ASTR 205: Stellar Astrophysics. Midterm: 2017.
Please write on the space provided after each question.
There are 8 questions – please attempt them all.
Maximum total 28 points.
Student Name_______________Number_____________

Some of the following constants and equations might prove useful:
speed light $c = 3x10^8$ m/sec;
1 AU = $1.5x10^8$ km; 1 light year = $9.5x10^{12}$ km;
1 parsec = 3×10^{13} km
Wien's Law $\lambda_{max} = 3x10^6/T$, λ_{max} is in nanometers, T in degrees K;
Stefan-Boltzman law $F = \sigma T^4$; $\sigma=5.67x10^{-8}$ W/m^2K^4;
Kepler's Third Law $p^2 = 4\pi^2a^3/G(M_1+M_2)$;
Gravitational Force $F=GM_1M_2/r^2$;
206265 arcsec/radian;
$F = L/4\pi d^2$ where F is the flux, L the Luminosity and d the distance;
$G=6.67x10^{-11}$ Nm^2/kg^2;
1 micrometer $=10^{-6}$m;
1 nanometer (nm) = 10^{-9}m;
Mass of Sun=$2x10^{30}$ kg;
Radius of Sun=$7x10^8$ m;
Luminosity Sun=$4x10^{26}$ watts;
Latitude Vancouver = 49 Degrees;
Distance (pc) = $1/\pi$, π in arcsec;
Velocity-Redshift $\Delta\lambda/\lambda = v/c$, c is speed light, v is velocity;
m2 – m1 = 2.5 log(b1/b2), m's are magnitudes b are fluxes
(m-M) = 5 log (d/10) + A; m apparent mag, M absolute mag, d distance in pc,
A is extinction = 3E(B-V) where E is the colour excess;
$B_\lambda(T) = (2hc^2/\lambda^5)(1/e^{hc/\lambda kT} - 1)$ w m^{-2} m^{-1} $sterad^{-1}$ B(T) is Planck Function
$1/\lambda = R_H(1/4 - 1/n^2)$ R_H Rydberg Constant = 1.09677×10^7 m^{-1}
Boltzmann Formula: $N_B/N_A = D\, e^{-(X_{AB}/kT)}$ N's are the numbers in various states, D depends on state, X_{AB} is energy difference between states, k is Boltzmann constant = 1.38×10^{-23} m^2 kg s^{-2} K^{-1},T temperature
$F(m) = m_2^3 \sin^3 i / (m_1 + m_2)^2 = pv^3/2\pi G$ is mass function p is period v velocity

(1) Where on Earth are the following statements true? [1 mark each]

(a) The star Capella (δ = +46 degrees) is circumpolar.

north of latitude +44°

(b) The star Antares (δ = -26 degrees) rises to an altitude of +10 degrees.

latitude 54°

(c) Altair (δ = +9 degrees) is seen at the zenith at some time during the year.

latitude = 9°

(2) A star has a parallax of 0.0046 arcsec. Its apparent magnitudes are $m_V = V = 6.95$ and $m_B = B = 7.34$. It has an effective temperature of 13,000 K and the intrinsic colour of the star is $(B-V)_0 = -0.2$.

(a) What is the distance to the star in parsecs? [1 mark]

$D = 1/\pi = 217.4\ pc$

(b) What is its absolute visual magnitude M_V and its visual luminosity in Solar units? [2 marks]

$(V - M_V) = 5 \log D/10 + A_V$; $A_V = 3 \times E(B-V) = 1.77$

$\therefore M_V = -0.91$

Sun $M_V = 4.83$ $\therefore M_2 - M_1 = 2.5 \log b_1/b_2$

$\therefore b_*/b_{sun} = 197.7$

(c) Estimate its radius in Solar radii. [1 mark]

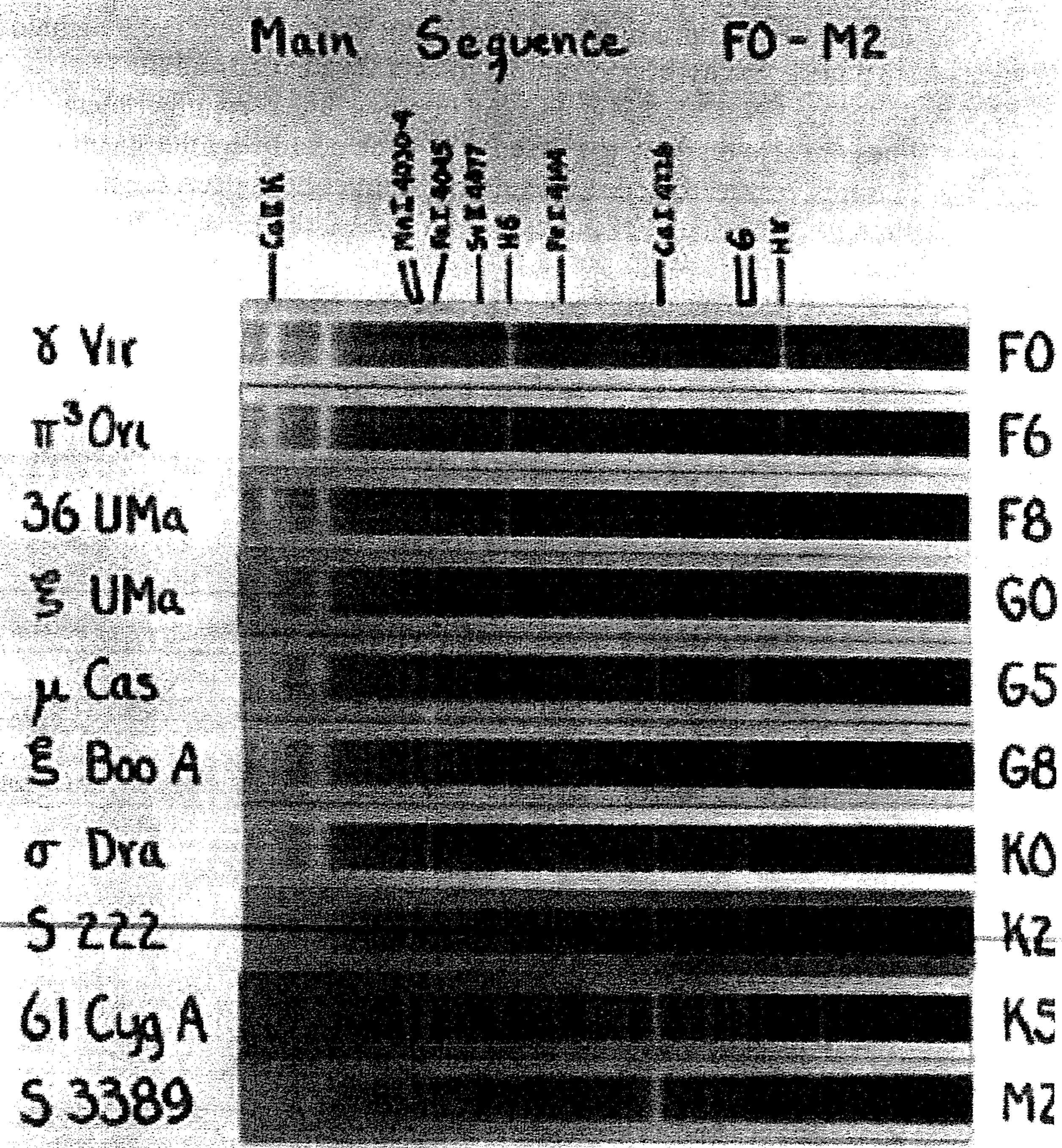
$$\frac{L_*}{L_\odot} = \frac{R_*^2 T_*^4}{R_\odot^2 T_\odot^4}$$

$$\therefore \left(\frac{T_\odot}{T_*}\right)^4 \times 197.7 = \left(\frac{R_*}{R_\odot}\right)^2 \Rightarrow \left(\frac{R_*}{R_\odot}\right) = 2.18$$

(3) Following are spectra for main sequence stars in the range of spectral class from F0 to M2. (a) What is the approximate range in surface temperature of these stars? Some spectral lines are indicated. Hδ and Hγ are hydrogen Balmer lines, G is a band from the molecule CN, CaI is a neutral calcium line and a few other metal lines are also marked. (b) Discuss in some detail why the spectral features vary as they do. Why do the Balmer lines decrease in strength, why does CN increase, why does CaI increase? [total 5 marks]

(a) F0 ≃ 7000K, M2 ≃ 3500K.

(b) Balmer decrease in strength as T_{eff} falls. This is because fewer atoms in 1st excited state where they can absorb a Balmer photon. CN increases as it is a molecule. Molecular bonds are weak ∴ as you get hotter fewer molecules are present - broken up into constituent atoms. Ca I lines increase to cooler temperature because it is a line from neutral Calcium. As you get hotter more Ca is ionized ∴ fewer neutral Ca lines ∴ weaker line.


Main Sequence F0 - M2
CaII K
MnI 4030-4
FeI 4045
SrII 4077
Hδ
FeI 4144
CaI 4226
G
Hγ
γ Vir
π³ Ori
36 UMa
ξ UMa
μ Cas
ξ Boo A
σ Dra
S 222
61 Cyg A
S 3389
F0
F6
F8
G0
G5
G8
K0
K2
K5
M2

(4) The temperature of the microwave background is 2.72548K and it is a nearly perfect blackbody. Suppose the motion of the Sun around the galactic centre at 220 km/s were our only motion with respect to the microwave background. What would you measure for the temperature of the microwave background in the direction of the Sun's motion? [3 marks]

$\lambda_{max} = \frac{3\times10^6}{T}$, $\frac{\Delta\lambda}{\lambda} = \frac{V}{c} = \frac{220}{3\times10^5} \Rightarrow \Delta\lambda = 807.20\, nm$

and this is negative

$\lambda_{max} = 1,100,723.54\, nm$

$\therefore$ new $\lambda_{max} = 1,099,916.34$

$\therefore \frac{1,099,916.34}{3\times10^6} = \frac{1}{T} \Rightarrow$ $T = 2.72748\,°K.$

(5) A giant star expands increasing its radius by a factor of 10. Assuming that its effective temperature stays the same at all times, what is the change in the apparent magnitude of the star due to the expansion (it is important that you get not only the magnitude but also the sign of the change right!)? [2 marks]

$\frac{L_2}{L_1} = \frac{R_2^2 T_2^4}{R_1^2 T_1^4}$

$\therefore \frac{L_2}{L_1} = 100.$

$m_2 - m_1 = 2.5 \log \frac{L_1}{L_2}$

$m_2 - m_1 = 2.5 \log \frac{1}{100}.$

$m_2 - m_1 = -5$

6) A star lying 20 parsecs from the Sun has a proper motion of 0.5"/year. What is the star's transverse velocity (in km/sec)? If the star's spectral lines are observed to be redshifted by 0.01 percent, calculate its three-dimensional velocity relative to the Sun. (3 marks)

$\theta = \frac{\text{arc}}{\text{radius}}$

arc = radius × θ $= \frac{20\,pc \times 3\times10^{13}\,\frac{km}{pc} \times 0.5''/yr}{206265}$ km/year

= ~~48.5~~

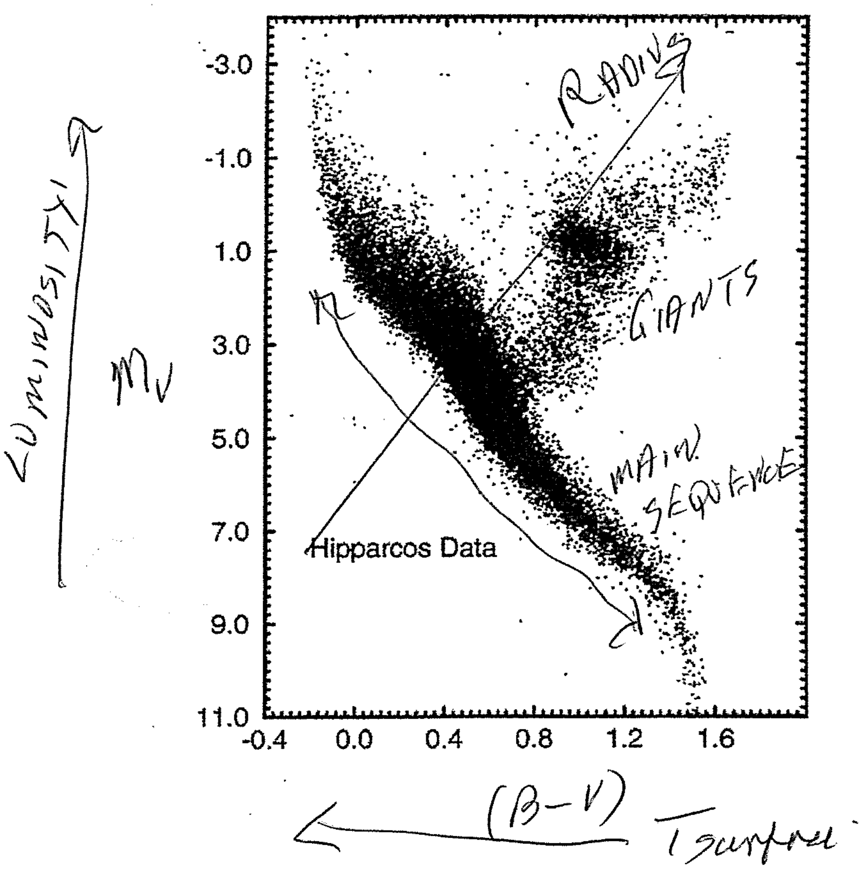
48.5 km/s = transverse

Radial vel $= 10^{-4} \times c = 30$ km/s

$\therefore \sqrt{V_T^2 + V_R^2}$ = 3-D velocity

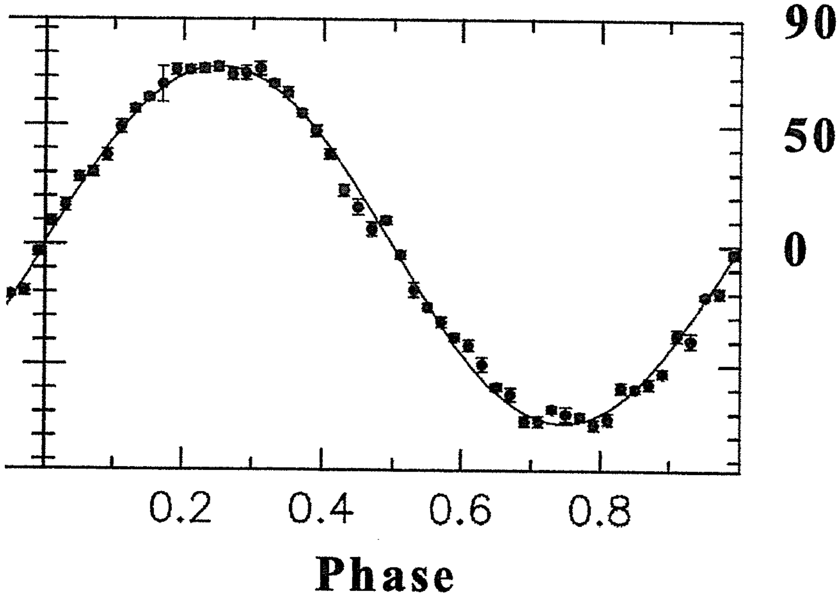
= 57 km/sec

7) Below is a colour-magnitude diagram obtained by the Hipparcos Parallax Satellite. a) Insert plausible axes labels on the diagram. b) Label right on the figure 2 major sections of the diagram. c) Indicate the direction of increasing surface temperature, increasing luminosity and increasing stellar radius on the diagram. (3 marks)



(8) A single-lined spectroscopic binary has a radial velocity curve as shown below. The period is 5.6 days and the radial velocities are indicated in units of km/sec along the y-axis. The visible star is an O-supergiant with a mass of 19.2 solar masses. The orbit of the binary is inclined by 20 degrees to the line of sight.

What can you say about the unseen secondary object? [5 marks]



Use mass fn $\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2} = PV^3/2\pi G$

$P = 5.6$ days $= 4.84 \times 10^5$ sec

$V = 75$ km/sec $= 75{,}000$ m/sec

$\therefore$ RHS $= \frac{4.84 \times 10^5 \times (75 \times 10^3)^3}{2\pi \times 6.67 \times 10^{-11}}$ Kg.

$\cong 4.87 \times 10^{29}$ Kg.

$\therefore \frac{m_2^3 (0.34)^3}{(m_1+m_2)^2} = 4.87 \times 10^{29}$ kg or $\frac{m_2^3}{(m_1+m_2)^2} = 1.2 \times 10^{31}$ Kg

$m_1 = 19.2 \times 2 \times 10^{30}$ Kg $= 3.84 \times 10^{31}$ Kg.

Try a few values for m_2 e.g $10 M_\odot = 2 \times 10^{31}$ Kg

$10 M_\odot$; LHS $= 2.35 \times 10^{30}$ (Too small)

$40 M_\odot$; LHS $= 3.7 \times 10^{31}$ (too large)

$\therefore M_2 \simeq 30 M_\odot$ $\therefore$ for sure a Black Hole.