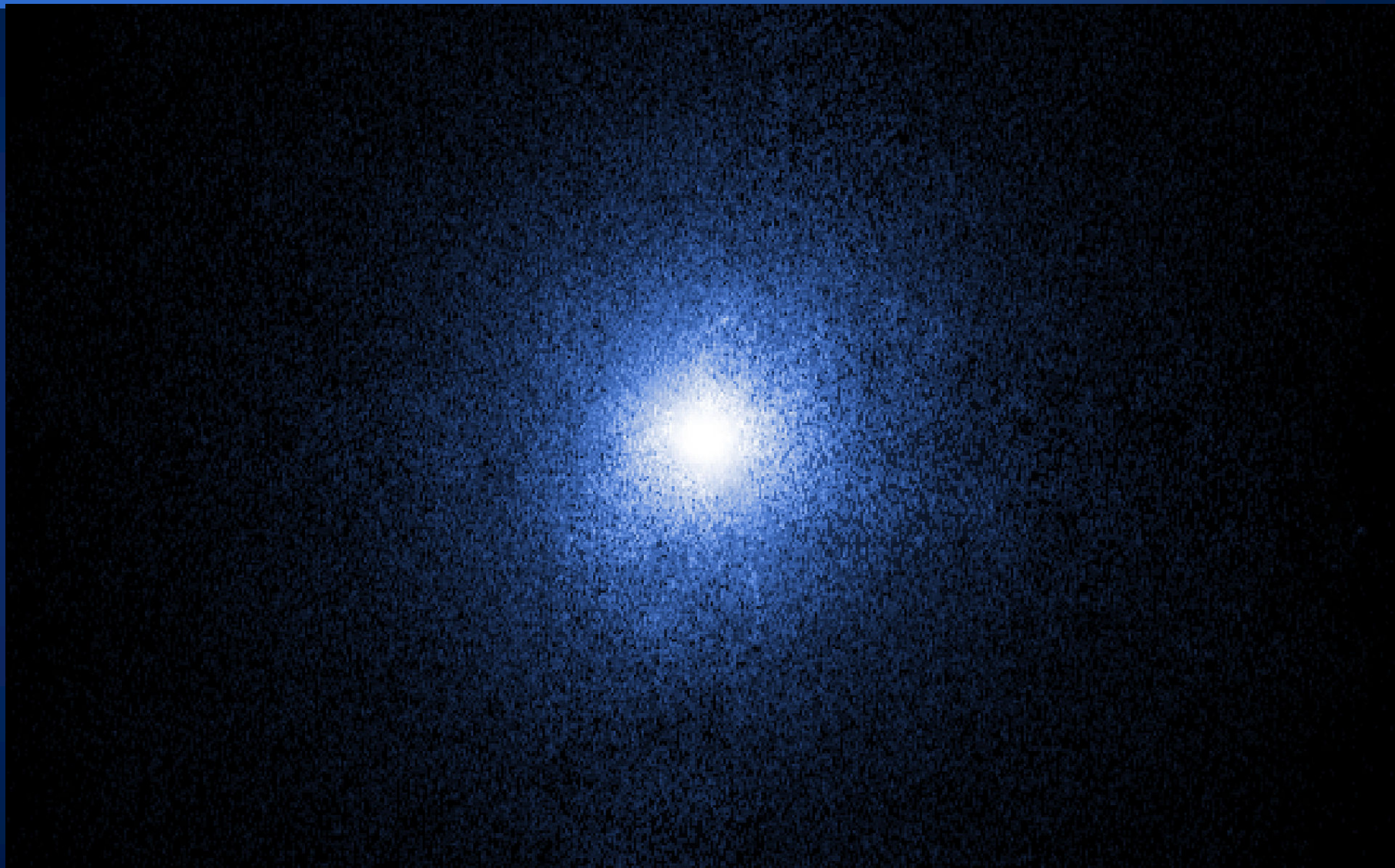


Cygnus X-1





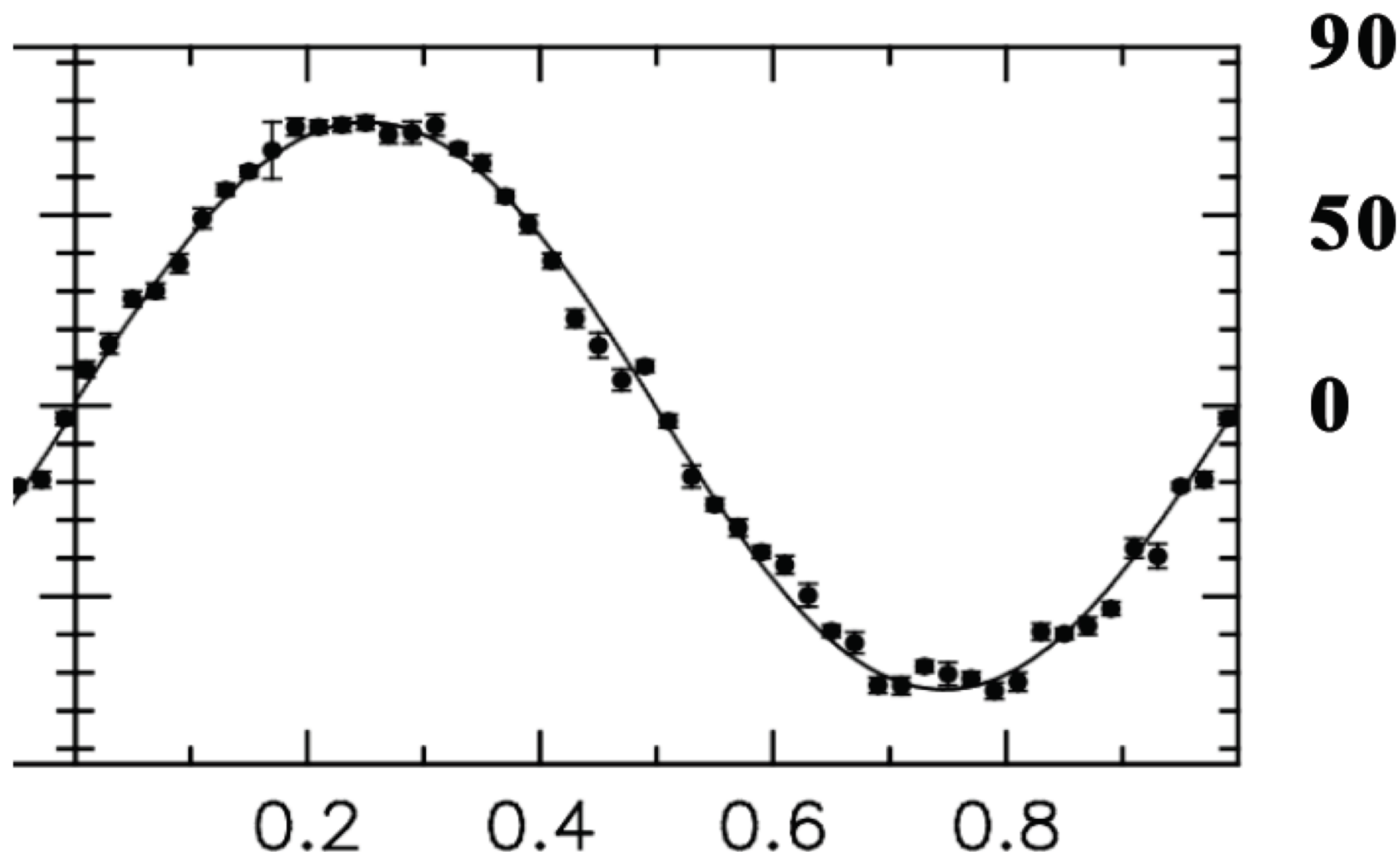
Cygnus X-1



X-Ray Image Cyg X-1 taken with Chandra Satellite

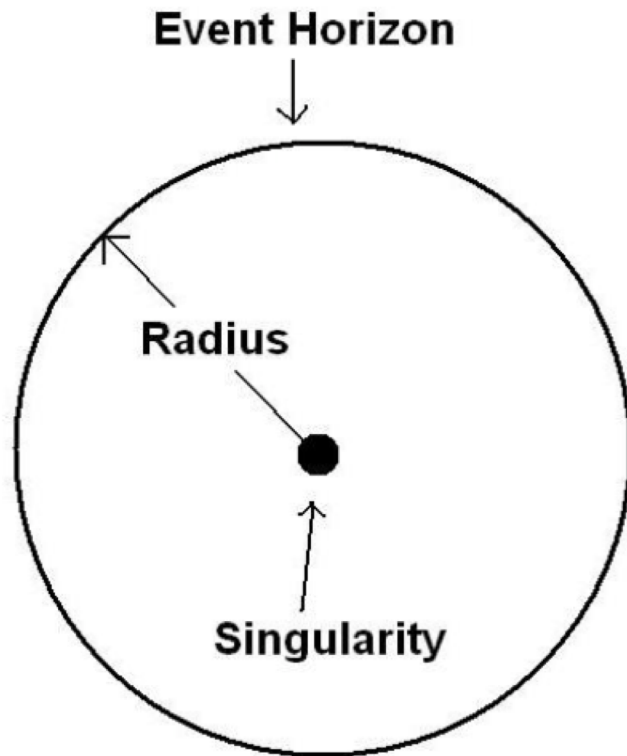


Cygnus X-1 16 Solar Mass BH



X axis Phase: $P=5.6$ Days: Y axis Velocity in km/s

Parts of a Black Hole



There are three parts to a simple black hole:

Event Horizon - Also called the Schwarzschild radius, that's the part that we see from the outside. It looks like a black, spherical surface with a very sharp edge in space.

Interior Space - This is a complicated region where space and time can get horribly mangled, compressed, stretched, and otherwise a very bad place to travel through.

Singularity - That's the place that matter goes when it falls through the event horizon. It's located at the center of the black hole, and it has an enormous density. You will be crushed into quarks long before you get there!

Black holes can, in theory, come in any imaginable size. The size of a black hole depends on the amount of mass it contains. It's a very simple formula, especially if the black hole is not rotating. These 'non-rotating' black holes are called Schwarzschild Black Holes.



Colliding Black Holes

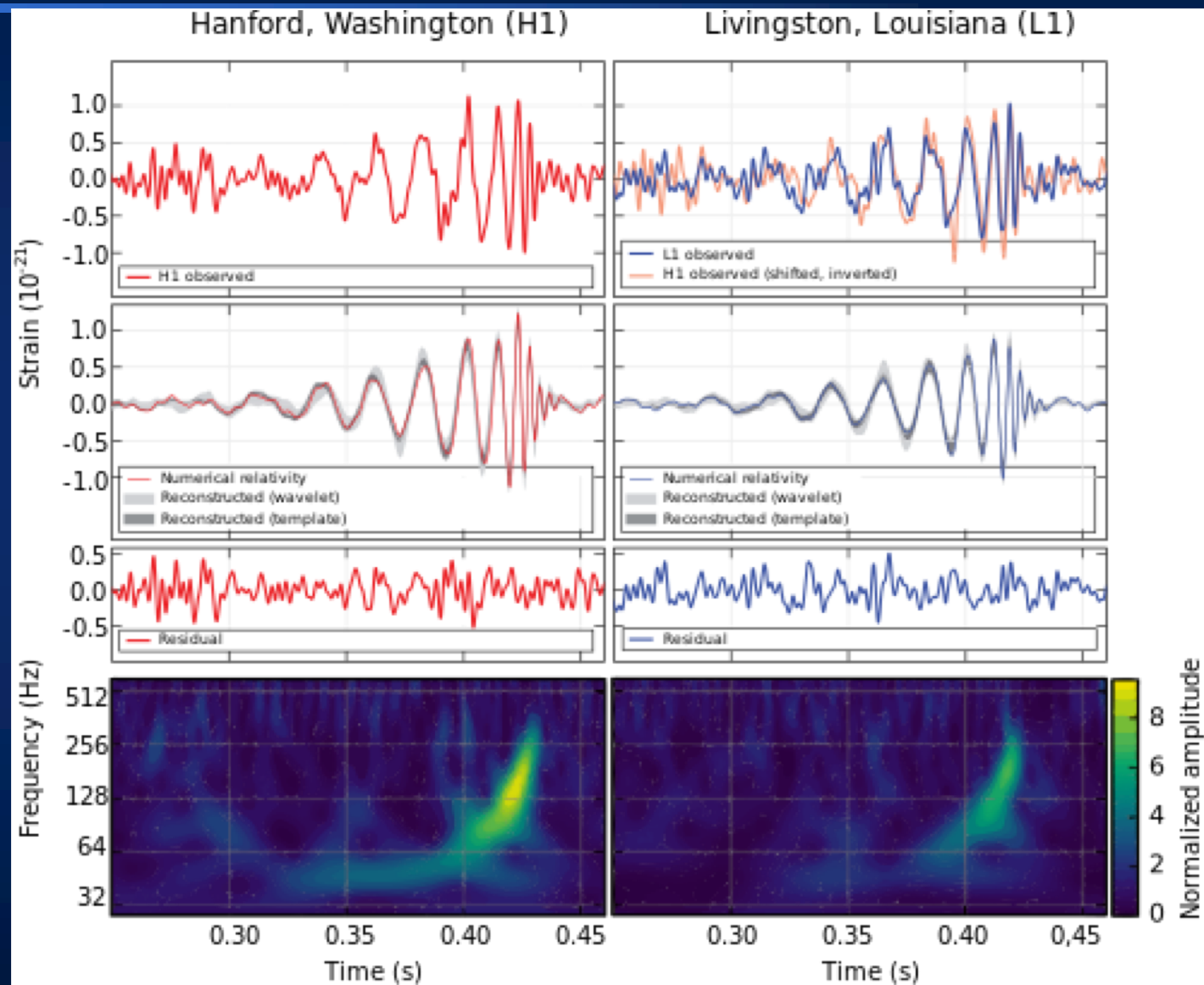


Gravitational Wave Detectors





36 and 29 Solar Masses



Properties of a Few Black Holes

A Short List of Known Black Holes

Stellar-Mass

Name	Constellation	Distance (Light years)	Mass (in solar units)
Cygnus X-1	Cygnus	7,000	16
SS 433	Aquila	16,000	11
Nova Mon 1975	Monoceros	2,700	11
Nova Persi 1992	Perseus	6,500	5
IL Lupi	Lupus	13,000	9
Nova Oph 1977	Ophiuchus	33,000	7
V4641 Sgr	Sagittarius	32,000	7
Nova Vul 1988	Vulpecula	6,500	8
V404 Cygni	Cygnus	8,000	12

Galactic - Mass

Name	Constellation	Distance (Light years)	Mass (in solar units)
NGC-205	Andromeda	2,300,000	90,000
Messier-33	Triangulum	2,600,000	50,000
Milky Way SgrA*	Sagittarius	27,000	3,000,000
Messier-31	Andromeda	2,300,000	45,000,000
NGC-1023	Canes Venatici	37,000,000	44,000,000
Messier-81	Ursa Major	13,000,000	68,000,000
NGC-3608	Leo	75,000,000	190,000,000
NGC-4261	Virgo	100,000,000	520,000,000
Messier-87	Virgo	52,000,000	3,000,000,000