Anisotropies in the Cosmic Microwave Background

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1 Introduction

In 1964, Penzias & Wilson (1965) serendipitously detected the microwave background as anomalous excess instrumental noise, coming from all directions and corresponding to a temperature of $\sim 3\,\mathrm{K}$. This was immediately interpreted as being a relic of the Primeval Fireball by Dicke et al (1965), who had already been preparing an experiment in the hope of detecting it. Recently the remarkable success of the FIRAS instrument on the COBE satellite has confirmed that the cosmic microwave background radiation has a Planck spectrum with (Mather et al 1994)

$$T_0 = 2.726 \pm 0.010\,\mathrm{K}\ (95\%\,\mathrm{CL}). \tag{1}$$

Since thermalization occurred in the standard Big Bang model at an epoch $t \lesssim 1\,\mathrm{yr}$ or $T \gtrsim 10^7\mathrm{K}$, the blackbody nature of the cosmic microwave background strongly suggests an origin in the early universe. The remote origin of the CMB was further supported by the high degree of isotropy apart from the detection, in 1975, of the dipole anisotropy (Corey & Wilkinson 1976, Smoot et al 1977). The best-fitting dipole is $D_{\mathrm{obs}} = 3.347 \pm 0.016\,\mathrm{mK}$ (95% CL) towards $(\ell, b) = (265^\circ\!.6 \pm 0^\circ\!.8, 48^\circ\!.3 \pm 0^\circ\!.5)$ (Smoot et al 1991, 1992, Kogut et al 1993, Fixsen et al 1994). After correction for the motion of the Earth around the Sun, the Sun around the Galaxy, and the Galaxy relative to the center of mass of the Local Group, one infers (Smoot et al 1991, Kogut et al 1993) that our Local Group of galaxies is moving at a velocity of $627 \pm 22\,\mathrm{km\,s^{-1}}$ in a direction $(\ell, b) = (276^\circ \pm 3^\circ, 30^\circ \pm 3^\circ)$. Convergence of the local velocity vectors to the CMB dipole does not occur until a distance of $> 100\,h^{-1}\mathrm{Mpc}$ according to the dipole measured in the IRAS all–sky galaxy redshift survey (Strauss et al 1992) and possibly to $> 150\,h^{-1}\mathrm{Mpc}$ if the recent claim of a dipole in the nearby Abell cluster frame is confirmed (Lauer & Postman 1992, 1993). Theoretical arguments actually suggest that convergence may only be logarithmic (Juszkiewicz et al 1990) if the large–scale density fluctuation spectrum has the Harrison-Zel'dovich form, $\delta\rho/\rho \propto \lambda^{-(n+3)/2}$ with $n = 1$ (Harrison 1970, Peebles & Yu 1970, Zel'dovich 1972).

Detection of anisotropy on smaller scales than that of the dipole has proved extremely difficult. The original detection paper set limits of about 10% on any anisotropy (Penzias & Wilson 1965). By 1968, the first simplistic theoretical predictions suggested that galaxy formation implied fluctuations in the CMB of the order of 1 part in 10^2 (Sachs & Wolfe 1967) or 10^3 (Silk 1967, 1968). As experimental sensitivity improved, the theoretical calculations grew more sophisticated (e.g. Peebles & Yu 1970, Doroshkevich et al 1978, Wilson & Silk 1981), predicting $\Delta T/T \sim 10^{-4}$ for universes containing predominantly baryonic matter. Claims of an electron neutrino mass of about 30eV (Lyubimov et al 1980) stimulated interest in non-baryonic dark matter–dominated universes (e.g. Bond et al 1980, Doroshkevich et al 1980). Neutrinos as dark matter failed to account for structure formation (e.g. Kaiser 1983, White et al 1983, 1984) despite the fact that the neutrino window (now closed for ν_e but still open for ν_μ or ν_τ) allows neutrinos to be a plausible dark matter candidate (e.g. Steigman 1993).

After 1980, the inflationary cosmology (see Narlikar & Padmanabhan 1991 for a review) revived interest in non–baryonic dark matter, now considered more likely to be of the cold variety (e.g. Peebles 1982a, Blumenthal et al 1984, Frenk et al 1990). The hot/cold classification (Bond & Szalay 1983) amounts to the velocity dispersion of the candidate particle being much greater or much less than the canonical escape velocity of a typical galaxy: $\sim 300\,\mathrm{km\,s^{-1}}$ at the epoch of equal densities of matter and radiation, $1 + z_{\mathrm{eq}} = 23{,}900\,(\Omega_0 h^2)$. Only at later times does substantial sub-horizon fluctuation growth occur.

As limits improved on small-scale fluctuations, to $\Delta T/T \sim 10^{-4}$ (Uson & Wilkinson 1984), refined theoretical estimates showed that, with the aid of dark matter, one could further reduce $\Delta T/T$ by an order of magnitude. (For a summary of the pre-COBE experimental situation see Partridge 1988, Readhead & Lawrence 1992). The experimental breakthrough came in 1992 (Smoot et al 1992) with the first detection of large angular scale anisotropies of cosmological origin in the CMB by the COBE DMR experiment (Smoot et al 1990). This has since been confirmed by at least one other experiment (Ganga et al 1993). The sky coverage and frequency range spanned means that one can now, with confidence, eliminate any galactic explanation, as well as the possibility that nearby superclusters containing diffuse hot gas are imprinting Sunyaev–Zel'dovich fluctuations on the CMB (Hogan 1992, Rephaeli 1993a,b, Bennett et al 1993). This is further strengthened by the lack of correlation with the X-ray background (Boughn & Jahoda 1993). Fluctuations are telling us about density perturbations at $z \sim 1000$. The first year DMR data represent a $> 7\sigma$ detection, with the best determined measurement being the sky variance on scales of 10°, $\sigma_{\mathrm{obs}}(10^\circ) = 30 \pm 5\,\mu\mathrm{K}$ (Smoot et al 1992).

Several other experiments have subsequently reported detections on intermediate angular scales, $\sim 1^\circ$ (with 9 separate claims of detection of fluctuations by the end of 1993). These are all generally consistent with the COBE amplitude, given plausible extrapolation from the large angular scales, as described below, although there is cause for serious concern about foreground galactic contamination as a consequence of limited sky and frequency coverage.

We are certainly now on the verge of a quantum leap in cosmological modelling. Large–scale 'seed' power has been discovered at a level of $\Delta T/T \sim 10^{-5}$. These fluctuations are the fossil precursors of the largest structures we see today, on scales of $\leq 50\,h^{-1}$Mpc. On angular scales $\geq 10^\circ$, they are also relics of the apparently acausal initial conditions in the Big Bang, which are accounted for by inflationary cosmology, and hence provide a possible verification of inflation. Gravity waves are another legacy from inflation, and can leave a distinguishable signature imprinted on the CMB (see Burke (1975) and Doroshkevich et al (1977) for a pre-inflation view). Indeed, there have been recent proposals to utilize the CMB fluctuations on large scales to reconstruct the inflaton potential.

The connection between large-scale power in the matter distribution and that in the CMB is conceptually simple, if at early epochs one is in the linear regime. The direct comparison in scales is simply that at large redshift, a comoving scale of 100 Mpc projects to an angular scale of approximately $(\Omega_0 h)$ degrees. Complications arise for several reasons. Inflation predicts the fluctuations are Gaussian. However, in non-inflationary cosmologies, especially likely if $\Omega_0 < 1$ as favoured by observations of the local universe, the intial conditions are non–Gaussian. Moreover, one may have non–linear topological defects as the source of seed density fluctuations. We cannot yet predict with much confidence the likely implications of such models for CMB anisotropies, largely because the connection with large–scale structure observations, to which the theory must ultimately be normalized, is tenuous.

Given an initial spectrum of density fluctuations, $\delta(k)$, one can calculate the transfer function to obtain the radiation power spectrum $P_{\rm rad}(k)$. The scale $z_{\rm eq} \propto (\Omega h^2)^{-1}$ is imprinted, thereby inevitably guaranteeing a dependence on both Ω_0 and, when normalized to an observed scale, H_0. Curvature can complicate the matter further since, in a low Ω_0 universe, on scales large than the curvature radius there is no unique definition of the matter fluctuation power spectrum.

Other cosmological model parameters that enter less directly are Ω_B, Ω_Λ and Ω_ν, the contributions of Ω_0 in baryons, vacuum and massive neutrinos, respectively. These all modify the detailed transfer function for a given $\delta(k)$. The ionization history of the Universe is yet another unknown. The intergalactic medium is highly ionized at $z = 5$. If it were even ninety percent ionized at $z \gtrsim 20$, the modification of the predicted $\Delta T/T$ can become significant, at the 10-20 percent level, on angular scales of a few degrees. If ionization occurred much earlier, there is strong smoothing of degree scale fluctuations, but at the cost of re-generating them together with subarcminute scale fluctuations in second order on the new last scattering surface.

This review is arranged as follows. Section 1 presents an overview of recombination, and introduces the various sources of temperature fluctuations. Section 2 summarizes structure formation theory, and the different fluctuation modes. The power spectrum formalism is described in Section 3. In Section 4, we review Gaussian autocorrelation function fitting, window functions, and alternative approaches to data analysis. Higher order effects (such as reionization) are described in Section 5. Problems arising from various type of uncertainties we summarize in Section 6. Section 7 discusses alternatives to the 'standard' model and Section 8 describes some issues that a new generation of experiments will have to address.

1.1 Recombination

The photons we observe from the microwave background have travelled freely since the matter was highly ionized and they suffered their last Thomson scatterings. If there has been no significant early heat input from galaxy formation, then this happened when the Universe became cool enough for the protons to capture electrons (the recombination epoch). If the Universe was reionized early enough, then the photons will have been scattered more recently, the effects of which we discuss in section 5.1. To understand the CMB fluctuations we observe, it is crucial to have a good picture of the recombination process.

The process of recombination would proceed via the Saha equation (see e.g. Lang 1980), except that recombinations to the ground state are inhibited by the recombination process itself (Novikov & Zel'dovich 1967). Thus recombination is controlled by the population of the first excited state, and the physical processes which either populate or depopulate it in the expanding Universe. This problem was first worked out in detail by Peebles (Peebles 1968) and at about the same time by Zel'dovich et al (1968).

Any modification in our understanding of recombination would be crucial for microwave background anisotropies, but in fact little has changed since the seminal work of the late 1960's, the only significant improvement being that of Jones & Wyse (1985), who refined some of the earlier assumptions and included the possibility of non-baryonic matter. Matsuda, Sato & Takeda (1971) considered the effects of collisional processes, which are negligible, and Krolik (1989,1990) showed that two previously unconsidered scattering effects in the Ly α line almost completely cancel one another.

Solving the coupled equations for ionized fraction and matter temperature gives the evolution of the ionized fraction $x_e(z) \equiv n_e/n_B$ and the visibility function $g(z) \equiv e^{-\tau} d\tau/dz$, for Thomson scattering optical depth τ. This function measures the probability that the radiation was last scattered in a redshift interval dz. It is reasonably well approximated by a Gaussian with mean $z_{\rm rec} \simeq 1100$ and width $\Delta z \simeq 80$, largely independent of Ω_0, Ω_B and H_0, as shown in Table 1.

Thus the epoch and thickness of the last scattering surface can be assumed to be independent of the cosmological model, although the amount of scattering will depend on $\Omega_B h^2$, the angular scales will depend on Ω_0 etc. Useful approximations to $x_e(z)$ and $g(z)$ are given by (Sunyaev & Zeldovich 1970, Zabotin & Nasel'skiĭ 1982, Jones & Wyse 1985, Grachev & Dubrovich 1991, Fink 1993). Note that $z_{\rm rec} = 1100$ corresponds to $T_{\rm rec} = 0.26\,{\rm eV}$, and $t_{\rm rec} = 5.6 \times 10^{12}(\Omega_0 h^2)^{-1/2}\,{\rm sec}$. The thickness of the last scattering surface $\Delta z = 80$ at this epoch corresponds to a comoving scale of $6.6\,\Omega_0^{-1/2}\,h^{-1}{\rm Mpc}$ and an angular scale of $3\overset{\prime}{.}8\,\Omega_0^{1/2}$.

Ω_0	0.1					0.2			1				
Ω_B	0.1		0.05		0.01	0.05		0.01	0.1		0.05		0.01
h	0.5	1	0.5	1	1	0.5	1	1	0.5	1	0.5	1	1
$z_{\rm rec}$	1060	1110	1070	1060	1100	1080	1060	1100	1080	1070	1100	1080	1170
Δz	81.8	85.1	85.8	82.5	98.4	89.5	84.4	106	91.5	85.5	104	92.1	135

Table 1: Parameters for the redshift of recombination for a range of cosmologies. The location and width are obtained by fitting a Gaussian to $\exp(-\tau)d\tau/dz$.

It is also worth pointing out that in the expanding Universe the fractional ionization approaches a constant, which is significantly different from zero $x_e(\text{residual}) \propto (\Omega_0 h^2)^{1/2}(\Omega_B h^2)^{-1}$. For models with significant reionization, radiation drag may also be important (see e.g. Rees 1977, Peebles 1993). The fluctuations cannot grow until the photons release their hold on the matter which happens at $1 + z_{\rm drag} \simeq 120\,(\Omega_0 h^2)^{1/5} x_e^{-2/5}$.

There is a prediction that there must be broad lines in the CMB spectrum due to the photons produced during H (and He) recombination at $z_{\rm rec}$. These distortions may be large in the Wien region (Peebles 1968, Zel'dovich et al 1968, Lyubarski & Sunyaev 1983, Fahr & Loch 1991), but there are few photons out there, so this effect will be swamped by the background at $\sim 100\mu$m. In the Rayleigh-Jeans region, where there are more photons, the distortions are small (Dubrovich 1975, Bernstein et al 1977), although they may be enhanced if there is some extra energy injection during recombination (Lyubarski & Sunyaev 1983). If such lines could ever be detected they would be a direct probe of Δz and the physics of recombination. Recombination can also lead to trace amounts of the primordial molecules H_2, HD, LiH etc. (Lepp & Shull 1984, Puy et al 1993). Resonance scattering by intergalactic LiH molecules at $z \lesssim 400$ may possibly result in smoothing of CMB fluctuations up to degree scales (Dubrovich 1993, Melchiorri 1993, Maoli et al 1993) at long wavelengths, as the resonance line is redshifted.

1.2 Sources of $\Delta T/T$

In the standard recombination picture, the cosmic plasma becomes neutral, and the microwave background photons are last scattered, at redshifts $z \sim 1100$. Hence any observed variation in the intensity of these photons gives us direct information about the Universe at that epoch, and potentially much earlier when the fluctuations were initially laid down. The theory behind microwave background anisotropies is reviewed by Kaiser & Silk (1986), Bond (1988), Efstathiou (1990) and others. Several effects contribute to fluctuations in the observed temperature of the radiation (in rough order of importance with decreasing angular scale):

- $\Delta T/T = V_\odot/c$ — dipole anisotropy, where $V_\odot$ is our motion relative to the radiation;
- $\Delta T/T = -\delta\phi$ — gravitational potential or Sachs-Wolfe fluctuations;
- $\Delta T/T = {}^1\!/_3\,\delta\rho/\rho$ — if the perturbations are adiabatic;
- $\Delta T/T = -{}^1\!/_3\,\delta S$ — if the perturbations are isocurvature;
- $\Delta T/T = v/c$ — Doppler shifts, when the photons were last scattered;
- $\Delta T/T = -2kT_e/(m_e c^2)$ — Sunyaev-Zel'dovich fluctuations caused by scattering off hot electrons.

We are not concerned here with the dipole contribution (which up until COBE was the only measured temperature variation), since it is 'extrinsic', being caused by our local motion (Peebles & Wilkinson 1968). [The 'intrinsic' dipole is expected to be of similar amplitude to the quadrupole, i.e. ~ 100 times smaller; similarly the 'extrinsic' quadrupole has amplitude $\frac{1}{2}(v/c)$ of the dipole]. Sachs-Wolfe (1967) fluctuations dominate at the largest scales, above that subtended by the horizon at last scattering. Large-scale anisotropies are higher in the case of isocurvature perturbations because the potential and intrinsic fluctuations add rather than partially cancelling as in the case of adiabatic perturbations (Efstathiou & Bond 1986, Kodama & Sasaki 1986. See section 2.4). These large-scale fluctuations are largely independent of any reionization, simply because no causal process during the scattering epoch can affect scales larger than the horizon size. At scales smaller than this, the radiation perturbations relax to a state of pressure equilibrium; therefore there are essentially no fluctuations caused by the peculiar gravity at small scales (Kaiser 1984).

Whether Sunyaev-Zel'dovich (Zel'dovich & Sunyaev 1969, Sunyaev & Zel'dovich 1970, 1972) fluctuations will be important depends on details of the distribution of hot electrons during a possible reionization. They will be largest in pancake (Szalay et al 1983, SubbRao et al 1994), explosion (Hogan 1984) or other models with significant re-heating. However $\Delta T/T \sim 10^{-5}$ is expected in nearby rich clusters on arc minute scales (e.g. Sunyaev 1978, Rephaeli 1981, Schaeffer & Silk 1988, Cole & Kaiser 1988, Trester & Canizares 1989, Cavaliere et al 1991, Bond & Myers 1991, Makino & Suto 1993), irrespective of reionization, with fluctuations in arbitrary directions an order of magnitude smaller (e.g. Markevitch et al 1991, Scaramella et al 1993). This effect causes a spectral distortion, which is easiest to observe as a decrement in the Rayleigh-Jeans part, and hence can be distinguished from the other effects, which are primeval in origin (Sunyaev & Zel'dovich 1972, Zel'dovich et al 1972). S-Z distortions have been seen in a few of the richest clusters (e.g. Gull & Northover 1976, Birkinshaw et al 1981, 1984, Uson 1986, Birkinshaw 1990, Klein et al 1991, Herbig et al 1992, Birkinshaw et al 1993. Wilbanks et al 1993, Jones et al 1993).

At small angular scales, the dominant mechanisms are the adiabatic fluctuations (Silk 1967) and scattering off moving electrons (Sunyaev & Zel'dovich 1970). In addition there is a complication at small scales, caused by gravitational lensing of the microwave background radiation by clustered matter (Chitre et al 1986, Blanchard & Schneider 1987, Kashlinsky 1988, Linder 1988a,1990, Feng & Liu 1992), but Cole & Efstathiou (1989), Tomita & Watanabe (1989) and Sasaki (1989) have shown that the effects are negligible for most models of galaxy formation, except perhaps at sub-arcminute scales (Cayón et al 1993a,b,c). Lensing can enhance fluctuations on the smallest scales; the effect can be considered as observing the CMB with a beamwidth of order the dispersion of deflection angles. Furthermore, discrete sources at high redshift, with their radiation reprocessed by one of several emission mechanisms, could produce frequency-dependent fluctuations which would increase towards small angular scales (e.g. Dautcourt 1977, Sunyaev 1977, 1978, Hogan 1980, 1982, Korolëv et al 1986, Bond et al 1986, 1991a). Of course, faint radio sources, as well as diffuse emission from our own Galaxy, can give anisotropies, although they are normally regarded as a contaminant rather than a source (e.g. Danese et al 1983, Banday & Wolfendale 1990, 1991a,b, Banday et al 1991, Franceschini et al 1989, Masi et al 1991, Brandt et al 1993).

Schematically, calculations of the contribution from different sources are performed via an integral through the scattering surface (see however section 5.1). For example, for the Doppler-induced variations, the total perturbation to the radiation temperature can be expressed as:

$$\frac{\Delta T}{T} = \int \left(\frac{v_\parallel}{c}\right) e^{-\tau} \frac{d\tau}{dt} dt, \qquad (2)$$

where the integrand is evaluated at time t, with τ the optical depth measured from the present back to t, and $v_\parallel$ the component of peculiar velocity along the line of sight. The exponential factor is an approximation which allows for the effect of multiple scatterings, a very good approximation since the predicted fluctuations are so small.

In fact to obtain accurate estimates of the fluctuations, and to determine the angular power spectrum, numerical calculations need to be done (see section 3.2). However, there are semi-analytical methods for either standard recombination, using the tight-coupling limit [which gives both the matter and radiation spectra] (Doroshkevich et al 1978, Bonometto et al 1983, 1984, Starobinskiĭ 1988, Doroshkevich 1988, Artio-Barandela et al 1991, Dodelson & Jubas 1993b, Atrio-Barandela & Doroshkevich 1994a) or for reionization, using the free-streaming approximation [which assumes a given $T_m(k)$] (Vishniac 1987, Efstathiou 1988, Hu et al 1994, Atrio-Barandela & Doroshkevich 1994b).

2 Theory

2.1 Inflation

Perhaps the most well studied paradigm for producing density fluctuations in the early universe is the inflationary mechanism. A review of the inflationary predictions for primordial power spectra has recently been completed by Liddle & Lyth (1993). We will include here a brief overview for completeness (see also Narlikar & Padmanabhan 1991).

An inflationary phase drives the Universe towards flat spatial hypersurfaces, i.e. $\Omega + \Lambda/3H^2 = 1$, or $\Omega_0 = 1$ if there is no cosmological constant today (which is the standard assumption, also generally adopted in this review). In the inflationary paradigm, the fluctuations which cause temperature anisotropies in the CMB are generated by fluctuations in quantum fields during the inflationary phase. The wavelength of the fluctuations is stretched by the general expansion until they represent modes outside the horizon. For such modes the equation of motion is simple: the amplitude is a constant or, in the common jargon, they are "frozen in". When the period of inflation ends, the horizon grows faster than the scale factor, and eventually (today) these fluctuations "reenter" the horizon. For a massless field, since the amplitude of the fluctuation is frozen in while the energy of each quantum is redshifting with the general expansion, the number of quanta describing each state has to increase dramatically. Thus when the fluctuation reenters the horizon it can be considered to evolve classically.

During inflation, when the perturbations were generated, the scale is set by the Hubble constant, so we expect that the power spectrum of fluctuations is proportional to $(H_k/m_{Pl})^2$ where H_k is the Hubble constant at the time mode k left the horizon during inflation. This is the entire story for isocurvature fluctuations. For fluctuations in the inflaton field, which are adiabatic and the primary source of density perturbations from inflation, a solution of the perturbed Einstein equations shows that the energy density associated with the fluctuation grows while it is outside the horizon. The growth depends on the details of the inflaton potential V. The final result is that adiabatic fluctuations are enhanced over their isothermal counterparts by a factor of $(V/V')^2$ (see e.g. Kolb & Turner 1990).

If, during inflation, the Hubble constant changes only slowly, then we expect that the spectrum should be nearly scale invariant (in horizon crossing coordinates) or Harrison-Zel'dovich. If H_k is nearly constant then so is V (using the Friedman equation) and we expect the enhancement of the adiabatic fluctuations to be large. As the potential becomes steeper the spectrum becomes more tilted, and the isocurvature fluctuations *can* become comparable to their adiabatic counterparts. Since in general V decreases with time, the slope of the power spectrum from inflation generically has more power on larger scales (see however Mollerach et al 1993).

The inflationary paradigm generally assumes that the inflaton field, or other fluctuating fields, are weakly coupled during the epochs under consideration. In this case the fluctuations are predicted to be Gaussian. Models in which the inflaton field couples strongly to other fields or has non-negligible self interactions can give rise to non-Gaussian fluctuations (Allen et al 1987). Also if the evolution equations for the inflaton field are non-linear, it is possible that fluctuations can be non-Gaussian (Kofman et al 1991, Moscardini et al 1991, Scherrer 1992).

2.2 Structure formation theories

The standard model of structure formation, which we shall use to explore the CMB fluctuations on degree scales, has been the Cold Dark Matter (CDM) model (e.g. Peebles 1982a, Blumenthal et al 1984, Davis et al 1985, Ostriker 1993). In this model $\Omega_0 = 1$, with a variable fraction Ω_B residing in baryons and the rest in massive (non-relativistic) dark matter. The initial fluctuations are assumed to be Gaussian distributed, adiabatic, scalar density fluctuations with a Harrison-Zel'dovich spectrum on large scales, i.e. $P_{\rm mat}(k) \propto k^n$ with $n = 1$.

Under these assumptions the matter power spectrum can be calculated. It depends only on Ω_B and the Hubble constant $H_0 = 100\,h\,{\rm km\,s^{-1}Mpc^{-1}}$. The main feature is a turnover from the $P_{\rm mat}(k) \propto k$ form at large scales (small k) to a k^{-3} falloff at small scales (large k). The turnover occurs at $k \simeq 0.01\,h\,{\rm Mpc}^{-1}$, the horizon size at matter-radiation equality. The reason for this turnover is that perturbations which enter the horizon before the universe becomes matter dominated do not grow. This retards the growth of fluctuations on small scales, which spend longer inside the horizon during the radiation dominated era. It is assumed the baryons fall into dark matter potentials that formed due to gravitational instability once the photon drag becomes small enough. The standard ($\Omega_0 = 1$, $n = 1$, $h = 0.5$) CDM matter spectrum (Holtzman 1989) has been plotted as the solid line in Figure 1 along with the power spectrum for the other models discussed in this section.

An alternative to CDM, now out of favour, is the Hot Dark Matter (HDM) model (e.g. Zel'dovich 1970, Bond et al 1980, Bond & Szalay 1983, Centrella et al 1988, Anninos et al 1991, Cen & Ostriker 1992). In this model the difference between Ω_B and $\Omega_0 = 1$ is made up of relativistic particles. These relativistic particles have a minimum scale on which gravitational instability can cause overdensities due to their free streaming. This leads to a small scale (large k) cutoff in the power spectrum as shown by the short-dashed line in Figure 1 (Holtzman 1989) for $\Omega_\nu = 0.94$ and $h = \frac{1}{2}$ ($m_\nu = 22$eV). In this model galaxies form by the fragmentation of larger structures unless there is some extra small-scale power (e.g. Dekel 1983, Brandenberger et al 1990, Gratsias et al 1993).

A popular idea currently (e.g. Schaefer & Shafi 1992, M Davis et al 1992, van Dalen & Schaefer 1992, Taylor & Rowan-Robinson 1992, Holtzman & Primack 1993, Klypin et al 1993), is the Mixed Dark Matter (MDM) model (Shafi & Stecker 1984, Achilli et al 1985, Umemura & Ikeuchi 1985, Bardeen et al 1987, Schaefer et al 1989, Holtzman 1989) which combines the above two scenarios, and may account for the extra power in large-scale structure measurements over the predictions of CDM (e.g. Maddox et al 1990, Kaiser et al 1991) The current favourite has a HDM component (of relic, massive neutrinos say, with $m_\nu \simeq 7\,$eV) of 30% of closure density and a CDM component (plus baryons) making up the rest. The radiation power spectrum for this model is very similar to that of CDM except on the very smallest scales where it is already strongly damped by the thickness of last scattering (see section 3.2). An approximation to the

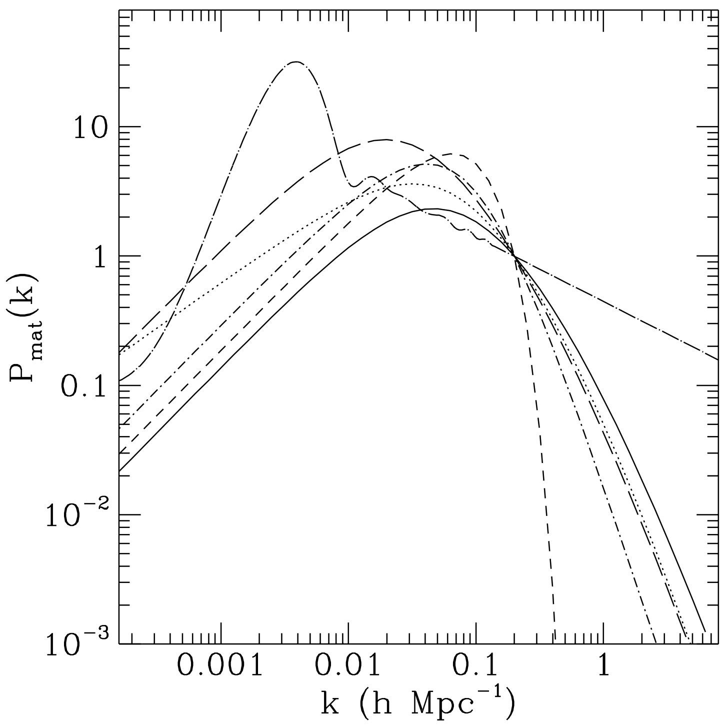

Figure 1: The matter power spectrum for a range of models of structure formation. The models are: CDM (solid), tilted CDM (dotted), HDM (short dashed), ΛCDM (long dashed), MDM (dot-short-dashed) and BDM (dot-long-dashed). All models have been arbitrarily normalized to 1 at $k = 0.2\,h\,\mathrm{Mpc}^{-1}$, which corresponds roughly to σ_8 normalization.

matter power spectrum is shown by the dot-short-dashed line of Figure 1 (Holtzman 1989).

Alternatively, the difference between the CDM component and $\Omega_0 \simeq 1$ could be made up of a cosmological constant (e.g. Peebles 1984, Turner et al 1984, Efstathiou et al 1990, Kofman et al 1993). Including a non-zero Λ has little effect on large-scale structure, other than the enhancement of large-scale power due to lowering Ω, an effect that is used to account for the galaxy-galaxy correlations found in deep surveys (Maddox et al 1990, Loveday et al 1992) as well as to allow an older universe (e.g. Gunn & Tinsley 1975). The matter power spectrum for $\Omega_\Lambda = 0.8$ and $h = 1$ (Efstathiou et al 1992) is shown in Figure 1.

On all but the smallest scales, the CDM, HDM, ΛCDM and MDM models give very similar radiation power spectra, showing that large and intermediate scale CMB fluctuations are not very sensitive to the form of the dark matter which makes $\Omega_0 = 1$ in these models. Since the shape of the matter spectra are changed however the relation between the large-scale power and the 'bias' is changed, affecting comparison with large-scale structure studies.

Instead of modifying the matter content of the model, one variant sticks with $\Omega_0 = 1$ in CDM, but modifies the initial power spectrum away from the Harrison-Zel'dovich form (e.g. Vittorio et al 1988, Salopek et al 1989, Cen et al 1992, Lucchin et al 1992, Fry & Wang 1992, Liddle et al 1992, Adams et al 1992, Cen & Ostriker 1993, Gottlober & Mucket 1993, Muciaccia et al 1993, and Suto et al 1990 for CMB constraints). Such "tilted" spectra can be (but are not always) associated with a stochastic background of gravitational waves. The most commonly discussed model has a spectral slope of $n \simeq 0.7$ (to be compared with the standard $n = 1$ slope) and is shown by the dotted line in Figure 1.

The Baryonic Dark Matter model (BDM, also known as PBI or PIB, see section 7.2) differs from its cold and hot dark matter rivals in that it does not assume $\Omega_0 = 1$ and postulates isocurvature rather than adiabatic fluctuations (Peebles 1987a,b, Cen et al 1993). The fluctuation spectrum is not taken to be Harrison-Zel'dovich but the slope of the spectrum of entropy fluctuations m is promoted to a free parameter that is adjusted to fit observational data. The transfer function $T(k) \to k^2$ at small k and $\to 1$ at large k. The principal feature of the (matter) power spectrum is a large 'bump' at the matter-radiation Jeans length (Doroshkevich et al 1978, Hogan & Kaiser 1983, Jørgensen et al 1993), the height of which will depend on the ionization history and the values of Ω_B and h. The 'bump' at large scales may help explain some of the large-scale velocity measurements (Groth et al 1989). Figure 1 shows a particular model (Cen et al 1993) which has $m = -0.5$, $\Omega_0 = \Omega_B = 0.1$, $h = 0.8$ and $x_e = 0.1$ (dot-dash line).

An orthogonal approach to structure formation is that of the defect models e.g. global monopoles and textures or cosmic string models. In these models, field configurations known as 'defects' which arise due to a phase transition in the early universe form the seeds for matter and radiation fluctuations. The prototypical defect model is the cosmic string model where the 'defect' is 1 dimensional. In this model all the string properties are described by the mass per unit length of the string μ. Cosmologically interesting strings have $G\mu \sim 10^{-6}$. In this model most of the fluctuations are imprinted at high redshift. The fluctuations are non-Gaussian, having strong phase correlations which lead to sharp line discontinuities on the sky (Kaiser & Stebbins 1984, Brandenberger & Turok 1986, Stebbins 1988, Bouchet et al 1988). Most defect models being discussed now (e.g. D Bennett et al 1992, Bouchet et al 1992, Veeraraghavan & Stebbins 1992, Hara et al 1992, Perivolaropoulos 1993a, Bennett & Rhie 1993, Pen et al 1993, Hindmarsh 1993, Coulson et al 1993, Brandenberger 1993) will produce a roughly scale-invariant CMB fluctuation spectrum as required. The major difference lies in the non-Gaussianness. Since on large scales (e.g. COBE), many 'defects' contribute to the observed fluctuations, the central limit theorem suggests the fluctuations will look Gaussian. One has to go to smaller scales (e.g. 1°) to observe significantly non-Gaussian structure. It is not clear whether there will be an easily detectable difference between the Gaussian and non-Gaussian models currently being considered.

2.3 The correlation function

It is conventional in CMB work to expand the temperature fluctuations in spherical harmonics $\Delta T/T(\theta,\phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$, and work in terms of the multipole moments $a_{\ell m}$. One can define the sky correlation function

$$C_{\rm sky}(\theta_{21}) = \left\langle \frac{\Delta T(\hat{n}_1)}{T}\frac{\Delta T(\hat{n}_2)}{T} \right\rangle_{21} \tag{3}$$

where the average is taken over the (observed) sky with the separation angle θ_{21} held fixed. Using the properties of $Y_{\ell m}$ the correlation function $C_{\rm sky}(\theta_{21})$ (hereafter we will drop the subscript 'sky') is

$$C(\theta) = \frac{1}{4\pi}\sum_\ell a_\ell^2 P_\ell(\cos\theta), \tag{4}$$

where we have introduced the rotationally symmetric quantity $a_\ell^2 \equiv \sum_m |a_{\ell m}|^2$. The a_ℓ^2 are not to be confused with $(2\ell+1)C_\ell$ (see later) which is often used to define the power spectrum of fluctuations in a Gaussian theory. At this point we have made no assumption about the underlying theory of fluctuations or the model of structure formation, the a_ℓ^2 are purely measured quantities on our sky. [Note that other authors' definitions of various quantities can differ by factors of 4π, $(2\ell+1)$ or similar.] The COBE team (C Bennett et al 1992, Kogut et al 1993, Fixsen et al 1994, Bennett et al. 1994) quote results for the first two moments i.e. $a_\ell T_0/\sqrt{4\pi}$ for $\ell = 1, 2$

$$D_{\rm obs} = 3.347 \pm 0.008\,{\rm mK} \qquad \text{and} \qquad Q_{\rm obs} = 6 \pm 3\,\mu{\rm K}. \tag{5}$$

A common way of comparing theory and experiment is through the $a_{\ell m}$. Of course any actual measurement of a temperature difference on the sky involves finite resolution and experimental measurement strategies. These are usually included in the theoretically predicted correlation function through a "window" or "filter" function, W_ℓ, which will be described in section 4.2.

2.4 Fluctuations

Both inflation and defect models predict that the fluctuation spectrum should be stochastic in nature. Thus we live in one sample of an ensemble of "possible universes" which was drawn from a distribution specified by the underlying theory. Due to the weak coupling nature of most inflationary theories, the distribution of fluctuations is predicted to be Gaussian. [The assumption of Gaussian *fluctuations* is not to be confused with the further restrictive assumption of a Gaussian *shape* for the power spectrum.] In contrast most defect models predict a non-Gaussian character for the fluctuations.

As discussed in section 2.2 on large (e.g. COBE) scales fluctuations in almost all theories will look Gaussian. On smaller scales there is the possibility of detecting non-Gaussianness. There have been several tests for non-Gaussian fluctuations proposed, e.g. the total curvature or genus of isotemperature contours (Coles 1988b, Gott et al 1990, Brandenberger et al 1993), the distribution of peaks (Sazhin 1985, Zabotin & Nasel'skiĭ 1985, Bond & Efstathiou 1987, Vittorio & Juszkiewicz 1987, Coles & Barrow 1987, Coles 1988a, Gutiérrez de la Cruz et al 1993, Cayón et al 1993c, Kashlinsky 1993b), skewness and kurtosis of the observed temperature distribution (Scaramella & Vittorio 1991, 1993a, Luo & Schramm 1993a, Perivolaropoulos 1993b, Moessner et al 1993) or the 3-point function (Falk et al 1993, Luo & Schramm 1993b, Srednicki 1993, Graham et al 1993). Of these, the 3-point function currently seems the most promising, although more work is needed to see if it can find departures from Gaussianness at the level predicted by the 'defect' theories.

In general the predictions of a theory are expressed in terms of predictions for the $a_{\ell m}$. If the fluctuations are Gaussian, the predictions are fully specified by giving the 2-point function for the $a_{\ell m}$ (just as the matter spectrum $P_{\rm mat}(k)$ is predicted by giving $\xi(r)$). Using rotational symmetry it is conventional to write

$$\langle a^*_{\ell m} a_{\ell' m'} \rangle_{\rm ens} \equiv C_\ell \, \delta_{\ell'\ell}\delta_{m'm} \tag{6}$$

where the angled brackets here represent an average over the ensemble of possible universes. The prediction for CMB anisotropy measurements of a theory can thus be expressed as a series of C_ℓ's. Alternatively one can work in k-space and define a (3 dimensional) power spectrum of fluctuations per wavelength interval. We describe this, and its relation to the C_ℓ's, in section 3.2. For large ℓ, the C_ℓ's are approximately the same as the 2-dimensional power spectrum of fluctuations (Bond & Efstathiou 1987).

2.4.1 Adiabatic

Any initial density perturbation may be decomposed into a sum of an adiabatic and an isocurvature perturbation. Since inflation naturally predicts adiabatic density perturbations we will consider these first.

Adiabatic modes are fluctuations in the energy density, or the number of particles, such that the "specific entropy" is constant for any species i (assumed non-relativistic here):

$$\delta\left(\frac{n_\gamma}{n_i}\right) \propto \frac{\delta\rho_i}{\rho_i} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} = 0 \tag{7}$$

In terms of the perfect fluid stress-energy tensor of general relativity the assumption of an adiabatic perturbation is equivalent to assuming that the pressure fluctuation is proportional to the energy density perturbation. For a discussion of the classification and behaviour of adiabatic perturbations in relativity see Kolb & Turner (1990), Efstathiou (1990), and Liddle & Lyth (1993).

It has been the standard assumption that inflation predicts a "flat" or Harrison-Zel'dovich ($n = 1$) spectrum of adiabatic density fluctuations. Recently, with the advent of the COBE measurement, it has been emphasized that in fact inflation generically predicts departures from the simple $n = 1$ form. In "new" inflation the departure is logarithmic, viz. $P_{\rm mat}(k) \propto k \log^3(k/k_0)$, with $k_0 \sim e^{60}{\rm Mpc}^{-1}$ – this can lead to an effectively tilted spectrum between say COBE scales and more traditional normalization scales (typically $n \approx 0.95$) even though the spectrum is "flat" on large scales. In "power-law", "chaotic", "natural" or "extended" inflation, one can have power law spectra with $n < 1$. Exotic models of inflation even allow $n > 1$ (Mollerach et al 1993).

On scales larger than the horizon size at last scattering (i.e. $\sim 2°$), the generation of temperature fluctuations from density inhomogeneities is straightforward to analyze. In addition to fluctuations in the temperature on the surface of last scattering (due to fluctuations in the radiation energy density), the matter perturbations give rise to potentials on the scattering surface and possibly time dependent perturbations in the metric. Any time dependence (such as gravity waves, to be discussed later) leads to energy non-conservation along the photon line of sight. The potentials give rise to a "red"-shifting of photons as they leave the last scattering surface. To first order in the perturbing quantity, the total energy change of a photon (above and beyond the cosmological redshifting) from the time it leaves the last scattering surface (emission) is the integral along the unperturbed path of the (conformal) time derivative of the metric perturbation ($h_{\mu\nu}$), plus the change in the potential between last scattering and observation (see appendix B):

$$\left(\frac{\Delta T}{T}\right)_{SW} = \Phi\Big|_e^o - \frac{1}{2}\int_e^o h_{\rho\sigma,0}\; n^\rho n^\sigma \, d\zeta. \tag{8}$$

where n^ρ is the direction vector of the photon and ζ is a parameter along the line of sight. If $h_{\mu\nu}$ is soley due to density perturbations the integrand is basically $4\dot\Phi$ where overdot represents a (conformal) time derivative. Either or both of these terms are known as the Sachs-Wolfe effect. The simplest part is the potential difference between the last scatterers and the observer. [The other (integral) term is usually associated with a background of gravitational waves or non-linear effects or $\Omega_0 < 1$ universes – see later.] To this energy shift must be added the temperature fluctuation on the last scattering surface itself. For fluctuations in the radiation field we have

$$\frac{\Delta T}{T} = \frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} = \frac{1}{3}\frac{\delta\rho}{\rho} \tag{9}$$

where the second equality follows from the adiabatic condition. Since an overdensity gives a larger gravitational potential ($\delta\rho/\rho = 2\delta\phi + \mathcal{O}[(k/H)^2]$) that a photon must climb out of, for adiabatic fluctuations the two terms partially cancel. One finds that $\Delta T/T = -{}^1\!/_3\,\delta\phi$. The minus sign means that CMB hot spots are matter *under*-densities.

2.4.2 Isocurvature

Isocurvature modes are fluctuations in the number density of particles which do not affect the total energy density. They perturb the specific entropy or the equation of state,

$$\delta s \equiv \delta\left(\frac{n_\gamma}{n_B}\right) \propto \frac{\delta\rho_B}{\rho_B} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} \neq 0 \tag{10}$$

While such perturbations are outside the horizon, causality precludes them from becoming an energy density perturbation. Inside the horizon however pressure gradients can convert an isocurvature perturbation into an energy density fluctuation.

In addition to isocurvature density (scalar) perturbations, there is the possibility of isocurvature gravity wave (tensor) perturbations. We will discuss the possibility of tensor perturbations below, for now we will concentrate entirely on the density perturbations. The possibility of scalar isocurvature fluctuations is not well motivated by usual inflation models, although if more than one field contributes significantly to the energy density during inflation one can get isocurvature fluctuations (the energy density fluctuation is no longer proportional to the pressure fluctuation).

For isocurvature fluctuations, a positive fluctuation in the matter density (and therefore the gravitational potential) is compensated by a negative fluctuation in the photon temperature. The Sachs-Wolfe effect and the initial temperature fluctuation therefore add (rather than cancel as in the adiabatic case), giving rise to 6 times more large-scale $\Delta T/T$ for a given "matter" perturbation. For this reason isocurvature cold dark matter models that are normalized to give the observed peculiar velocities predict too large a temperature anisotropy in the CMB.

Specifically CDM isocurvature models with roughly scale-invariant (i.e. $m = -3$) power spectra (e.g. in the axion model Axenides et al 1993) are probably ruled out (Efstathiou & Bond 1986). The situation is similar for HDM (Sugiyama et al 1989). Scale-invariant baryon-dominated models are also in serious conflict with the microwave background anisotropies (Efstathiou 1988), and cannot be saved even by invoking a cosmological constant (Gouda & Sugiyama 1992). However, models with larger n are not as yet ruled out (see also Efstathiou & Bond 1987). Isocurvature fluctuations are these days only discussed in terms of the Baryonic Dark Matter model. This is an observationally-motivated model, with low Ω_0 in baryons only. The large fluctuations generated at small scales have to be erased by the re-heating due to some early collapsed objects. The effects of such a reionization will be discussed later. Constraints from anisotropies on scales $\gtrsim 1°$ (Peebles 1987b, Sugiyama & Gouda 1992), from the Vishniac effect at small scales (Efstathiou 1988, Hu et al 1994), from spectral distortions (Daly 1991, Barrow & Coles 1991) and from the clustering properties of galaxies (Cen et al 1993) imply that only models with $-1 \lesssim m \lesssim 0$ are viable. High values of Ω_0 and high values of h which enhance the 'bump' also tend to be ruled out.

2.4.3 Gravitational waves

Until now, we have focussed on the anisotropies in the cosmic microwave background arising from density perturbations in the early universe. In many models, there is also the possibility that a stochastic background of long-wavelength gravitational waves (GW) can be produced (Starobinskiĭ 1979); for a discussion of inflationary models in this context see (Rubakov et al 1982, Adams et al 1992, Liddle & Lyth 1993). If such a background were to exist, it would leave an imprint on the CMB at large scales through the Sachs-Wolfe effect (Fabbri & Pollock 1983, Abbott & Wise 1984c, Starobinskiĭ 1985, Abbott & Schaefer 1986, Fabbri et al 1987, Linder 1988b, White 1992). With the advent of the COBE measurement of the power at large scales, many authors addressed the question of the interpretation in terms of scalar and tensor contributions (Krauss & White 1992, Liddle & Lyth 1992, Adams et al 1992, Salopek 1992, Lucchin et al 1992, Dolgov & Silk 1993).

If there is a sizable contribution from GW in the COBE-detected anisotropies, this would lower the predicted value of $(\Delta T/T)_{\rm rms}$ on smaller scales. This should be kept in mind when comparing degree scale experiments or large-scale structure studies to power spectra normalized to COBE on large scales.

Unlike the anisotropies generated by scalar fluctuations (section 3) those generated by (isocurvature) tensor perturbations, or GW, damp at scales comparable to the horizon (see e.g. Starobinskiĭ 1985, Turner et al 1993, Atrio-Barandela & Silk 1993), which means $\ell \sim \sqrt{1+z_{\rm rec}} \simeq 30$ (see appendix A). This can be understood as due to the redshifting of GW which entered the horizon before recombination. The maximal contribution to the anisotropy on some scale comes from GW with wavelengths comparable to that scale. GW begin to redshift after they enter the horizon, thus scales which are smaller than the horizon at last scattering are dominated by GW which have redshifted before the photon begins to travel to us. The different behaviour at small scales leads one to hope that the two contributions could be disentangled. A detailed numerical analysis of the anisotropy generated by GW on both large and small scales has been carried out by Crittenden et al (1993a).

In general, GW provide a small contribution to $\Delta T/T$ on top of the scalar anisotropy. One requires a comparison of both large and small scale temperature anisotropies to isolate them. On large scales, one must deal with cosmic variance while on small scales one has sample variance and uncertainties due to cosmological parameters and history which are far from orthogonal. The situation with regard to disentangling a GW signal is somewhat confused. White et al (1993) claim that cosmic variance and cosmological model uncertainty makes such a detection extremely difficult, while Crittenden et al. (1993a) predict that a definitive detection is possible. [The analysis assumed a specific form for the relation between the spectral index and the ratio of the scalar to tensor ratio: $T/S = 7(1-n)$ (R Davis et al 1992). This form requires significant correction for most theories (Liddle & Lyth 1992, Kolb & Vadas 1993) and also biases the fit towards "detection" of a tensor component. In addition, recent work (Bond et al 1994) suggests that including uncertainties in cosmological history may alter their conclusions regarding gravity waves.] This question is of some importance, since any possible GW signal will affect the power spectrum normalization inferred from COBE. Since the GW production predicted in most theories is very small, (for $n \gtrsim 0.9$ as required by COBE and Tenerife (Hancock et al 1994) $T/S \lesssim 1$) perhaps their only observable effect for some time will be in generating large angular scale CMB anisotropies (Sahni 1990, Krauss & White 1992, Souradeep & Sahni 1992, Liddle 1993, Turner et al 1993). The possibility that GW lead to an observable polarization in the CMB (Polnarev 1985) has been shown to be very small (Crittenden et al 1993b but see however Frewin et al 1993).

In theories of inflation, the normalization of the spectrum of scalar fluctuations depends on both the inflaton potential and its derivative at the epoch of fluctuation generation. In contrast, the tensor spectrum depends only on the value of the inflaton potential at the same epoch. This fact coupled with the COBE measurement can be used to limit the scale of inflation (Rubakov et al 1982, Lyth 1985, Krauss & White 1992, Liddle 1993). In principle, one can also derive information about the inflaton potential from both the tensor and scalar components of the CMB anisotropy (Liddle & Lyth 1992, R Davis et al 1992, Salopek 1992). Recently, several authors have considered the possibility of reconstructing the "inflaton" potential from CMB observations (Hodges & Blumenthal 1990, Copeland et al 1993a,b,c, Lidsey & Tavakol 1993, Turner 1993, see also Carr & Lidsey 1993) or of using relations between observable parameters as "tests" of inflation (R Davis et al 1992, Bond et al 1994 but see Liddle & Lyth 1992, Kolb & Vadas 1993).

3 Power spectrum

3.1 Power spectrum on large scales

Let us take the power spectrum of primordial fluctuations to be a power law in comoving wavenumber k. In the "processed" radiation power spectrum, this simple power law is multiplied by a transfer function $T^2(k)$. On COBE scales $T(k) \approx 1$, and we can write the temperature fluctuation power spectrum as

$$P_{\rm rad}(k) = A\,(k\eta_0)^{n-1}\,, \tag{11}$$

where A is the amplitude for scalar perturbations and $\eta_0 \simeq 3t_0 = 2H_0^{-1}$ (for $\Omega_0 = 1$) is the conformal time today with scale factor normalized to unity. By using $k\eta_0$ as our fundamental variable, we have A as a dimensionless number multiplied by something of order 10^{n-1} on COBE scales. The connection between A and the normalization of the matter power spectrum is discussed in section 3.3. [Another common convention is to define the matter power spectrum as $P_{\rm mat}(k) = Bk^n$ on large scales, which means that the dimensions of B will depend on n (and will be length^4 for $n=1$), see equation (23).]

We can write the average over universes of the moments of the temperature anisotropy as

$$\begin{aligned} C_\ell &\equiv \left\langle |a_{\ell m}|^2 \right\rangle & (12)\\ &\simeq 16\pi \int_0^\infty \frac{dk}{k} A(k\eta_0)^{n-1} T^2(k)\; j_\ell^2(k\eta_0) & (13)\end{aligned}$$

$$= \quad 2^n \pi^2 A \, \frac{\Gamma(3-n)\Gamma\left(\ell + \frac{n-1}{2}\right)}{\Gamma^2\left(\frac{4-n}{2}\right)\Gamma\left(\ell + \frac{5-n}{2}\right)}, \qquad \text{if } T(k) = 1. \tag{14}$$

(see e.g. Peebles 1982c, Bond & Efstathiou 1987). For the special case of $n = 1$, we have $C_2/A = 4\pi/3$ and $C_\ell^{-1} \propto \ell(\ell+1)$. This is often referred to as 'flat' since potential fluctuations (and the amplitude of $\delta\rho/\rho$ at horizon crossing) are independent of scale, and it also makes $\ell(\ell+1)C_\ell = \text{constant}$.

In some older literature, the normalization of the power spectrum is given in terms of ϵ_H, the dimensionless amplitude of matter fluctuations at horizon crossing. For a flat spectrum this quantity is simply $\epsilon_H^2 = (4/\pi)A$.

The normalization convention used by the COBE group is $Q_{\rm rms-PS}$. This is obtained by a best fit to the correlation function assuming a flat spectrum of fluctuations and allowing the normalization to vary. In terms of C_2, this corresponds to

$$Q_{\rm rms-PS} \simeq \left\langle Q_{\rm rms}^2 \right\rangle^{0.5} = T_0 \left(\frac{5C_2}{4\pi} \right)^{1/2}. \tag{15}$$

(For $n = 1$ the factor in parenthesis is $\frac{5}{3}A$, which allows a simple conversion from "quadrupole normalization" to our normalization in terms of A.) We would like to stress that $Q_{\rm rms-PS}$ is the COBE group's best estimate, measured from our sky, of the power spectrum normalization. It is *not* the quadrupole measured by the COBE team from their maps. The value quoted for $Q_{\rm rms-PS}$, including the effects of systematic error, is (Smoot et al 1992, Wright et al 1994a, Bennett et al. 1994)

$$Q_{\rm rms-PS} = 17.6 \pm 1.5\ \mu{\rm K} \Rightarrow A = (2.3 \pm 0.4) \times 10^{-11}, B = (5.9 \pm 1) \times 10^5 (h^{-1}{\rm Mpc})^4. \tag{16}$$

Since the analysis for $Q_{\rm rms-PS}$ assumed a flat spectrum one should not use (16) to normalize other spectra, although $\left\langle Q_{\rm rms}^2 \right\rangle^{0.5}$ is still a valid way of quoting the normalization of the power spectrum.

For the first year data a fit to the correlation function gives $n = 1.1 \pm 0.5$ (Smoot et al 1992). Including the second year data gives $n \approx 1.5 \pm 0.5$ (Bennett et al. 1994, Wright et al. 1994b) [by combining both COBE and Tenerife data, a stronger limit $n \gtrsim 0.9$ has been obtained (Hancock et al 1994)] and the inferred value of $Q_{\rm rms-PS}$ is quite correlated with n (Seljak & Bertschinger 1993, Watson & Gutiérrez de la Cruz 1993). For $n \neq 1$ the best value for the normalization is (Smoot et al 1992, Bennett et al. 1994)

$$\left. \frac{\Delta T}{T} \right|_{10^\circ} = (1.1 \pm 0.1) \times 10^{-5} \tag{17}$$

which probes a range of ℓ centred around $\ell \approx 4$ (Wright et al 1994a). Note that for $n = 1$ these two normalizations differ by $\sim 10\%$, since the fit to $Q_{\rm rms-PS}$ uses the full correlation function.

Another normalization sometimes used is the "bias", defined through

$$b_\rho^{-2} = \sigma_8^2 = \int_0^\infty \frac{dk}{k}\ A(k\eta_0)^{n+3}\ T_m^2(k)\ \left. \left(\frac{3j_1(kr)}{kr} \right)^2 \right|_{r=8\,h^{-1}{\rm Mpc}}, \tag{18}$$

where $T_m(k)$ is a matter transfer function (see later section) not to be confused with $T(k)$, and $\sigma^2(r)$ is the variance of the density field within spheres of radius r. The variance of galaxies, possibly biased relative to the matter ($\delta_{\rm gal} = b\delta_\rho$), is roughly unity on a scale of $8\,h^{-1}$Mpc (Davis & Peebles 1983). Equation (18) is non-trivial to evaluate numerically because of the "ringing" of the j_1 and the final result is dependent on the transfer function assumed. For CDM we will take (Efstathiou 1990)

$$T_m(k) = \left[1 + \left(ak + (bk)^{3/2} + (ck)^2 \right)^\nu \right]^{-1/\nu} \tag{19}$$

with $a = 6.4\Omega_0 h^{-2}$Mpc, $b = 3\Omega_0 h^{-2}$Mpc, $c = 1.7\Omega_0 h^{-2}$Mpc and $\nu = 1.13$. We will set $\Omega_0 = 1$ and $h = 1/2$ unless otherwise noted. For $n = 1$ the COBE best-fit gives $\sigma_8 \simeq 1.2$, i.e. an unbiased model. However this depends on the values of Ω_0, h etc. which are adopted. It is possible to have a non-standard (e.g. $\Omega_\Lambda \simeq 0.8$) CDM model with the galaxies significantly biased on small scales as seems to be required (e.g. Davis et al 1985, Bardeen et al 1986, Frenk et al 1990, Bardeen & Silk 1993, Carlberg 1991).

Large-scale flows also provide a measure of the power spectrum (Peebles 1993):

$$v_{\rm rms}^2(r) \propto \int \frac{dk}{k} A(k\eta_0)^{n+1} T_m^2(k) e^{-k^2 r^2}, \tag{20}$$

where $v_{\rm rms}$ is the 3D velocity dispersion smoothed with a Gaussian filter of width r. This tends to probe scales similar to the degree-scale CMB experiments. Whether there is agreement between the two measures for a particular theory is still a matter of debate (see e.g. Vittorio & Silk 1985, Juszkiewicz et al 1987, Suto et al 1988, Atrio-Barandela et al 1991, Kashlinsky 1991, 1992, 1993a, Górski 1991, 1992).

3.2 Power spectra on smaller scales

While on large angular scales the transfer function is $T(k) \approx 1$, there is significant structure on small to intermediate scales. Generally the Sachs-Wolfe effect dominates the power spectrum on scales larger than the horizon size at last scattering and the power spectrum can be taken to be a power law. On smaller scales, however, causal interactions become important and the spectrum is modified.

For a given cosmological model, the shape of the fluctuation spectrum is fixed. It depends on the primordial spectrum (e.g. k^n) and its evolution as the waves enter the horizon. This makes the spectrum at smaller scales dependent on Ω_0, Ω_B, H_0 and the dark matter. Given a cosmological model, however, both the radiation and matter power spectra are well defined. For a fixed Ω_0, the radiation power spectrum depends only on the type of dark matter at the high k end.

The calculation proceeds by considering perturbations in the photon distribution function, $f(x,q,t)$, which can be written in terms of

$$\Delta = \int d^3q\ q\,\delta f \Big/ \int d^3q\ q\bar{f}, \tag{21}$$

with $\bar{f}$ the Planck function and q the comoving photon momentum. Δ is the total energy, or brightness, perturbation, which is also $4\,\Delta T/T$ for a uniform shift in temperature. Liouville's theorem tells us that the total phase space density is conserved for collisionless particles, i.e. the distribution function is constant along particle paths: $Df/Dt = 0$. Source terms ('collisions') add on the right hand side; the important sources are baryon velocities and photon density perturbations, coupled through Thomson scattering. The Boltzmann equation (or equation of radiative transfer, written here in the synchronous gauge) for Δ is

$$\dot{\Delta} + c\gamma_i \frac{\partial \Delta}{\partial x^i} - 2\gamma_i\gamma_j \dot{h}_{ij} = \sigma_T c n_e a(\Delta_0 - \Delta + 4\gamma_i v_B^i/c) \tag{22}$$

(see Peebles & Yu 1970, Hu et al 1994, Dodelson & Jubas 1993b), where the dot denotes differentiation with respect to conformal time η $(= \int dt/a)$, γ_i are the direction cosines defined by $\hat{q}$, n_e is the number density of free electrons, $a(t)$ is the cosmological scale-factor, v_B is the baryon peculiar velocity, Δ_0 is the isotropic part of Δ, h_{ij} is the metric perturbation and isotropic scattering has been assumed. The Boltzmann equation is generally Fourier transformed, since in the linear approximation the different k-modes evolve independently, which makes the calculation tractable. It is also assumed that all fluctuations are still in the linear regime, which is a reasonable approximation for the relevant scales at the scattering epoch. Since the effects of the radiation on the matter cannot be ignored (except for late reionization) the equation of motion for the baryons needs to be solved simultaneously; this is the continuity equation for matter evolving freely, but also takes into account the Compton drag at early times.

The radiation and baryons evolve as a coupled fluid at early times, but need to be followed more accurately as the Universe recombines, and eventually the baryons decouple from the photons entirely. After this point the photons can be assumed to free-stream to the observer, and the subsequent behaviour of the anisotropies is often treated analytically. Dark matter evolves collisionlessly throughout, although its gravity feeds back into the baryon and photon evolution. Detailed calculations along these lines have been carried out for many different cosmological models, e.g. the work of Peebles & Yu (1970), Wilson & Silk (1981), Bond & Efstathiou (1984,1987), Vittorio & Silk (1984,1992), Holtzman (1989), Sugiyama & Gouda (1992), Dodelson & Jubas (1993a), Stompor (1993) and Crittenden et al (1993ab) amongst others. The Boltzmann equation tends to be solved by expanding the Δ's into Legendre polynomials up to some high enough ℓ and numerically integrating the coupled equations.

We show in figure 2 the power spectrum for CDM models as a function of wavenumber k, for a range of Ω_B consistent with BBN (Krauss & Romanelli 1990, Walker et al 1991, Smith et al 1993). To calculate the C_ℓ from these power spectra approximately one integrates the power spectrum with measure $j_\ell^2(k\eta_0)dk/k$. The C_ℓ are thus in effect an average of the power spectrum around $k \approx \ell H_0/2$. We have also included an $x_e = 1$ reionized BDM power spectrum (arbitrarily normalized) on the figure for comparison.

The plateau in figure 2 at low k is the contribution from the Sachs-Wolfe effect (or gravitational redshift), which damps on the scale of the scattering surface thickness. The bumps and wiggles reflect the phase of the oscillating baryons and photons when recombination occurs, i.e. the number of oscillations a given mode has undergone in the time between entering the horizon and the switch-off of radiation pressure when the matter becomes neutral. These so-called "Doppler" peaks (physically they come from a combination of v and Δ sources which are difficult to separate), rise at $k \sim 0.01$Mpc which is the size of the horizon at the time of last scattering. In ℓ-space the rise to the first Doppler peak is quite gradual, which can lead to an effective $n > 1$ even on relatively large scales, e.g. $n_{\rm eff} \simeq 1.15$ on COBE scales (Bond 1994). The height of the Doppler peaks is dependent on the number of scatterers or $\Omega_B h^2$ (scaling roughly as $\Omega_B^{1/3}$, which would be the dependence for an experiment like MAX, see also Fukugita et al 1990). The second and third

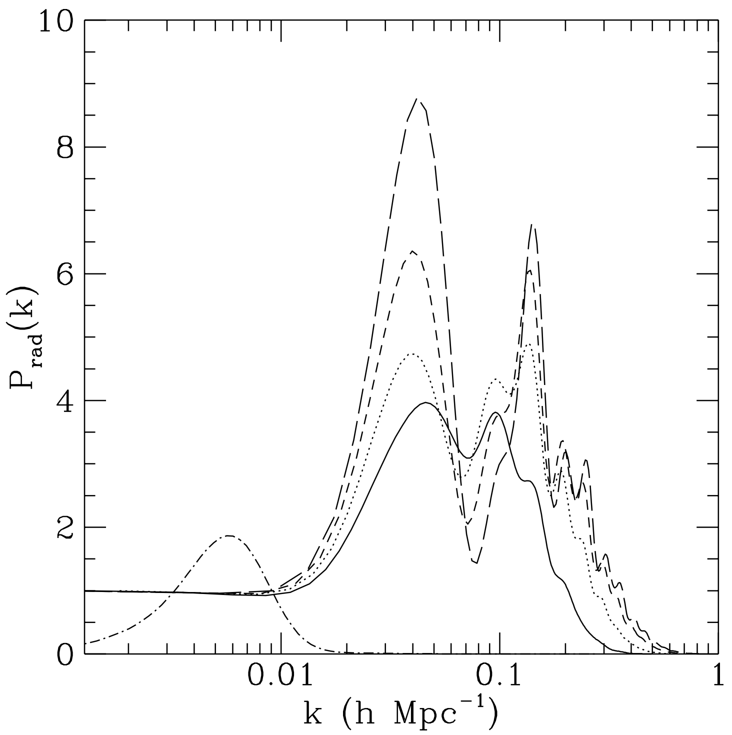

Figure 2: Power spectrum for "standard" CDM models ($h = \frac{1}{2}$, $\Omega_0 = 1$ and $\Omega_\nu = \Omega_\Lambda = 0$) with $\Omega_B = 0.01$ (solid), 0.03 (dotted), 0.06 (short-dashed) and 0.10 (long-dashed) consistent with the range from BBN, from Sugiyama & Gouda (1992). The curves have been normalized to unity at small k. For comparison we also show a fully-ionized BDM model (dot-dashed line) with $\Omega_0 = 0.1$, $n = 0$, and $h = 0.5$, chosen (arbitrarily) to match at $k = 0.002\, h\,\mathrm{Mpc}^{-1}$.

peaks are harmonics of the first which is the fluctuation which has had half an oscillation since it entered the horizon. There is also an effect due to the baryons falling into the dark matter potential wells after recombination, although this happens largely after the photons have been scattered. CDM fluctuations that first entered the horizon during radiation domination suffer growth suppression, which partly accounts for the differing heights of the peaks. Also there is an exponential cutoff in the power spectrum at large k, due to the effects of the thickness of the last scattering surface, as well as the familiar damping (Silk 1968) of baryon and photon fluctuations prior to decoupling. The damping scale is set by the thickness Δz.

For a reionized model (see 5.1) the visibility function is centred around some z_* and has width $\Delta z \sim z_*$, so that the two important scales above are almost the same. The effect is that there is only one Doppler peak, which is at the scale of the horizon at z_* (i.e. smaller k or ℓ), with damping for higher wavenumbers. There are no extra bumps and wiggles, since at the time of last scattering the photons and baryons were no longer oscillating.

3.3 The observed power spectrum

In this section, we consider the current status of measurements of the matter power spectrum (see Peacock 1991, Peacock & Dodds 1994), and the radiation power spectrum (see Bond 1993) in the context of inflation (see Liddle & Lyth 1993). In figure 3 we compare the radiation power spectrum with the matter power spectrum measured by IRAS-selected galaxies (Fisher et al 1993, Feldman et al 1993), the CfA redshift survey (Vogeley et al 1992) and the APM galaxy survey (Baugh & Efstathiou 1993a,b). One cannot make a theory-independent extrapolation from the radiation power spectrum (probed by CMB measurements) to the matter power spectrum (probed by large-scale structure work). In the following, we will assume CDM and write the matter power spectrum as

$$\begin{aligned} P_{\rm mat}(k) &= 2\pi^2\eta_0^4 A\; kT_m^2(k) \\ &= 2.55\times 10^{16} A\,(k/h\,\mathrm{Mpc}^{-1})\,T_m^2(k)\;(h^{-1}\mathrm{Mpc})^3, \end{aligned} \tag{23}$$

where A is as in (11) and T_m is a matter transfer function (see equation (19)), not to be confused with $T(k)$. The change in units and the index of the power law from k^0 to k^1 is a matter of convention. We follow the usual convention in large-scale structure (LSS) work that a "flat" spectrum has $P_{\rm mat}(k) \propto k$ (see Liddle & Lyth 1993 for more discussion on the various definitions of power spectra) and work in units of h^{-1}Mpc. To convert from our normalization to that of the bias or σ_8 conventionally used in LSS work see equation (18), but note that these would be different for any variation in the theory. The large-scale structure and CMB data are shown in figure 3.

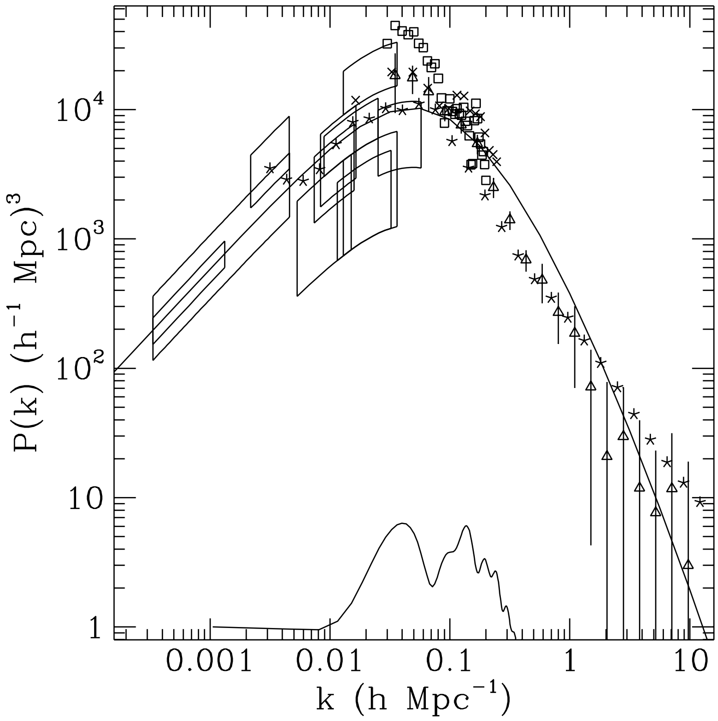

Figure 3: The CDM matter power spectrum on a range of scales as inferred from CMB and LSS data. The solid line is CDM normalized to COBE. Stars, crosses, squares and triangles are the APM, CfA, IRAS-QDOT & IRAS-1.2Jy surveys respectively, with IRAS surveys scaled to $\sigma_8^{DM} = 1$. Apart from scaling the IRAS surveys we have avoided complicated issues related to normalization and redshift corrections. We show error bars on the IRAS-1.2Jy survey to indicate the approximate accuracy involved. The boxes are $\pm 1\sigma$ values of the matter power spectrum inferred from CMB measurements assuming CDM with $\Omega_B = 0.06$, with the horizontal extent taken to be between the half-peak points of the window functions for each experiment. From left to right the experiments are COBE, FIRS, Tenerife, SP91-13pt, Saskatoon, Python, ARGO, MSAM2, MAX-GUM & MAX-MuP, MSAM3. The radiation power spectrum for CDM assuming $\Omega_B = 0.06$ is shown at bottom.

For the CMB anisotropy measurements, we have chosen some recent experiments for which we could estimate the best fit normalization. Specifically we show results from COBE (Smoot et al 1991,1992, Wright et al 1994a), FIRS/MIT (Page et al 1990, Ganga et al 1993, Bond 1993), Tenerife (Davies et al 1992, Watson et al 1992, Hancock et al 1994), Python (Dragovan et al 1993), ARGO (de Bernardis et al 1993, 1994), SP91/ACME (13-point) (Schuster et al 1993), Saskatoon (Wollack et al 1993), MAX (MuP (Meinhold et al 1993) and GUM (Gundersen et al 1993)) and MSAM (Cheng et al 1994). We have concentrated here on those experiments which quote a detection, leaving out those which give upper limits, e.g. Relikt (Klypin et al 1992), 19.2 GHz (Boughn et al 1992), SP89 (Meinhold & Lubin 1991), SP91-9pt (Gaier et al 1992), ULISSE (de Bernardis et al 1992), White Dish (Tucker et al 1993), OVRO (Readhead et al 1989, Myers 1993), VLA (Fomalont et al 1993). Note we have also avoided complicated issues involving redshift space corrections (e.g. Kaiser 1987) to the large scale structure data.

Note that the $P_{\rm mat}(k)$ inferred from CMB anisotropies depends on the assumed theory (Ω_0 for example shifts the correspondence between θ and k, and shifts the amplitude for a given $\Delta T/T$). So the boxes in figure (3) would have to be re-drawn for each theory.

In addition to the survey data shown in figure 3, there is information on large-scale flows (e.g. Kashlinsky & Jones 1991). Bertschinger et al (1990a) estimated the 3D velocity dispersion of galaxies within spheres of radius $40\,h^{-1}$Mpc and $60\,h^{-1}$Mpc. After smoothing with a Gaussian filter on $12\,h^{-1}$Mpc scales they found $\sigma_v(60) = 327 \pm 82\,\mathrm{km\,s^{-1}}$ and $\sigma_v(40) = 388 \pm 67\,\mathrm{km\,s^{-1}}$. The unbiased CDM estimate for these quantities are $224\,\mathrm{km\,s^{-1}}$ and $287\,\mathrm{km\,s^{-1}}$ respectively. Translating this information into the power on scales of $40\,h^{-1}$Mpc and $60\,h^{-1}$Mpc gives roughly $\sigma_8 \simeq 1.15$ (Efstathiou et al 1992).

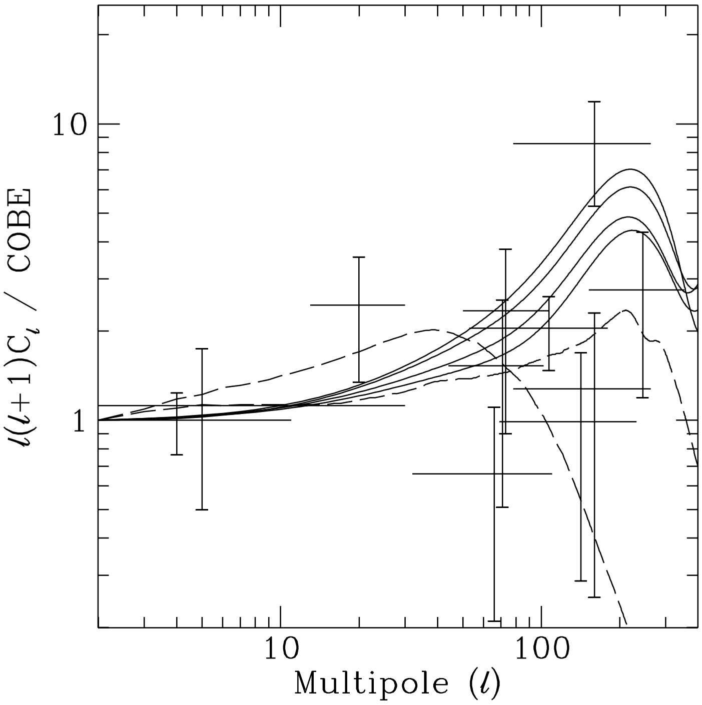

Figure 4: The normalization of a Harrison-Zel'dovich power spectrum of fluctuations required to reproduce the $\Delta T/T_{\rm rms}$ quoted for each experiment. The horizontal error bar on each point gives the range of ℓ between the half-peak points of the window function. These points should be interpreted as only loose approximations to the results of a full analysis. From left to right the experiments are COBE, FIRS, Tenerife, SP91 (13 point), Saskatoon, Python, ARGO, MSAM (2-beam), MAX GUM & MuP, MSAM (3-beam). Also plotted are theoretical power spectra for CDM with $\Omega_B = 0.01, 0.03, 0.06, 0.10$ and standard recombination (solid) and for $\Omega_B = 0.12$ and $z_{\rm rec} = 150$ and 50 (dashed).

One can also consider the CMB data independently of theories of structure formation. In figure 4, we show the current situation with regard to experiments which have quoted detections on degree scales or larger. We plot the normalization which, for an $n = 1$ power spectrum, would reproduce the quoted $\Delta T/T_{\rm rms}$ for each experiment. If there is a Doppler peak in the power spectrum on degree scales, then this would show up as a higher required normalization for a flat spectrum to fit the data. In calculating these numbers for MSAM the value of $\Delta T/T_{\rm rms}$ used was $C_0^{1/2}$ from the GACF fit. All of these points should be interpreted as approximations to the results of a full analysis.

4 Data analysis

4.1 The Gaussian auto-correlation function

It has become common in analyzing data from small-scale experiments to employ a Gaussian Auto-Correlation Function (GACF) as the assumed underlying "theory". The GACF is parameterized by two numbers, its amplitude C_0 and correlation angle θ_c

$$C_{\rm GACF}(\theta) \equiv C_0 \exp\left[-\frac{\theta^2}{2\theta_c^2}\right]. \tag{24}$$

This "theory" is then convolved with the observing strategy and the predictions compared to the data. Usually limits or best fit values are quoted on C_0 for a range of θ_c. In the language of the multipole moment expansion the assumption of a GACF is equivalent to assuming

$$C_\ell = 2\pi C_0 \theta_c^2 \exp\left[-{}^1\!/_2\ell(\ell+1)\theta_c^2\right]. \tag{25}$$

Note that this is very different from CDM where $\ell(\ell+1)C_\ell$ has a Sachs-Wolfe plateau followed by Doppler/adiabatic peaks.

Commonly a plot of C_0 vs θ_c is used to describe the sensitivity of a particular experiment to fluctuations on various scales, under the assumption that the sky correlation function is really a GACF. The GACF approximation is a simple way to understand the sensitivity of an experiment. By varying θ_c, one can match the "power spectrum" to the window function of the experiment (especially multi-beam experiments where the GACF power spectrum and the window function have similar shape, which we will call "Gaussian"). The experiment is most sensitive to a GACF whose peak $\ell(\ell+1)C_\ell$ occurs at essentially the same place as the peak of its window function W_ℓ. The amplitude of fluctuations to which one is sensitive (or the "area" under the window function) is then parameterized by the minimum C_0.

For experiments in which the window function is similar to the GACF power spectrum, the approximation made in fitting with a GACF is numerically quite good. This is because, unless the underlying power spectrum is varying very rapidly on scales probed by the experiment, once the power spectrum is convolved with the window function, it has a "Gaussian" shape. The GACF retains its "Gaussian" shape when convolved with a Gaussian window function. Hence the two convolved spectra will look very similar (Bunn et al 1994).

An analysis using a GACF will (roughly) take into account the peak and area of the window function of the experiment. The minimal values of $C_0^{1/2}$ will therefore be more comparable between experiments than the measured rms temperature fluctuations (which could be defined in an observer-dependent way). The correlations between nearby points will also be approximately correct for experiments with "Gaussian" window functions (and in which the window function approach is applicable, see section 4.2). Although for some experiments, the GACF approximation may be used to give a fit to the data, the GACF assumption should be viewed with caution. One should bear in mind that the quoted $C_0^{1/2}$ is the best fit amplitude of fluctuations for a power spectrum that *is* a GACF with some fixed correlation angle θ_c. There is also little meaning in the values of $C_0^{1/2}$ at any point other than the minimum of the likelihood curve.

4.2 Window functions

It has become conventional to describe the details of the instrument and the observing strategy in terms of a "window function" W_ℓ which describes the sensitivity of the experiment to the modes of the spherical harmonic decomposition of the CMB temperature fluctuations. The signal seen by any experiment can then be considered as the convolution of the sky power and the window function

$$\left(\frac{\Delta T}{T}\right)^2_{\rm rms} \equiv C(0) = \frac{1}{4\pi}\sum_{\ell=2}^{\infty} a_\ell^2 W_\ell. \tag{26}$$

If one takes an ensemble average (over universes) of this expression, then $a_\ell^2 \to \left\langle a_\ell^2 \right\rangle = (2\ell+1)C_\ell$. Often this ensemble average is assumed when the window function is computed.

The simplest and most common window function is that due to finite beam resolution. As expected, finite resolution introduces a high-ℓ cutoff. If the beam has a Gaussian response with a Gaussian width of σ, the window function is (see e.g. Silk & Wilson 1980, Bond & Efstathiou 1984, White 1992)

$$W_\ell = \exp\left[-\ell(\ell+1)\sigma^2\right]. \tag{27}$$

For an experiment which measures temperatures by differencing 2- or 3-beam setups the window functions, in addition to the beam smoothing factor, are (see e.g. Bond & Efstathiou 1987)

$$\exp\left[\ell(\ell+1)\sigma^2\right] W_\ell = \left\{ \begin{array}{cl} 2\left[1-P_\ell(\cos\theta)\right] & 2-\text{beam} \\ {}^1\!/_2\left[3-4P_\ell(\cos\theta)+P_\ell(\cos 2\theta)\right] & 3-\text{beam} \end{array} \right. \tag{28}$$

where θ is the angle between the beams. Note that these types of experiments are not sensitive to the low ℓ modes of the multipole expansion because of the differencing. Since the high ℓ cutoff is controlled by the beam width while the separation (or "chop") controls the low ℓ behaviour, one can increase both the width and height of the window function by separating these scales as much as possible.

Such a double- or triple-beam differencing strategy is often called a "square wave chop". There are however other scan strategies which have been used. Several experiments (in particular South Pole, Saskatoon and MAX) use a "sine wave chop", moving the beam continuously back and forth across the sky, sinusoidally in time. Additionally, the temperature is weighted by ± 1 or by a harmonic of the chop frequency. The resulting time-integrated, weighted temperature is then the "difference" assigned to that point on the sky. Window functions for these experiments can be found in Bond et al (1991b), Dodelson & Jubas (1993a), White et al (1993) and Bunn et al (1994). [The window function for MAX, given in White et al (1993), should be multiplied by 1.13 to account for the finite size of the beam on the calibration: see Srednicki et al (1993)] There are also several interferometer experiments which make maps of the intensity of the radiation on small patches of the sky (e.g. ATCA (Subrahmanyan et al 1993), VLA (Fomalont et al 1993) and Timbie & Wilkinson (1990)). The window function for these experiments can be measured as the Fourier transform of the beam pattern and for accuracy needs to be supplied by the experimenters.

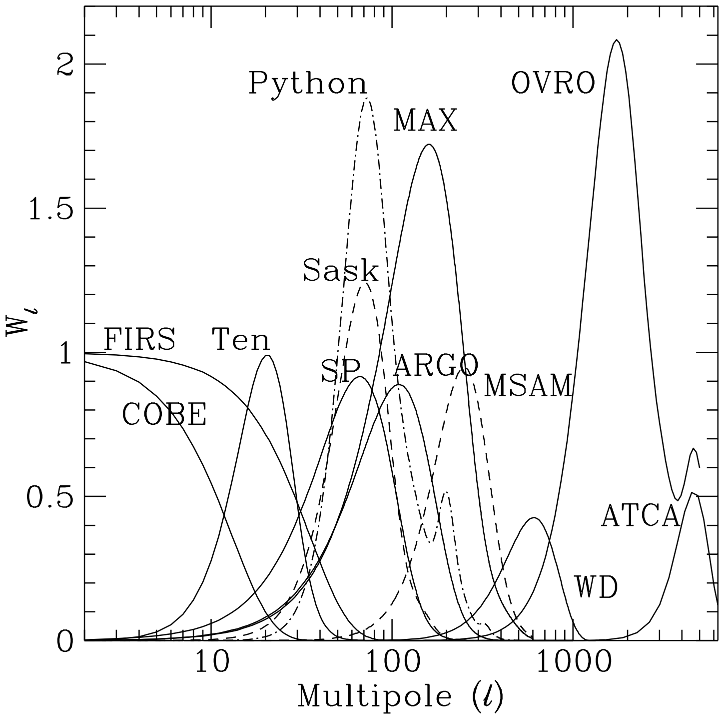

Figure 5: The window functions for large and medium scale experiments as a function of multipole. From left to right the experiments are COBE (with 10° smoothing), FIRS, Tenerife, SP91, Saskatoon (dashed), Python (dot-dashed), ARGO, MAX, MSAM (3-beam, dashed), White Dish (Method II, neglecting binning), OVRO and ATCA. Some parameters of the window functions are displayed in table 4.

We show the window functions vs ℓ for several experiments in figure 5. Some numbers describing the functions shown here are given in table 2. Note that the relative heights can have as much to do with the treatment of the data as with the sensitivity, i.e. the window function that is convolved with theory should be consistent with the observers' $\Delta T/T$. It is worth stressing an example to illustrate this. Consider a triple-beam set-up, which consists of the difference of a difference of two temperatures. The experimenters could choose to assign a measurement of $T_1 - \frac{1}{2}(T_2 + T_3)$ to a point in direction '1', or they could have chosen to take $2T_1 - (T_2 + T_3)$. In the latter case, the window function would be 4 times larger and the 'measured' $(\Delta T/T)_{\rm rms}$ would be two times bigger. The difference in height for the window function would be artificial. While in this case the difference can be quite obvious, in some instances the effects can be more subtle. It is important that experimentalists be explicit about their sampling, weighting and calibrations before the correct window functions can be computed.

Experiment	ℓ_0	ℓ_1	ℓ_2	Max
COBE	–	–	11	1.0
FIRS	–	–	30	1.0
Ten	20	13	30	1.0
SP91	66	32	109	0.9
Sask	71	44	102	1.2
Pyth	73	50	107	1.9
ARGO	107	53	180	0.9
MAX	158	78	263	1.7
MSAM2	143	69	234	2.1
MSAM3	249	152	362	0.9

Table 2: Parameters for the window functions: ℓ_0 represents the multipole at the maximum, ℓ_1 and ℓ_2 are the "half peak" points. The maximum value of the window function is also given. For MSAM we present results in 2-beam and 3-beam mode.

Common approximate formulae for the window functions or analysis procedures assume a square wave chop (e.g. Górski 1993, Gundersen et al 1993). This approximation usually does not reproduce the beam pattern on the sky all that well, although it works better for the window function. Even so, such approximations differ from the exact results, e.g. for MAX the difference between the exact result and (28) is $\sim 10\%$ near the peak, and larger off-peak.

Given both a theory and the window function, it is straightforward to compute the expected rms temperature fluctuation. In table 3, we show the predicted $\Delta T/T_{\rm rms}$ for various experiments, normalized to $A = 1$. The predictions assume full sky coverage and an "average universe" though actual experiments may measure different values due to incomplete sky coverage or cosmic variance (to be discussed later).

	Ω_B				
Experiment	0.00	0.01	0.03	0.06	0.10
COBE	2.57	2.62	2.63	2.63	2.64
FIRS	3.23	3.38	3.41	3.44	3.47
Ten	1.92	2.10	2.14	2.18	2.20
SP91	2.24	2.84	3.00	3.22	3.38
Sask	2.15	2.81	2.98	3.24	3.40
Pyth	2.69	3.84	4.10	4.49	4.76
ARGO	2.20	3.27	3.49	3.84	4.09
MAX	3.08	5.15	5.49	6.09	6.49
MSAM2	3.46	5.64	6.02	6.65	7.06
MSAM3	1.88	3.49	3.69	4.09	4.32

Table 3: Predictions for small scale experiments in a CDM-dominated universe. We show the predicted $\Delta T/T_{\rm rms}$ for various experiments, normalized to $A = 1$. The predictions are for an all sky average and an "average universe", individual experiments may measure different values due to incomplete sky coverage or cosmic variance. For MSAM the predictions are shown for two-beam and three-beam mode. The column $\Omega_B = 0$ refers to an $n = 1$ power spectrum. All values assume CDM with $\Omega_0 = 1$ and $h = 0.5$.

It is sometimes possible to define window functions which correspond to off-diagonal elements of the correlation matrix or averages of the form

$$\langle T(\theta_1,\phi_1)T(\theta_2,\phi_2)\rangle_{\rm ens} = \frac{1}{4\pi}\sum_{\ell=2}^{\infty}(2\ell+1)C_\ell W_\ell(\theta_1,\phi_1;\theta_2,\phi_2) \tag{29}$$

which are required when fitting data. [Note that this is different from the sky-averaged correlation function of the COBE group. It is not an average over our observed sky, but the covariance matrix required when computing likelihood functions, assuming Gaussian statistics for the temperature fluctuations]. In general, the window function approach works well for computing $\Delta T/T_{\rm rms}$ or for experiments in which the data span only one dimension (such as the individual linear scans of the ACME South Pole experiment). In other cases however the data are two-dimensional on the sky and there can be strong anisotropies in the theoretical covariance matrix which are difficult to include in this manner. Alternative approaches are then preferable (see e.g. Srednicki et al 1993). Also, if the scanning strategy or data analysis procedure is sufficiently tortuous, the window function approach is extremely complicated and simulations of the scanning, binning and analysis become necessary. Coarsely binned data in an experiment which scans smoothly (rather than "stepping") across the sky is one example of this, where correlations introduced by the binning will be important.

4.3 Fitting data

In comparing the theory of CMB fluctuations to the new measurements of the anisotropy on various scales, there are two primary techniques that are used. These are the Bayesian 'likelihood function' analysis and the frequentist 'likelihood ratio' method (which is often calibrated using Monte-Carlo simulations). Both of these techniques have been discussed and compared in the review of Readhead & Lawrence (1992), and we will not discuss them in detail here.

It is perhaps important to emphasize, however, that the methods will lead to the same conclusions when the data set is "well-behaved" but can differ when the data are 'unlikely' or atypical. Both methods have ways of testing for possible breakdowns in statistical assumptions (which are not always used). For example, conclusions of the Bayesian approach should be robust under changes of prior, and conclusions of the frequentist approach should take into account the type-II error (the probability of accepting the hypothesis when the null hypothesis is true).

The computation of the likelihood function simplifies considerably in the case that the fluctuations are assumed to be Gaussian. This is because all the non-vanishing moments of the distribution can be related to the variance. For Gaussian-distributed fluctuations, the likelihood function is

$$\mathcal{L} \propto \frac{1}{\sqrt{\det C}} \exp\left[-{}^1\!/\!_2 T_i C_{ij}^{-1} T_j\right] \tag{30}$$

where T_i are the measured temperatures and C_{ij} is the auto-correlation matrix which includes a theoretical and experimental piece. In the limit that the experimental errors σ_i are uncorrelated we have

$$C_{ij} = C_{ij}^{\rm th} + \sigma_i \delta_{ij} \tag{31}$$

where $C_{ij}^{\rm th}$ is computed using equation (29) or its generalization (see e.g. Bond et al 1991b, Dodelson & Jubas 1993a, Bunn et al 1994, Srednicki et al 1993, Bond 1993 and Vittorio et al 1989, 1991, Vittorio & Muciaccia 1991, Górski et al 1993). Notice that the term in the exponent is just the χ^2.

In the Bayesian approach, this function has to be multiplied by the 'prior' to obtain the (relative) probability distribution for the parameters being fitted, while in the frequentist approach the final distribution comes from ratios of likelihood functions or from Monte-Carlo simulation (see e.g. Berger (1985) for a discussion of these two philosophies).

It is also possible to fit more than one component to the data, in order to obtain the best constraints on the cosmic anisotropies. One example is in fitting an extra white noise component to the data (e.g. Bond 1993). Another example is the simultaneous fitting of a foreground signal of some given form (e.g. Dodelson & Stebbins 1993) to multi-channel data. It is relatively straightforward to implement this idea (for either statistical approach), although there is some subjectivity in the choice of the number of parameters, which of them should be fixed (the frequency dependence of the foreground signal perhaps) and which to fit or integrate over. The fitting process becomes more computationally time consuming the more parameters are included.

5 Beyond linear theory

5.1 Reionization

A knowledge of the variation of optical depth (or equivalently x_e) and of the peculiar velocities associated with different scales are enough to calculate $\Delta T/T\,(\theta)$. Reionization will erase the fluctuations generated at $z \sim 1000$, by making the Universe optically thick at much lower redshifts, but in the process extra fluctuations will be generated due to the motions of the new last scatterers. Because reionization can be used to erase the primordial fluctuations in a model which would otherwise conflict with experimental upper limits, it is important to be able to calculate the secondary fluctuations so that the minimal level of anisotropy can be estimated.

The dominant contribution to anisotropies generated during reionization are Doppler shifts of the scatterers (equation 22). The first (analytic) considerations of the velocity-induced perturbations in a reionized model were by Sunyaev (1977, 1978), Davis (1980) and Silk (1982). However it was pointed out (Kaiser 1984) that an important term contributing to the fluctuations had been neglected in the previous work. The correct expression for $\Delta T/T$ could not be written quite as straightforwardly as in equation (2). The evolution of the radiation fluctuations needs to be followed more accurately, requiring a numerical solution of the collisional Boltzmann equation for the photons (see section 3.2).

Ostriker & Vishniac (1986) extended the discussion to second-order fluctuations (see Section 5.2), detailed calculations being presented by Vishniac (1987). Efstathiou (1988) made further calculations of the effects of reionization, including details of the pattern of polarization and giving approximations valid for small angular scales. The important effects of reionization on anisotropies are at arc-minute scales, in particular, where primary anisotropies are expected to be erased, and approximate calculations can be used which are valid only for small angles and correspondingly for small wavelength perturbations. More recent calculations have been carried out for reionization in BDM models (e.g. Hu et al 1994), generic open universe models (Persi & Spergel 1993), CDM models (Hu et al 1994, Dodelson & Jubas 1993b, Sugiyama et al 1993, Chiba et al 1993) and for the case of decaying dark matter (Scott et al 1991).

An important epoch is the redshift at which the optical depth for photons becomes unity. Since Thomson scattering is independent of frequency, the evaluation of z_* (or equivalently η_*) is relatively simple:

$$\tau = -\int_0^z \bar{n}_e(z)\sigma_T c \left(\frac{dt}{dz}\right) dz, \tag{32}$$

which, for an $\Omega_0 = 1$ universe, with a constant ionized fraction, becomes

$$\tau = \frac{\sigma_T c H_0}{4\pi G m_H} x_e \Omega_B (1 - Y_p) \left\{ (1+z)^{\frac{3}{2}} - 1 \right\} \simeq 0.04 \Omega_B h x_e z^{3/2}, \tag{33}$$

where Y_p is the primordial fraction of the baryonic mass in helium, which is assumed to be all neutral. Hence, for a particular model of the ionization history, the redshift for which $\tau = 1$ can be calculated. For example, for constant ionized fraction $z_* \simeq 69(h/0.5)^{-2/3}(\Omega_B/0.1)^{-2/3} x_e^{-2/3}$. For an open universe $z_* \propto \Omega_0^{1/3}$ very approximately, so that the last scattering surface can be at significantly lower redshift.

Reionization need not be 'complete', i.e. the optical depth may not become very large back to the new scattering surface. Obviously if $\tau \lesssim 0.1$ say, then reionization has negligible effect. This would be the case for a standard CDM model fully ionized even up to $z \sim 20$. However if $0.1 \lesssim \tau \lesssim 1$, then a good approximation to the effects of erasure of the primary anisotropies (i.e. those generated at the standard recombination scattering surface) is that $\Delta T/T$ is reduced by the fraction of photons scattered at $z \sim 1000$ rather than at the new scattering surface. If $1 - e^{-\tau_i}$ is the fraction of photons scattered since the Universe was reionized at z_i (i.e. this is the integral of the visibility function), then we simply have $(\Delta T/T)_{\rm obs} \simeq (\Delta T/T)_{\rm prim} e^{-\tau_i}$. There will also be *secondary* anisotropies generated on the new last scattering surface, although these will generally be smaller (see below). However, second-order fluctuations at much smaller angular scales also need to be considered.

Early reionization is likely to have been inevitable for COBE-normalized CDM (Tegmark et al 1993, Sasaki et al 1993, Fukugita & Kawasaki 1993, following the early studies by Couchman 1985 and Couchman & Rees 1985). On scales of order the horizon at z_* there will still be Sachs-Wolfe fluctuations and the polarization would be expected to be higher.

5.2 Second order anisotropies

The idea that reionization would erase the primary anisotropies, while generating smaller secondary Doppler anisotropies (since the new scattering surface was so thick), was overturned by the realization that the Vishniac effect played an important role at small angular scales. The first-order Doppler effects mainly cancel when the red-shifts and blue-shifts are integrated through the thick last scattering surface. However there is a second-order term in the Boltzmann equation (equation 22) coming from the $v\delta$ cross term in $\sigma_T \bar{n}_e(1+\delta)v$. In the solutions to the Boltzmann equation at small scales, this term is a convolution over velocity and density perturbations, which does not suffer from the cancellation effect. This leads to some models giving *larger* secondary anisotropies than the primary anisotropies which the reionization was invoked to erase.

We can obtain a scaling relation for the secondary Doppler fluctuations as follows. Notice that the last scattering shell is something which is at η_* with width $\sim \eta_*$. The number of independent regions of scale λ lying across the thickness of the shell is $N \sim \eta_*/\lambda$. The optical depth through each region of comoving scale λ is $\tau_\lambda \sim 1/N \sim \lambda/\eta_*$. The fluctuations on scale $\theta\,(\propto \lambda)$ are about $N^{1/2}$ times the fluctuation due to a single lump. The fluctuation for each lump is second-order because of the approximate cancellation of red- and blue-shifts through a lump, leading to an effect

$\sim \tau_\lambda^2 v$ (see Kaiser 1984). From the continuity equation, the peculiar velocity is $v \sim \lambda\delta_k(\eta_*)/\eta_*$ and overdensities evolve as $\delta \propto \eta^2$. Therefore an order-of-magnitude estimate for the temperature fluctuations (i.e. $k^2\Delta$, see also Kaiser 1984, Vishniac 1987) is

$$(\Delta T/T)^2_\theta \sim N(\Delta T/T)^2_{\rm lump} \sim k^{-3}P(k)/(\eta_*\eta_0^4) \propto (1+z_*)^{1/2}. \tag{34}$$

Hence earlier reionization generally leads to larger large-scale (first-order Doppler) fluctuations from the primary last-scattering surface. Similarly for the second-order (e.g. Vishniac) fluctuations, the effect for each lump is $\sim \tau_\lambda v\delta$, giving $(\Delta T/T)^2_\theta \sim k^2P^2(k)(1+z_*)^{-5/2}$. Again, this argument is crude, but shows that second-order small-scale fluctuations from the secondary last scattering surface would be expected to be *smaller* for earlier reionization. There are several experimental limits at scales $\sim 1'$, which provide good constraints on these secondary fluctuations rather than the primary ones (e.g. Uson & Wilkinson 1984a,b,c, Readhead et al 1989, Fomalont et al 1993, Subrahmanyan et al 1993). The strong k dependence of the Vishniac effect means that the fluctuations will be very "spiky", so that a double- or triple-beam experiment will be dominated by the zero-lag autocorrelation, Note, however, that non-linear effects may also be important, e.g. if the universe is very clumpy, as may happen in BDM or other models with lots of small-scale power (Hu et al 1994), or if most of the baryons are trapped in galaxies at an early epoch (R Juszkiewicz & PJE Peebles private communication).

In general, there are *many* second-order terms to consider once a fully second-order Boltzmann equation is derived (Hu et al 1994, Dodelson & Jubas 1993b). For most of these terms, there will be a cancellation of red-shifts and blue-shifts (Kaiser 1984) through the last-scattering surface, so that the major contribution to the fluctuations comes from modes with k-vector almost perpendicular to the lines of sight. The only significant term which remains is the Vishniac term, which is a convolution of v and δ, coupling large-scale velocity perturbations with small-scale density perturbations. In a model with significant reionization the primordial anisotropies are erased, a new Doppler peak is generated at smaller ℓ and of lower amplitude, and a Vishniac bump is generated at arc-minute scales. For CDM-type models the Vishniac effect is negligible, but for models with lots of small-scale power and/or open models (e.g. BDM models, which also *require* reionization), the constraints can be restrictive. Generally the ionization history, $x_e(z)$ can be varied in such models, since there is no clear picture for the process of early reheating. The Vishniac anisotropies tend to become smaller for an earlier last scattering surface, while the first-order amplitude becomes bigger; so there are limits to how much the constraints can be avoided by invoking a different ionization history.

Another kind of second-order anisotropy is that coming from non-linear effects in the growth of perturbations, which causes the potential to change with time even in an $\Omega_0 = 1$ universe. This change in potential in the light-crossing time across a fluctuation (Zel'dovich 1965, Rees & Sciama 1968, Dyer 1976) is known as the Rees-Sciama or integrated Sachs-Wolfe effect. It is small in almost all models (e.g. Argüeso & Martínez-González 1989, Martínez-González et al 1990, 1992, 1993, Anninos et al 1991), although large voids (Nottale 1984, Scaramella et al 1989, Thompson & Vishniac 1987, Van Kampen & Martínez-González 1991, Arnau et al 1993), or structures such as the Great Attractor (Bertschinger et al 1990, Martínez-González & Sanz 1990, Hnatyk et al 1992, Goicoechea & Martin-Mirones 1992, Saez et al 1993) or the Great Wall (Atrio-Barandela & Kashlinsky 1992, Chodorowski 1992) etc. might give observable signatures.

6 Uncertainties

If the underlying fluctuation spectrum is assumed to be stochastic in nature, then one is faced with the problem of trying to compare a theory which defines probability distributions as functions of the underlying parameters with just one sample drawn from these (our Universe). Only observations over an ensemble of universes would allow one to determine the parameters of the underlying theory unambiguously, even in principle. Since we can observe *only* our universe there is an irremovable uncertainty in our ability to relate certain CMB measurements, *no matter how precise*, to parameters of the theory. The uncertainty introduced in determinations of theoretical parameters has been called "cosmic variance" or "theoretical uncertainty" (e.g. Abbott & Wise 1984a,b, Scaramella & Vittorio 1990, 1991, 1993b, Cayón et al 1991, White et al 1993).

In terms of the multipole moments the cosmic variance may be thought of as an uncertainty in relating

$$a_\ell^2 \leftrightarrow \left\langle a_\ell^2 \right\rangle_{\rm ens} = (2\ell+1)C_\ell. \tag{35}$$

In a theory with Gaussian fluctuations, the $a_{\ell m}$'s are independent Gaussian random variables and the a_ℓ^2 are thus $\chi^2_{2\ell+1}$ distributed (Abbott & Wise 1984a,b, Abbott & Schaefer 1986, Vittorio et al 1988). The uncertainty in relating a_ℓ^2 and $\left\langle a_\ell^2 \right\rangle_{\rm ens}$ is given by the width of the distribution, which scales as $(\ell + 1/2)^{-1/2}$. As one goes to smaller scales and probes larger ℓ, one thus becomes less sensitive to cosmic variance. On large scales, however, cosmic variance forms the limiting uncertainty in fixing (e.g.) the normalization of the power spectrum. For the quadrupole (with only 5 degrees of freedom), the actual amplitude in our Universe is likely to differ substantially from its expectation value (Gould 1993, Stark 1993).

While it is true that on smaller scales, which probe higher ℓ, the cosmic variance becomes negligible, this applies only to a full sky measurement. The variance from sampling only a fraction of the sky is larger than the cosmic variance one gets when sampling the whole sky. Since current small-scale experiments cover only small patches of the sky, this can be an important effect. In general for an experiment which samples a solid angle Ω, the cosmic variance is enhanced by a factor of $4\pi/\Omega$ (Scott et al 1994, see also Bunn et al 1993). We should emphasize that this "sample variance" is completely independent of the experimental precision, and simply reflects the fact that the experiment has not covered enough of the sky to provide a good estimate of the rms value of $\Delta T/T$, i.e. it simply does not have a big enough sample.

The spectrum of fluctuations depends not only on the primordial spectrum but on the evolution of perturbations as waves enter the horizon. This evolution introduces dependence on Ω_0, H_0 and the dark matter. In addition, a knowledge of the ionization history of the universe is important since re-ionization would lead to both a "reprocessing" of primary fluctuations and generation of (small-scale) secondary fluctuations. [See section 5.1.]. A non-negligible ionized fraction during much of the history of the universe would also contribute a radiation drag which would affect the growth of baryon perturbations and alter the observed CMB anisotropy. In general, the uncertainties in the cosmological parameters and history produce very correlated shifts in the radiation power spectrum (Bond et al 1994).

In addition, there are possible sources of 'foreground' contamination such as synchrotron and free-free emission (at low frequency) and dust (at high frequency). These non-cosmological sources have been discussed in detail in the review of Readhead & Lawrence (1992) and we will not mention them further except to say that to discriminate between these sources it is necessary to have good frequency coverage and a wide range of angular resolutions.

7 Miscellaneous

7.1 Polarization

Penzias & Wilson (1965) set a limit of 10% on the polarization of the CMB, and there have been several subsequent upper limits (e.g. Nanos 1979, Caderni et al 1978, Lubin & Smoot 1981, Lubin et al 1983, Partridge et al 1988, Wollack et al 1993), but so far none which have been low enough to be cosmologically interesting.

Any quadrupole anisotropy in the radiation field will give rise to a linear polarization when it is Thomson scattered. This was first suggested in the case of an anisotropic universe (Rees 1968, Anile 1974, Dautcourt & Rose 1978), but can also arise, generally of smaller amplitude, from any inhomogeneity, and can moreover be enhanced by reionization (Negroponte & Silk 1980, Basko & Polnarev 1980, Stark 1981, Tolman & Matzner 1984, Nasel'skiĭ & Polnarev 1987, Ng & Ng 1993). A transfer equation similar to equation (22) can be written for the polarization amplitude Δ_P (Bond & Efstathiou 1987). Solutions indicate that the level of anisotropy is generally small for standard recombination. Efstathiou (1988) has developed approximations for calculating the small angular scale autocorrelation function of Δ_P in reionization scenarios. Kaiser (1983) has shown that the r.m.s. polarization from density fluctuations is typically 10–20% of $\Delta T/T$ for adiabatic fluctuations on arc-minute scales. This means that it is still well below the detection levels of present-day experiments. The problem has been studied numerically, including both the contribution of density fluctuations and gravitational waves in Crittenden et al (1993a), where it was shown that the additional contribution due to gravity wave modes was too small to be resolved with current sensitivities (see also Frewin et al 1993). However, in future, as these experiments become more sensitive and it becomes more difficult to distinguish real fluctuations from those caused by confusion with faint radio sources and diffuse emission from the Galaxy, it may be that polarization information could be used as a discriminant.

The polarization pattern can also be a useful test of the ionization history of the universe. The important point is that the polarization pattern is expected to be coherent over scales corresponding to the thickness of the last scattering surface (Hogan et al 1982). Hence coherence on scales $\gg 10$ arc minutes would imply that reionization had indeed taken place. Note that all of the statements above are for linear polarization – circular polarization is not expected to be generated unless there are primordial magnetic fields or strong anisotropies at the scattering epoch.

7.2 Open universes

Until now, we have implicitly assumed that $\Omega_0 = 1$ as favoured by inflationary models. However astronomical evidence generally favours $\Omega_0 \sim 0.2(\pm 0.1)$, based on large-scale structure (<10 Mpc) studies. The negatively-curved spatial hypersurfaces of a low density ($\Lambda = 0$) Friedmann model greatly complicate the analysis of large angular scale anisotropy in the CMB. One expects that the curvature radius will introduce a feature into the low order multipoles, due to gravitational focussing of geodesics that shifts the power from lower to higher orders. The characteristic angular scale is $\sim \Omega_0$ radian. This effect is analogous to the "ring-of-fire" effect in weakly anisotropic cosmologies (Wilson & Silk 1981, Fabbri et al 1983). Early calculations include those of Kaiser (1982) and Peebles (1982b). This redistribution of the low-order multipole moments was realized in the elegant numerical computations of Wilson (Silk & Wilson 1981, Wilson & Silk 1981, Wilson 1983), who expanded the radiation power spectrum in terms of generalized wave-number $\sqrt{k^2+K}$, where K is the spatial curvature, defined as eigenvalues of the Laplace operator for density perturbations in a curved background.

Similar results to those of Wilson were subsequently obtained by Tomita & Tanabe (1983), and Abbott & Schaefer (1987). These latter authors used a gauge-invariant approach, expanded the power spectrum in wave-number k, and included cosmic variance in estimating the multipole moments. Górski & Silk (1989) computed the low order multipoles for the case of primordial isocurvature (or entropy) fluctuations. Their results were generalized by Gouda et al (1991a,b), who included both primordial adiabatic and entropy fluctuations, and computed the contribution from the integrated Sachs-Wolfe term. This line-of-sight term vanishes in flat $\Omega_0 = 1$ models but gives an important contribution to all scales in an open model (Hu & Sugiyama 1993).

Any simple interpretation of the results, however, is complicated by the fact that the concepts of wave number and power-law spectrum must be generalized to curved hyperspaces. They can no longer simply be interpreted in terms of constant curvature fluctuations associated with a spectrum of comoving scales if e.g. $n = 1$. Kamionkowski & Spergel (1993) have studied power spectra for primordial adiabatic fluctuations that are power-laws in either volume (most large scale power) or distance, or eigenvalues of the Laplace operator (most large scale power), and find that in all cases there is some suppression of the multipoles on scales larger than the curvature scale. In a flat universe, these 3 definitions of power spectrum would all coincide. There is large uncertainty in defining the spectrum because of the lack of any unique prescription for the initial conditions in a low-Ω universe. Indeed, much of the motivation for a Harrison-Zel'dovich spectrum is lost if $\Omega_0 < 1$, because one can no longer appeal to simple models of inflation. More complex models that "predict" fluctuation spectra in open models, whether for primeval curvature perturbations (Lyth & Stewart 1990) or for primordial entropy perturbations (Yokoyama & Suto 1991, Dolgov & Silk 1993), are not compelling.

Given the above, it appears that in low-Ω_0 universes, adiabatic CMB fluctuations are larger than their $\Omega_0 = 1$ counterparts for the same value of H_0 (of course when $\Omega_0 < 1$ the requirement of small H_0 is relaxed and the anisotropy can be somewhat reduced). The most detailed model of fluctuations in an open universe to date is the BDM model discussed in section 2.2. Analysis of the spectrum produced by this model is complicated, as radiation drag (from the non-zero x_e) can alter the shape, and the integrated Sachs-Wolfe effect is important on large scales in addition to the secondary fluctuations induced on small scales (Gouda et al 1991a). This model appears to be tightly constrained by current observations (Chiba et al 1993).

7.3 Effects of a cosmological constant

Due to the emerging, but still controversial, discrepancy between the lifetime of a matter-dominated $\Omega_0 = 1$ universe inferred from the Hubble constant, and the lifetime inferred by stellar evolution (see e.g. Demarque et al 1991) there has been interest in the possibility that the cosmological constant may be non-zero (see also Caroll et al 1992). This would allow $\Omega_0 < 1$ while retaining a "flat" cosmology and avoid the "Ω-problem" (see e.g. Kolb & Turner 1990). If a large vacuum energy drove an inflationary phase in the early universe, the idea that it may be non-zero today is not wildly implausible.

Note that a non-zero cosmological constant has little effect on the large-scale structure and dynamics of the Universe (e.g. Markevitch et al 1991, Lahav et al 1991), and will have negligible effect on the recombination process and the visibility function. However, microwave background fluctuations will be significantly different in $\Lambda \neq 0$ models. This is because of several effects.

Firstly, there is a change in angular scales. The angle at recombination corresponding to a proper length λ can be approximated as $\theta \simeq 30''\Omega_0^{1/3}(1-\Omega_0)^{-1/4}\lambda(h^{-1}\mathrm{Mpc})$ (Blanchard 1984, Stelmach et al 1990). Hence, for the same normalization, the $\Lambda \neq 0$ model measures anisotropies which correspond to smaller angular scales than in the $\Lambda = 0$ model. Secondly, since the growth of fluctuations is different in a $\Lambda \neq 0$ model, the potential fluctuations are no longer constant in time. Hence the Sachs-Wolfe temperature fluctuations pick up a piece due to the integrated potential term (see equation 8). This modifies the (scalar) Sachs-Wolfe formula (Görski et al 1992, Vittorio & Silk 1992, Sugiyama et al 1990, Hu & Sugiyama 1993, Stompor & Görski 1993). [The formula for the tensor mode contribution is essentially unchanged]. Generally the integrated Sachs-Wolfe effect for a flat $\Omega_0 < 1$ universe is less important than in the case of an open universe. Finally, the fluctuations (assuming they are linear) stop growing when the the cosmological constant becomes important, which happens at $1+z \sim (\Omega_0^{-1}-1)^{1/3}$. For fixed power spectrum today the potential in the early universe scales as Ω_0/D (Peebles 1984), where D is the growth factor and the extra factor of Ω_0 comes from $\Phi \propto \Omega_0$. [Similar conclusions can also be drawn (Sugiyama & Sato 1992) for models in which the cosmological constant decays with time (Freese et al 1987, Ratra & Peebles 1988, Overduin et al 1993).]

7.4 Topology

The fluctuations at large angular scales can also be used to constrain the topology of the Universe, which in principle could be non-trivial (Zel'dovich 1973, Sokolov & Starobinskiĭ 1976, Fang & Houjun 1987, Fang 1991). The Sachs-Wolfe spectrum of C_ℓ's is an integral over the power spectrum (equation 13) in an ordinary simply-connected universe, but becomes a sum over modes which are harmonics of the box-size in a universe which has the topology of a three-torus (i.e. periodic boundary conditions). This sum fails to accurately approximate the integral on scales approaching that of the box. Then the fact that COBE measures a roughly flat power spectrum on large scales means that the box must be roughly the horizon size or bigger, unless the initial power spectrum has a pathological increase for small multipoles. More detailed comparisons indicate that the scale of any such topology is $\gtrsim 80\%$ of the horizon size (Stevens et al 1993, Sokolov 1993, Starobinskiĭ 1993).

For open universes Gurzadyan and collaborators have argued that curvature effects may result in elongated shapes of anisotropies in the CMB (Gurzadyan & Kocharyan 1993, Gurzadyan & Torres 1993). The isotropy pattern of the CBR can also be used to place limits on the rotation of the Universe (Collins & Hawking 1973, Barrow et al 1985). Currently the limit on the dimensionless rotation is $\omega/H_0 \lesssim 10^{-6}$ (Smoot 1992), which is about $1''$ every 30 Gyr!

8 Conclusions

The CMB is a unique laboratory for studying the initial conditions that gave rise to the observed Universe. In particular, the temperature fluctuations on scales from arc-minutes to tens of degrees provide, at least in principle, a precise measure of the primordial density fluctuation power spectrum. The difficulties arise in large part because of the "dirty window" effect: foreground contamination, predominantly galactic but possibly atmospheric and extragalactic, obscures our view of the last scattering surface. Improved sky coverage, angular resolution, and frequency coverage will eventually help surmount these obstacles, but we do not anticipate an early answer. Another problem arises with the proliferation of astrophysical parameters: these include Ω_0, baryon density, ionization history, primordial power spectrum shape, isocurvature or adiabatic fluctuation contribution, tensor mode strength, fraction of cold, warm and hot dark matter, Λ, role of topological defects as seeds, non-gaussian fluctuations, and decaying dark matter. In the future we can hope that the sensitivity to all these parameters will be seen as a boon rather than as a 'problem'.

With post-COBE fluctuations being reported (in at least 8 other experiments at the time of preparing this review) plots like figures 3 and 4 are likely to become familiar. Several questions are immediately apparent: Are the experiments consistent with each other? Is there any evidence for a Doppler peak in the data? Problems with foreground contamination and other systematic effects may mean that some experiments should be assigned larger error bars. But given this extra leeway, and taking full account of the sample variance etc., will they prove to be consistent with Gaussian fluctuations on the sky? If the answer to this question is 'no', then perhaps we will require non-Gaussian fluctuations, or patchy re-ionization, or perhaps even some new component of cold galactic dust. At the moment all of these avenues are worth exploring, but it is premature to say that any such ideas are necessary.

At present, the number of definitive conclusions one can draw is depressingly few. There is also a vigorous debate as to the optimal method of confronting experiment and theory, whether by Bayesian or frequentist techniques. It is clear that from the CMB measurements alone, one model, that of adiabatic fluctuations in a baryon-dominated universe, can be discarded. Dark matter has proved an invaluable foil for resurrecting flat models, and at present one cannot even eliminate the canonical cold dark matter model from consideration. However we are at an exciting moment in the history of this data-starved subject: many results are being reported and we are on the verge of being able to eliminate, or to confirm, the $n = 1$ Harrison-Zel'dovich spectrum that was proposed in the earliest and simplest inflationary models. Almost all models predict Doppler/intrinsic fluctuation peaks over 10-60 arc-min: current experiments should be able to decide soon on the reality of these features as sky coverage is improved. If the Doppler peaks are not present then this would provide a strong argument in favour of reionization.

Theorists may rightly consider new directions. Late-time phase transitions (Hill et al 1989, Nambu et al 1991, Jaffe et al 1994) showed some brief promise of producing minimal fluctuations, but even this class of models is detectable at a level of $\Delta T/T \sim 10^{-5}$. Textures produce extreme hot spots, that can perhaps be alleviated with a high ($b \sim 4$) bias factor, allow the possibility of living with hot or even baryonic dark matter, but remain a relatively soft target for theories of large-scale structure. Significant, large-scale, non-Gaussian behaviour remains elusive in any model of structure formation, but might well be a useful weapon to bring to bear on the intermediate angular scale data. Perhaps a complex reionization history would generate non-Gaussian smoothing signatures. Also galaxies could contain unsuspectedly large amounts of cold dust in their halos, and provide an unavoidable contamination of the CMB fluctuation signal, a prospect that must seem less unlikely if halos indeed are baryonic. The fact that "point" sources are being reported in at least one CMB anisotropy experiment with spectra indistinguishable from that of the CMB must add to one's concern about foreground contamination that has not previously shown up in IRAS 100 μm maps. The large-scale bulk flows seen in the galaxy distribution, now observed to 15,000 km s^{-1}, imply observable signatures in the CMB on sub-degree scales: if these flows are confirmed and corresponding precursor $\Delta T/T$ fluctuations from the last scattering surface are not observed, a non-Gaussian fluctuation model would seem to be the only resolution.

The next few years promise to be a lively time in cosmology, both for theory and observation, as the $\Delta T/T$ measurements are refined. Observations on all angular scales, from tens of degrees to sub-arcminute scales, will undoubtedly play a role.

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A Some useful numbers

Converting from multipole space ℓ to angles θ is accomplished via the approximate formula

$$\frac{\theta}{1^\circ} \simeq \frac{60}{\ell}. \qquad (36)$$

It is this equality that is used to make statements that certain measurements are on a scale of θ. Furthermore at any $z \gg 1$ an angle θ degrees subtends a comoving distance of $105\,\theta\ (\Omega_0 h)^{-1}$Mpc.

To convert from ℓ to comoving wavenumber k we have $\ell \simeq (6{,}000\,h^{-1}\mathrm{Mpc})k$. Additionally we have the physical scales (at horizon crossing)

$$\begin{array}{lcrl} k_{\rm hor}^{-1} & = & 6000 & h^{-1}\mathrm{Mpc} \\ k_{\rm curv}^{-1} & = & 3000\,(1-\Omega_0)^{-1/2} & h^{-1}\mathrm{Mpc} \\ k_{\rm rec}^{-1} & = & 180\,\Omega_0^{-1/2} & h^{-1}\mathrm{Mpc} \\ k_{\rm eq}^{-1} & = & 39\,(\Omega_0 h)^{-1} & h^{-1}\mathrm{Mpc} \end{array}. \qquad (37)$$

The redshifts of equality, recombination and decoupling (Compton cooling) for standard recombination are

$$\begin{array}{lcl} 1+z_{\rm eq} & = & 23{,}900\,(\Omega_0 h^2) \\ 1+z_{\rm rec} & = & 1100 \\ 1+z_{\rm dec} & = & 500\,(\Omega_B h^2)^{2/5}, \end{array} \qquad (38)$$

and for reionization with constant ionized fraction x_e

$$\begin{array}{lcl} 1+z_{\rm dec} & = & 6.0\,(\Omega_0 h^2)^{1/5} x_e^{-2/5} \\ 1+z_{\rm drag} & = & 120\,(\Omega_0 h^2)^{1/5} x_e^{-2/5}. \end{array} \qquad (39)$$

Also the following angles can be useful:

$$\begin{array}{lll} \text{Hubble radius} & \Delta\theta(H^{-1}, z \gg 1) & = {}^1\!/\!_2\Omega_0^{1/2} z^{-1/2} & = 0^\circ\!.87\Omega_0^{1/2}(z/1100)^{-1/2} \\ \text{Length scale} & \Delta\theta(\lambda, z \gg 1) & = {}^1\!/\!_2\Omega_0 H_0 R_0 \lambda & = 34''\!.4\,(\Omega_0 h)\lambda_{\rm Mpc} \\ & & & = 65''\!.4\,(\Omega_0^{2/3} h^{1/3})(M/10^{12}\mathrm{M}_\odot)^{1/3} \\ \text{Curvature scale} & \Delta\theta(\mathcal{R}R_0/R, z \gg 1) & = {}^1\!/\!_2\Omega_0/(1-\Omega_0)^{1/2} & = 28^\circ\!.6\,\Omega_0(1-\Omega_0)^{-1/2} \end{array} \qquad (40)$$

B Sachs-Wolfe effect

In this appendix we give a brief derivation of the Sachs-Wolfe effect in a "flat" cosmology with no cosmological constant, using the metric perturbation approach. Our starting point is the metric (following Sachs & Wolfe 1967)

$$ds^2 = a^2(\eta)\left(g^{(0)}_{\mu\nu} + h_{\mu\nu}\right) dx^\mu dx^\nu = a^2(\eta) d\bar{s}^2, \tag{41}$$

where $a(\eta)$ is the scale factor, η is the conformal time and $g^{(0)}_{\mu\nu}$ is the unperturbed metric (Minkowski space). To understand the temperature fluctuations induced by the perturbations we need to study the photon trajectories in ds^2. For photon (null) geodesics $ds^2 = 0$ so by (41) there is a 1-to-1 correspondence between photon paths in ds^2 and $d\bar{s}^2$. This allows us to consider the problem first in $d\bar{s}^2$ and later translate our results in ds^2.

We can solve for the geodesics by extremizing the Lagrangian $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$; the geodesic (Euler-Lagrange) equations for $d\bar{s}^2$ are

$$\frac{d}{d\zeta}\left((g^{(0)}_{\mu\nu} + h_{\mu\nu})\dot{x}^\nu\right) = \frac{1}{2} h_{\nu\rho,\mu}\dot{x}^\nu\dot{x}^\rho, \tag{42}$$

where ζ is a parameter along the photon trajectory and the overdot represents differentiation w.r.t. ζ. The term in parenthesis is the 4-momentum k_μ. Integrating we find

$$k_0 = E + \frac{E}{2}\int_0^\zeta d\zeta' \; h_{\rho\sigma,0}\dot{x}^{(0)\rho}\dot{x}^{(0)\sigma} + \mathcal{O}(h^2) \qquad \text{and} \qquad \hat{k} = E\hat{e} + \mathcal{O}(h), \tag{43}$$

where E is the unperturbed energy and $x^{(0)} = (\mathrm{const} + \zeta', \zeta'\hat{e})$ is the unperturbed photon path.

The photon energy seen by an observer with 4-velocity u ($|u^2| = 1$) is $k \cdot u$. Using $u = (1 - \frac{1}{2}h_{00}, \vec{v})$ with $|\vec{v}| \ll 1$ and (43)

$$\frac{(k\cdot u)_e}{(k\cdot u)_r} = \left(1 - \frac{1}{2}\int_e^r h_{\rho\sigma,0}\dot{x}^{(0)\rho}\dot{x}^{(0)\sigma}\, d\zeta + \frac{1}{2}h_{00}\Big|_e^r - \vec{v}\cdot\hat{e}\Big|_e^r\right) + \mathcal{O}\left(h^2\right), \tag{44}$$

where e and r refer to "emission" and "reception" respectively. The corresponding expression in ds^2 comes from multiplying the whole expression by $a(\eta_r)/a(\eta_e)$ to account for the cosmological redshift.

Assuming a uniform source and using the correspondence $h_{00} = 2\Phi$ between the metric perturbation and the Newtonian potential the temperature fluctuation induced is

$$\left(\frac{\Delta T}{T}\right)_{SW} = \Phi\Big|_e^r - \vec{v}\cdot\hat{e}\Big|_e^r - \frac{1}{2}\int_e^r h_{\rho\sigma,0}\dot{x}^{(0)\rho}\dot{x}^{(0)\sigma}\, d\zeta. \tag{45}$$

The three terms can be identified as the gravitational potential redshift, the Doppler effect due to motion of the emitter and receiver, and an extra effect due to the time dependence of the metric (see also Stebbins 1993). In a flat $\Lambda = 0$ universe Φ is constant in time in linear theory so the last (integral) term vanishes and in the absence of Doppler shifts the potential change is known as the Sachs-Wolfe effect. In this limit the Sachs-Wolfe effect is simply the red-shifting of the photon as it climbs out of the potential on the surface of last scattering (assuming $\Phi = 0$ at the time of observation). In some cases, such as with gravitational waves, non-flat or Λ dominated cosmologies, or non-linear fluctuations, the integral term can also play a role.

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NOTE ADDED IN PROOF:

Since we submitted this article the field of CMB anisotropies has refused to stay still, and there have been several significant developments. Firstly, the situation with regard to the COBE value of n has been clarified. The best fit to the two year data is 1.10 ± 0.32 including the quadrupole, and 0.87 ± 0.36 excluding the quadrupole (Górski 1994). Similarly the value of $Q_{\rm rms-PS}$ is $20\,\mu$K, rather than the earlier $17\,\mu$K. Secondly, the new MAX data (Devlin et al 1994, Clapp et al 1994) show consistently high detections at the half-degree scale in all 3 regions scanned. This has added support to the reality of the Doppler peaks. There are also a couple of places where assumptions we made about experiments have turned out to have been wrong: the Python point in figures 3 and 4 is too low by a factor of approximately 3; and the window function for ARGO in figure 5 is too high by a factor of 4 (P. de Bernardis, private communication).

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