9.5 Power spectrum of signal and noise

Suppose the noise is additive; call it x and the signal, s. Then, using upper case for the transforms, the transform must be $S + X$. The power spectrum is this times its complex conjugate, so

$$(S + X)(S^* + X^*) = |S|^2 + |N|^2 + 2\mathrm{Re}(S^* X).$$

The first term on the right-hand site is the power spectrum of the signal. The second is the spectrum of the noise, which (for Gaussian noise) is a chi-square random variable. The last term contains X, which will be Gaussian (the real and imaginary parts separately and independently Gaussian), scaled up by S. Re denotes the real part of all this. So in the power spectrum, the noise level will in general depend on the power spectrum – the noise is no longer additive. If $S \to 0$ at some large value (the smoothing of a point-spread function often achieves this) then the level of the spectrum gives a guide to the noise – if it is white and additive in the original data.