9.4 Effect of surface density changes on $w(\theta)$

(a) Magnitude of offset in $w(\theta)$. From the text: 'Changing surface densities will spuriously enhance measured values of $w(\theta)$. This is because the number of close pairs of galaxies in any region depends on the local surface density ($DD \propto \overline{\varsigma^2}$), but the number of pairs expected over the sky by random chance depends on the global average surface density ($RR \propto (\overline{\varsigma})^2$). Systematic fluctuations mean that $\overline{\varsigma^2} > (\overline{\varsigma})^2$, increasing $w(\theta)$ by

$$\Delta w(\theta) = \frac{\overline{\varsigma^2}}{(\overline{\varsigma})^2} - 1 = \overline{\delta^2} \tag{1}$$

where $\delta = (\varsigma - \overline{\varsigma})/\overline{\varsigma}$ is the surface overdensity. Equation 1 applies on angular scales less than those on which the surface density is typically varying; on larger scales the estimate of DD in this model is wrong.'

The simple model of this exercise has a survey divided into two equal areas with a fractional offset in surface density of ϵ. Here the square of the surface overdensity $\overline{\delta^2}$ is

$$\overline{\delta^2} = \frac{1}{2}(\epsilon/2)^2 + \frac{1}{2}(-\epsilon/2)^2 = \epsilon^2/4 \tag{2}$$

Thus for $\epsilon = 0.2$, $\Delta w = 0.01$.

(b) Verification of the offset. The results of the exercise using the given data file are shown in Figure 1.

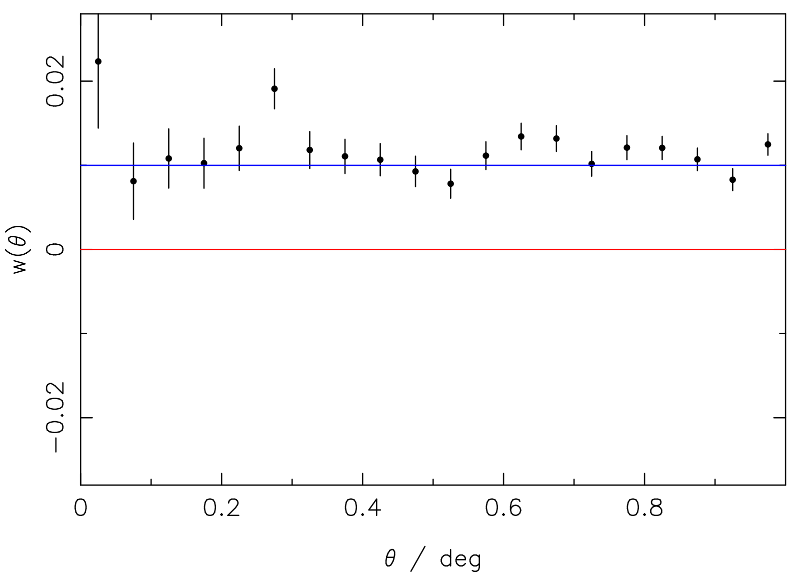

Figure 1: Effect of surface density change on $w(\theta)$. The toy sky generated for the data of this example has 100000 points in the region $0° < \alpha < 60°$, $-20° < \delta < 20°$, with a 20 per cent step in point surface density at $\delta = 0°$. The correlation function was calculated with the Landy-Szalay estimator w_3, using 10 random skies in each of which 100000 points were distributed uniformly over the area. The blue line represents the predicted offset of 0.01 in $w(\theta)$.

An offset of the predicted size does appear.

Could gradient steps such as these be ignored if they merely result in 'small' fixed offsets such as 0.01 in $w(\theta)$? No. Amplitudes of cosmological correlation functions are small. A step of 0.01 could mask signal at larger spacings, and if unrecognized will certainly lead to erroneous determinations of true slope and amplitude.

With the same formalism, investigate the effects of surface-density changes on scales smaller than that of the example, as well as the effects of large-scale gradients rather than steps.

Obviously in checking program development, initial runs will be done with sample sizes smaller than 100000, and with fewer than 10 random skies. Can you get a reasonable verification with less than 100000 points? (Unlikely.) In running 100000-point correlation functions, some optimization of the correlation-function loops will be essential to compute the result in reasonable time.