9.3 Integral constraint on $w(\theta)$

As described in the text, if there is correlation, the pair count *cannot* be enhanced in all separation bins while keeping the total number of pairs constant. The normalization must change, and we can formulate this in terms of an adjustment factor C as follows:

$$< DD(\theta) >= C \times \frac{1}{2}n(n-1)\,\delta G_p\,[\,1 + w(\theta)\ ] \tag{1}$$

where δG_p is the equal-area fraction of the surface between θ and $\theta + \delta\theta$.

(a) The factor C. The total number of pairs remains $\frac{1}{2}n(n-1)$. Integrating over all pairs in the above expression :

$$C \times \frac{1}{2}n(n-1)\int^{\theta} \delta G_p\,[\,1 + w(\theta)\ ] = \frac{1}{2}n(n-1), \tag{2}$$

$$C\int^{\theta} \delta G_p\,[\,1 + w(\theta)\ ] = 1, \text{ and so} \tag{3}$$

$$C = \frac{1}{1+W}, \text{ with } W = \int^{\theta} w(\theta) dG_p \tag{4}$$

Thus

$$< DD(\theta) >= \frac{1}{2}n(n-1)\,\delta G_p\left[\frac{1 + w(\theta)}{1+W}\right] \tag{5}$$

For all practical cases $W \ll 1$, and from (5) the estimated angular correlation function is related to the actual function via $1 + w_{est} \approx (1 + w_{true})(1 - W)$. With $W w_{true}$ negligible,

$$w(\theta) \ \approx w(\theta)_{\ est} + W, \tag{6}$$

i.e. the estimated $w(\theta)$ is in error by a constant offset of W.

(b) An approximation for W. Take a survey of angular dimension R. Recall the fractional element of equal area δG_p, which in this case will be $dG_p = 2\pi\theta\, d\theta/\pi R^2$. Assume a power-law angular correlation function $w(\theta) = (\theta/\theta_0)^{-b}$. Then

$$W = \int^{\theta} w(\theta) dG_p, \text{ and substituting :} \tag{7}$$

$$= \frac{2}{R^2}\int^{R} \theta\, d\theta (\theta/\theta_0)^{-b} \tag{8}$$

$$= \frac{2}{2-b}\left(\frac{\theta_0}{R}\right)^{b} \tag{9}$$

With $b \approx 1$ as it turns out to be for cosmological angular correlations, we get

$$W \sim 2(\theta_0/R)^b. \tag{10}$$

For example, from large-scale radio surveys, θ_0 turns out to be $\sim 0.001°$. For an all-sky survey, or even one of degrees in scale, W is clearly negligible. Several deep pencil-beam surveys on scales of arc minutes have been used in studies of clustering of radio sources, and for these the offsets due to the integral constraint may be significant.