6.7 Jackknife and Bootstrap

There is some sample data for this problem, 100 samples from a power law of index 2 and lower cut-off at 0.01.

The maximum-likelihood estimate of the index is 2.17 and the jackknife gives a standard deviation on this of 0.22. Our approximate estimate of the error, from maximum likelihood, is just $2.17/\sqrt{100}$ so the agreement is good. A bootstrap simulation, on the same data, gives a standard deviation on the answer of 0.24 (1000 bootstrapped samples) but the distribution of the estimated indexes is not very Gaussian. Caution is needed in estimating confidence regions, especially in the tails. See the histogram of estimates in the Figure.
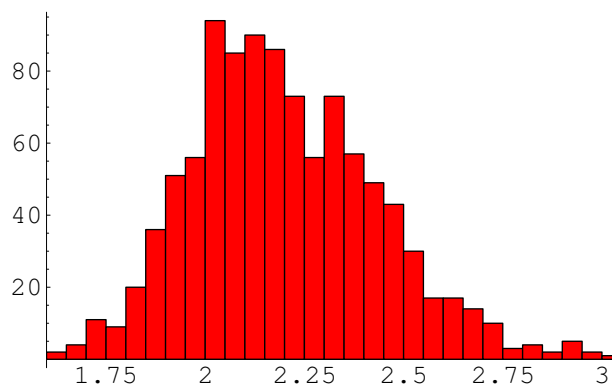

Figure 1: Distribution of estimates of the power-law index, for 1000 bootstrap repetitions on a set of 100 observations.

For small samples (take subsets of 10 in the data) the distribution is very non-Gaussian, although the bootstrap and jackknife (and indeed a direct Monte Carlo) agree on the variance. However the variance now cannot be interpreted as if it referred to a Gaussian.