Astronomy 205 Fourth Assignment – A Simple Solar Model: Due March 18, 2019 at Noon in the ASTR 205 Box.

This assignment is a theoretical project that has the student calculating a simple model of the Sun and comparing their derived model with the Standard Solar Model, which is a complete integration of the equations of stellar structure.

Much of what we do in this course is observational – that is using data to learn things about the stars. But theory is equally important. It is possible to construct a very simple theoretical model of the radial structure of the Sun using the equation of hydrostatic equilibrium, mass continuity and a few reasonable assumptions.

Constants Needed:
Radius Sun = 6.96e8 m
Mass Sun = 1.989e30 kgm
Mass hydrogen atom = 1.67e–27 kgm
μ (mean molecular weight Sun) = ? calculate it from X=0.74, Y=0.24, Z = 0.02
k (Boltzman Constant) = 1.3807e–23 JK^{-1}

Central Solar Values from the Standard Solar Model – Model provided in the associated table (note units in table are in cgs):
ρ_c = 1.505e5 kgm m^{-3}
P_c = 2.338e16 Nm^{-2} (Pa)
T_c = 1.548e7 K

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The first two equations of stellar structure

1) $dP(r)/dr = -GM(r)\rho(r)/r^2$
2) $dm(r)/dr = 4\pi r^2\rho(r)$

cannot be solved as they stand – they require further equations to determine the density ρ. There are 2 exceptions however:

(a) When ρ is a known function of r and
(b) When ρ is a known function of P.

We explore the first of these cases here.

If $\rho(r)$ is a known function of r, equation 2 can be integrated from $m(0) = 0$ to yield $m(r)$. Given $m(r)$, equation 1 may then be integrated starting from $r = R$ with the boundary condition $P(R) = 0$.

To illustrate this assume ρ is a linear function of r between $\rho = \rho_c$ (c for centre) at $r = 0$ and $\rho = 0$ at $r = R$ (the radius of the star). That is

$\rho = \rho_c(1-r/R)$.

a) Integrate equation 2 with this expression for ρ. You should obtain $m = M(4x^3 - 3x^4)$ where M is the total mass of the star and $x = r/R$.
b) Show that the central density is then $\rho_c = 3M/\pi R^3$
c) Use your expression from (b) above and the general expression for the density to show that
$P = (5/4\pi)(GM^2/R^4)[(1-(24/5)x^2+(28/5)x^3-(9/5)x^4)]$
Where $P=0$ at the surface $r=R$.
d) Assuming an ideal gas law with no radiation pressure
$P=\rho kT/um_H$
Show that the temperature is given by
$T = (5/12)(Gum_HM/kR)(1 +x -19/5x^2 +9/5x^3)$
What is the central temperature of the Sun in this model and compare it to the known value.
e) Make plots of m, P and T as a function of radius and compare these with the standard model of the Sun given

in the attached table (note that the table values are in cgs units).

For each plot, the radius should be normalized to the Solar Radius and the ordinate should be renormalized to total mass (for the mass plot) and to the central values for both T and P for the other plots. On each of these three separate plots over-plot (in a different colour or line type) a similar plot for the Standard Solar Model.

You should note that your VERY simple model does a decent job of modeling the Sun. The trends are the same, which is quite amazing given that we have not included any nuclear energy generation or radiative transfer.