Data on a surface - 2D

Elizabeth L Scott 1917-1988

- 1949 PhD (UC Berkeley) in Astronomy + Maths => Stats in Astro.

- 1951 permanent post Dept of Maths at Berkeley.

- 1949-1964: 30 papers in statistics + astronomy. Lasting collab with Jerzy Neyman, classic papers with Neyman and Shane.

- the 'Scott Effect' - distant clusters contain more galaxies, and galaxies of any rank will be intrinsically brighter.

- 30 papers in weather modification research analysis; pioneering studies of UV/ skin cancer.

- pioneering studies in stats to study gender-based inequities and to promote equal opportunities, pay for women in academia.

- President of the Institute of Statisticians, VP of the AAS, Scott prize, etc.

What We Did in Lecture 19

1. How a **digital correlation spectrometer worked** – and why.
   - example with wave packets => how power spectrum adds up to give a spectral line.
   - **one-bit digitization is crucial**, and loses remarkably little signal.

2. The **periodogram, and the Lomb-Scargle algorithm in period-searching**, the general basis of current searches in gappy data.
   - sampling **above the Nyquist frequency** is possible.
   - **aliasing** (phonies) is serious with bad gappiness.
   - true periodicities from tricks such as **period folding**.

3. **Wavelet analysis/decomposition** => spectacular improvements:
   - localized decomposition; **location preserved.**
   - adaptable to **different scales** at each location and to different types of features.

4. **1/f noise**, the bane of every physical experiment.
   - **why it leads (eventually) to infinite variance,** why integration no longer works.
   - filtering to minimize its ever-increasing variance?

Sky Projections

If we want to draw a piece of sky to plot out sources, **we must use an equal-area projection** to preserve density of points.

The following three are well known, given **right ascension *α*** and **declination *δ***:

1. The **Aitoff projection**:

$$x = 2\phi\frac{\cos\delta\sin\frac{\alpha}{2}}{\sin\phi},\;\; y = \phi\frac{\sin\delta}{\sin\phi},\;\; \text{where}\;\; \phi = \cos^{-1}(\cos\delta\cos\frac{\alpha}{2}).$$

2. The **Hammer-Aitoff projection**:

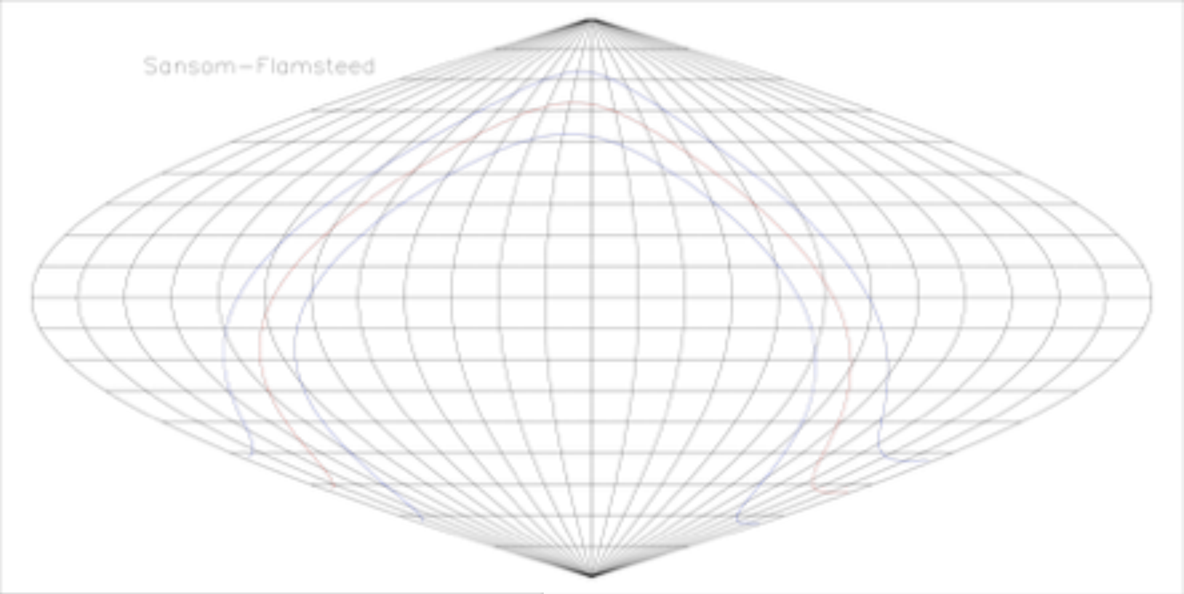
$$x = 2\phi\cos\delta\sin\frac{\alpha}{2},\;\; y = \phi\sin\delta,\;\; \text{where}\;\; \phi = \sqrt{2}/\sqrt{1+\cos\delta\cos\frac{\alpha}{2}}.$$

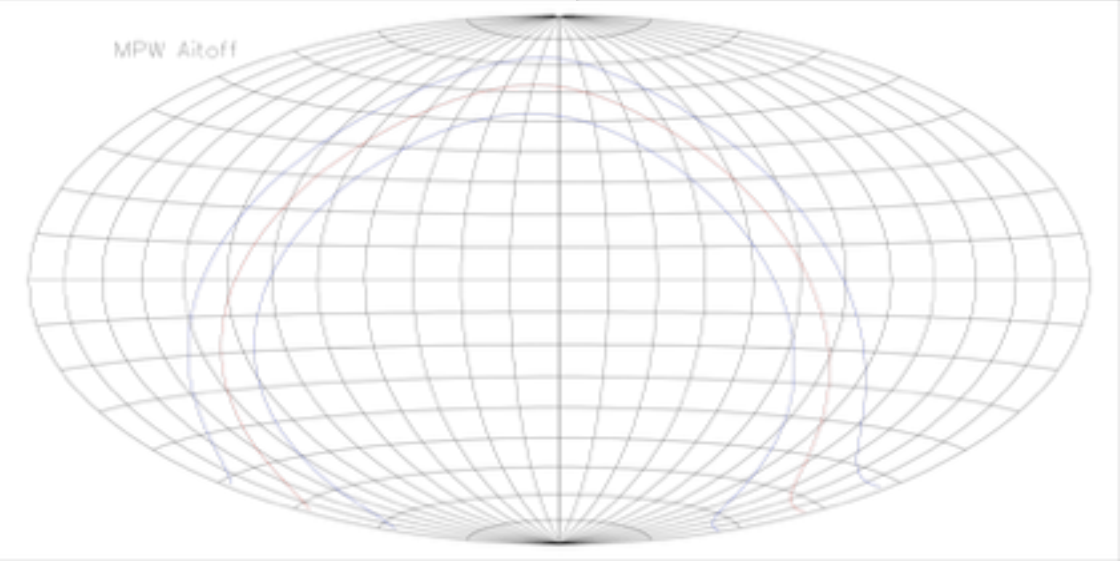
3. The **Sanson-Flamsteed projection**:

$$x = \alpha\cos\delta,\quad y = \delta.$$

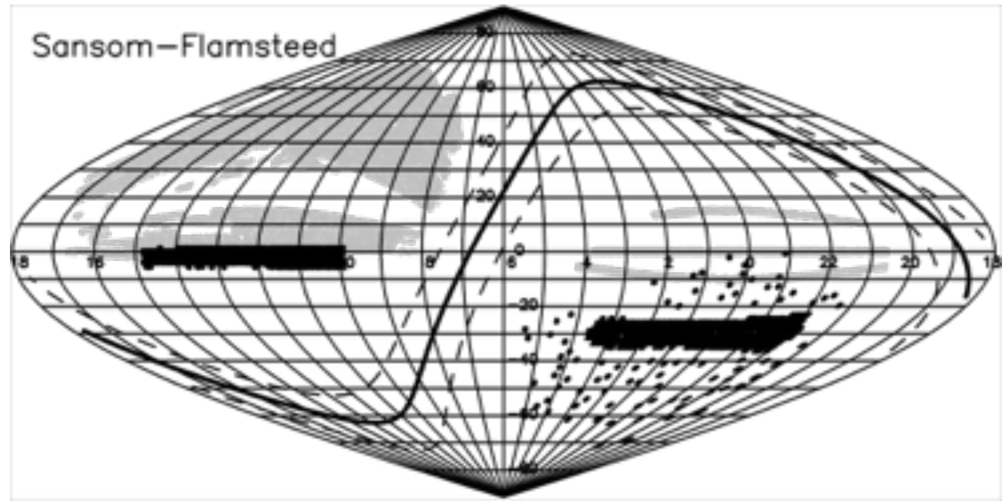
Sky Projections - Examples

Galactic Plane
±10° from Plane
Ra, dec



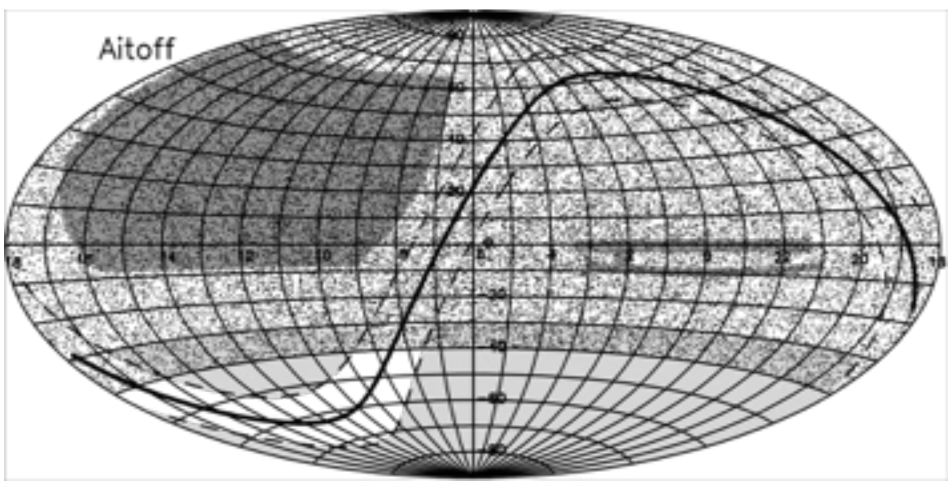


Sky Projection - Examples with Real Data



Radio surveys:

- **NVSS - dotty**
- **FIRST – blackish**
- **SUMSS – light gray**



Redshift surveys:

- **SDSS light gray**
- **2dF GRS black**

Sky Projection: Density Mapping

To map the density - compute **a weighted average of a set of points** ***$P_1, P_2, \ldots P_n$*** from your sky-sprinkled data as follows:

Useful weighting scheme is this one:

$$W_n(P, P_i) = \frac{C_n}{4\pi n \sinh(C_n)} \exp[C_n \cos(\theta_i)]$$

where $\boldsymbol{\theta_i}$ is the angular distance between ***P*** and data P_i. **The weight depends only on the angular distance of points from *P*.**

Smoothing is controlled by C_n and **varies inversely as C_n**.

Choose C_n to increase with ***n***.

Choose appropriate *P* levels for contouring, and find a contouring routine.

Sky Distribution - Quantitative Measures

1. Classical tests

- Rayleigh's test for randomness from a uniform distribution
- direction of a true median
- tests to determine if samples have same median direction
- bipolar clustering
- correlation, regression

2. Two-point angular correlation function

$$\delta P = \varsigma^2 \, [\, 1 + w(\theta) \,] \, \delta\Omega_1 \, \delta\Omega_2$$

- great for odd shaped areas
- statistical estimators are available

3. Counts-in-cells

$$y = \frac{\mu_2 - \overline{N}}{(\overline{N})^2}$$

can show equivalence to $w(\theta)$

4. Angular power spectrum

$$\varsigma(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} a_{\ell,m} \, Y_{\ell,m}(\theta, \phi)$$

can show equivalence to $w(\theta)$

Two-Point Angular Correlation Function $w(\theta)$

Clustering **increases the number of close pairs** and ***w(θ)*** quantifies this.

It is the **fractional increase relative to random** in the probability ***δP*** of finding objects in each of two solid angle elements $\boldsymbol{\delta\Omega_1}$ and $\boldsymbol{\delta\Omega_2}$ separated by ***θ***:

$$\delta P = \varsigma^2 \, [\, 1 + w(\theta)\,]\, \delta\Omega_1 \, \delta\Omega_2$$

In order to estimate ***w(θ)*** from a distribution of ***n*** objects:

1. measure the **angular separation *θ*** of all galaxy pairs,

2. bin these separations to form the **data-pair count *DD(θ)***, the number of galaxy separations having lengths ***θ*** to ***θ + δθ***, and

3. calculate ***RR(θ),*** the corresponding number in each of these bins **for a random sky** knowing the average surface density of the real sky.

(Note: the expected number of random pairs in the separation bin $\theta \rightarrow \theta + \delta\theta$ is $RR(\theta) = 1/2\, n\, \varsigma\, 2\pi\theta\, \delta\theta$, when ς **is the average surface density**.)

Two-Point Angular Correlation Function w(θ)

Hence an estimator for ***w(θ) -*** the fractional enhancement in pairs above random - is

$$w_0(\theta) = \frac{DD(\theta)}{RR(\theta)} - 1,$$

….. and this is ***w(θ)*** in its simplest form.

But need to consider:

- **excess variance**
- **bias**
- **cross pairs**
- statistical fluctuations and **random sample size**
- **edge effects**, important for small survey areas or weak clustering.

The best of the estimators for ***w(θ)*** is that with the **smallest bias and variance** in the angular range under investigation => **Poisson variance can be attained**.

$$\boxed{w_3 = \frac{r(r-1)}{n(n-1)}\frac{DD}{RR} - \frac{(r-1)}{n}\frac{DR}{RR} + 1}$$

- the Landy - Szalay Estimator

Two-Point Angular Correlation Function $w(\theta)$

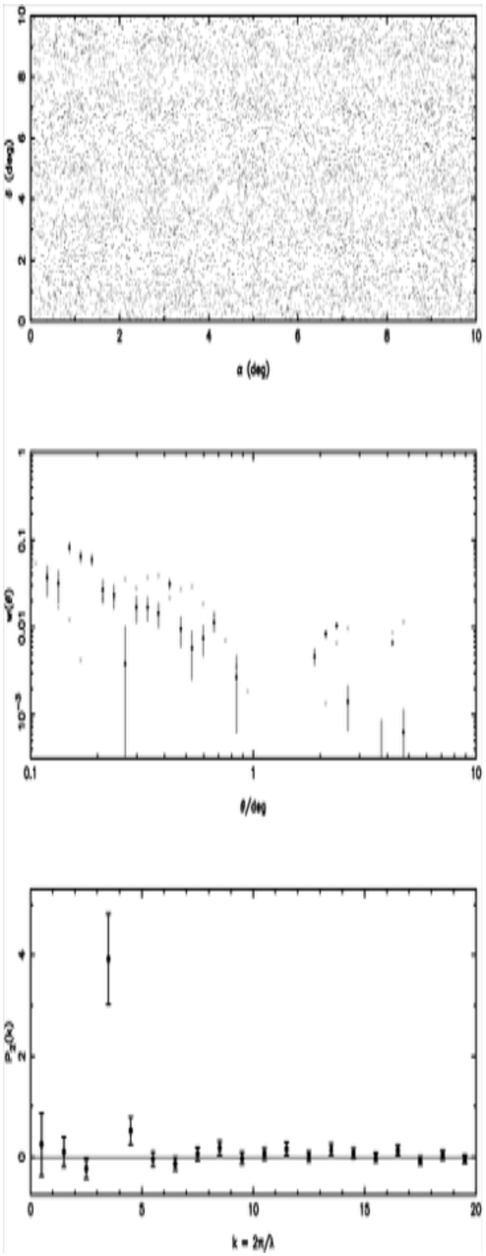
Errors in adjacent bins are correlated: a single object appears in many different separation bins because it participates in many pairs.

=> covariance matrix essential.
=> $\sqrt{N}$ errors are much too small => bootstrap?

Integral constraint: The total number of pairs over all bins is fixed at ***n(n-1)/2***; clustering shifts pairs from larger to smaller separations. If the surveyed area is small, ***w(θ)*** **appears positive for even the most distant separations** sampled. Normalize!

Instrumental effects:
- **surface gradients** from bad calibration => increase in ***w(θ)***
- **over-resolution** – resolving galaxies/sources into multiple components can totally overwhelm any real cosmological information.

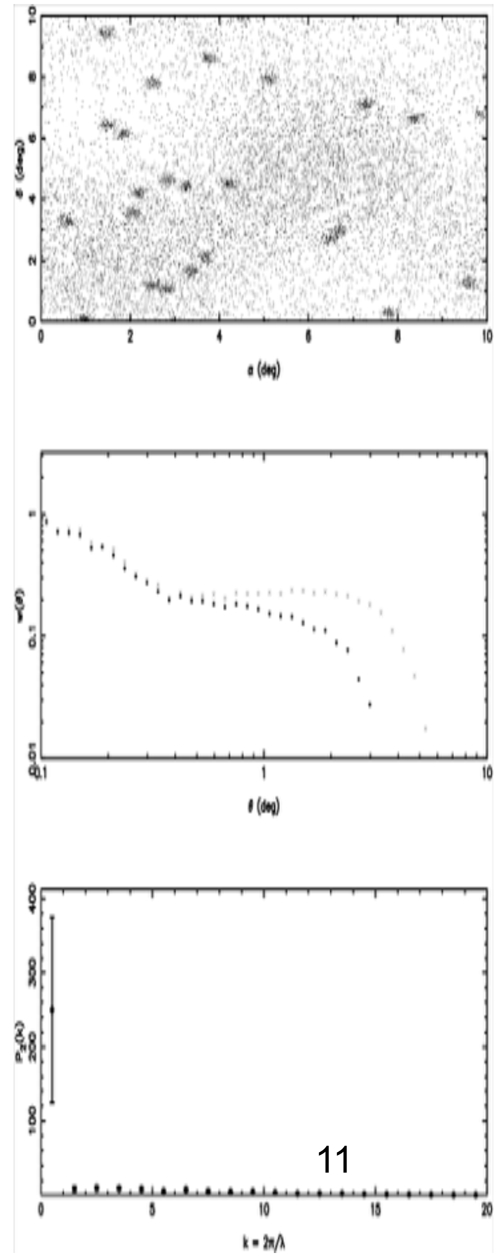
$w(\theta)$ - Example I



Left -- uniform 2° grid with 25 low-surface-brightness clusters (total 1500 points in Gaussians of width 0.4°) on a random background of 8500 points,

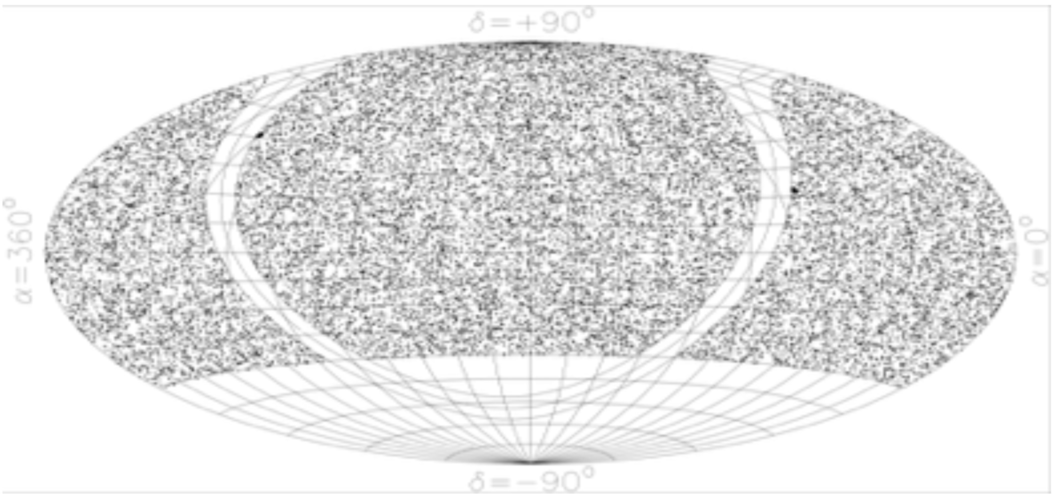
Right -- dipole background 10000 objects background, 2000 in 25 equal clusters of Gaussian width 0.1°, random positions.

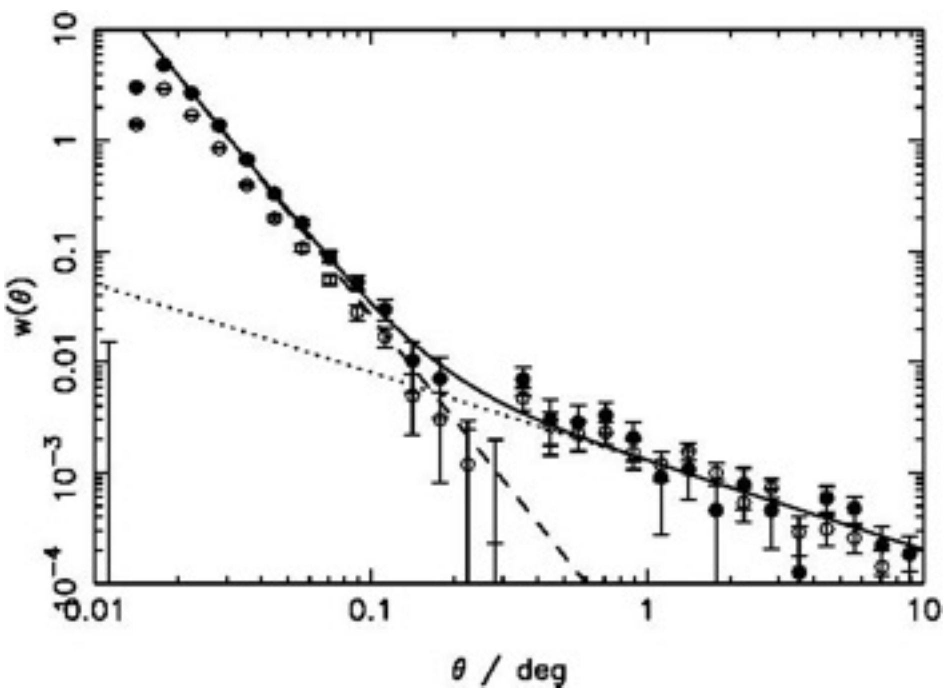
Measured **$w(\theta)$** and angular power spectra below each. ***$w(\theta)$*** evaluated with simple estimator (crosses) and the Landy-Szalay estimator (dots with error bars).



$w(\theta)$ - Example 2

Right: NVSS survey; catalogue entries with $S_{1.4\ GHz}$ > 200 mJy. Sources within $\pm 5^{\circ}$ of Galactic Plane masked out.





Left: Over-resolution! Angular correlation function **w(θ)** for the source catalogue of the NVSS survey, at $S_{1.4\ GHz}$ = 20 mJy (solid circles) and 10 mJy (open circles). Best-fitting sum of two power laws for the 20 mJy data is shown as the solid line; dashed due to multiple-component sources; dotted due to galaxy clustering.

Counts-in-Cells

- Set up cells on the sky; count! Form P(N) (Scott, Neyman, Shane ~1960). Szapudi (1998) – large number of randomly-placed cells over the sky, **heavy oversampling**. P**artially filled** cells? – MC to the rescue.

- **P(N) is Poisson**, if no clustering. Clustering => **higher variance** than a random distribution; cells may cover clusters or voids, **broadening P(N).**

- **Variance μ_2 as a function of cell size** quantifies clustering. Variance vs cell size is the standard plot of c-in-c.

- Simple relation exists between this function and 2-pt angular correlation function.

Counts-in-Cells

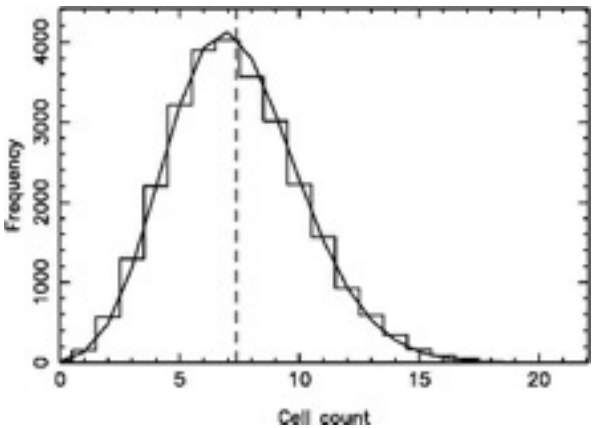
- Now just used an adjunct to $w(\theta)$ – and to search for **skewness μ_3, asymmetry.** **Tail** in the probability distribution to high cell-counts?

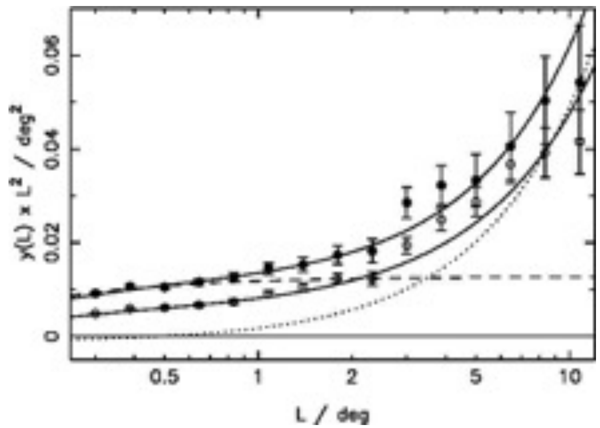
- Assuming Gaussian primordial perturbations and linear growth of clustering, the skewness of counts-in-cells remains zero (Peebles 1980).

- Measurement of a **non-zero skewness => (a) non-linear gravitational clustering or (b) non-Gaussian initial conditions**.

- As the growth of cosmic structure moves out of the linear regime, the expected skewness increases from zero.

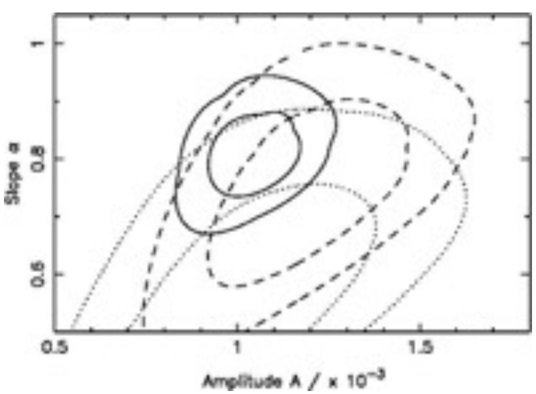
Counts-in-Cells; Variance Function - Example





Left: Counts of NVSS radio sources with S > 20 mJy in cells of diameter 1°. Solid curve - expected Poisson distribution.

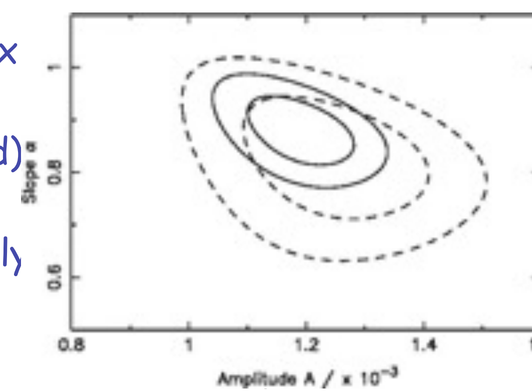
Right: Variance statistic y(L) plotted for thresholds 20 mJy (solid circles) and 10 mJy (open circles), with predictions of the double power-law w(θ) models at 20 mJy and 10 mJy (solid lines).



Clustering parameters A and α, $w(\theta) = A\ \theta^{-\alpha}$. Contours of constant χ^2; these are at approx at 2σ for flux-density thresholds 30 mJy (dotted), 20 mJy (dashed) and 10 mJy (solid).

Left: fitting the correlation function directly

Right: fitting the counts-in-cells variance function above right.



The Angular Power Spectrum

- imagines that the **object surface density field** over the sky is expressed as a sum of **angular density fluctuations of different wavelengths**.

- a **Fourier analysis around the sky**.

- basis functions are the **spherical harmonic functions**, the 2D analogues of sin, cos

- the quantity c_l is the amplitude of the l^{th} **multipole**, which produces fluctuations on angular scales **$\theta \sim 180^o / l$.**

- theoretically the **c_l spectrum** is entirely equivalent to the angular correlation function ***$w(\theta)$*** as a description of the galaxy distribution; connected by well-known relations.

The Angular Power Spectrum

But the angular scales on which the measured signal is highest are very different!

w(θ) can only be determined accurately at **small angles**, beyond which Poisson noise dominates.

c_l has highest signal at small l, corresponding to large angular scales **$\theta \sim 180^o/l$. Complementary!?**

The two statistics **quantify very different properties** of the galaxy distribution.

c_l quantifies the **amplitude of fluctuations on the angular scale** corresponding to l

The value of ***w(θ)*** is the average of the product of the galaxy overdensity at any point with the overdensity at a point distant by ***θ***

=> ***w(θ)*** depends on angular fluctuations on all scales.

Angular Power Spectrum, continued

Practical advantages in comparison with ***w(θ)***:

1. Measurements of $\mathbf{c}_l$ at different multipoles l are uncorrelated. (or can be, with care)

2. On **small scales**, structure-evolution is complicated by **non-linear effects**. Advantageous to investigate **larger scales** where **linear theory** applies.

3. Natural relation between the $\mathbf{c}_l$ spectrum and the **spatial power spectrum *P(k)*,** a **convenient means of describing structure** in the Universe - **primordial form is produced by models of inflation**, prescribing the pattern of **initial density fluctuations *δρ/ρ*.**

4. Linear theory for the growth of perturbations describes **fluctuations by different wavevectors k, evolving independently**. ***w(θ)*** is more **naturally related to the spatial correlation function ξ(r)**, the Fourier transform of **P(k).**

Angular Power Spectrum - Example

A large-area radio survey maps the galaxy distribution out to large distances **D > 10^3 Mpc** => ***P(k)*** on large scales ***$k \sim 1/D < 10^{-3}$*** Mpc^{-1}, where it is **unaltered from its initial form.**

So **c_l spectrum of radio galaxies** => constraints on **primordial pattern of density fluctuations** independent of measurements of fluctuations in the CMB radiation.

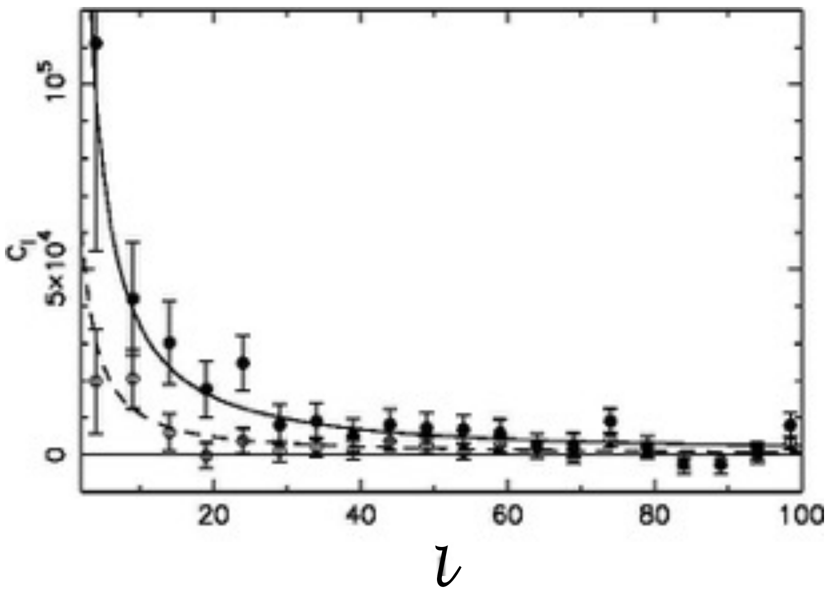
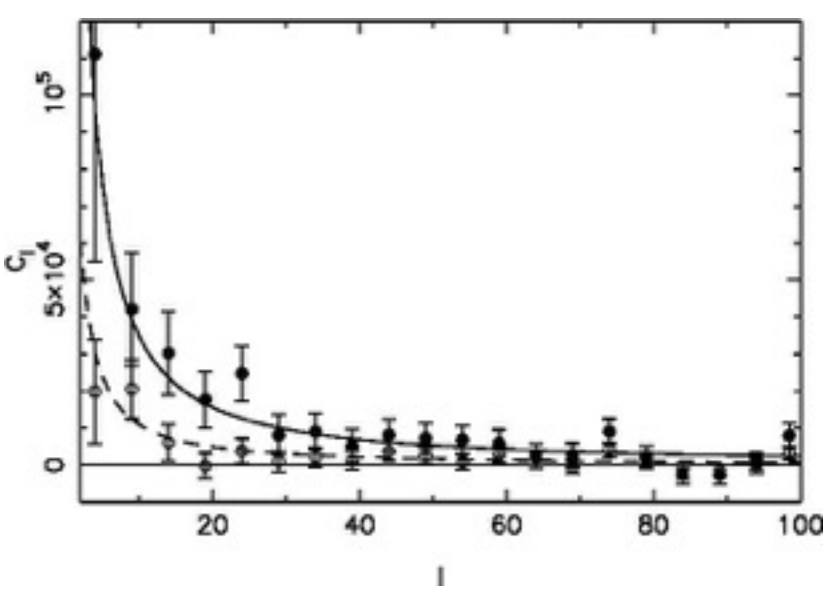
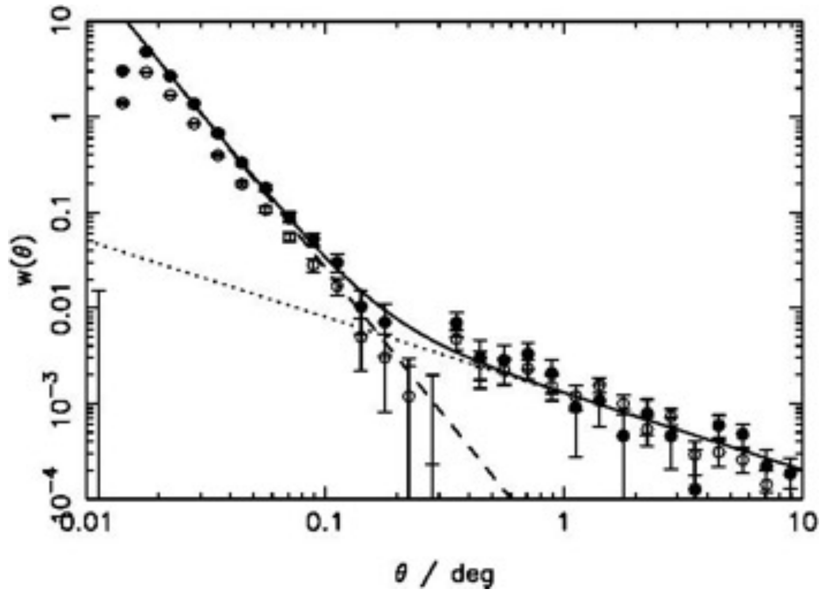


Figure: results for NVSS, with predictions from transforming ***w(θ),*** with ***$w(\theta) = 1.0 \times 10^{-3}\ \theta^{-0.8}$***. Results plotted at 10 mJy (solid circles) and 20 mJy (open circles).

Angular Power Spectrum - Example, concluded

A good match - with the exception of the dipole term ℓ**=1**, 'spuriously' high **due to the cosmic velocity dipole detected in this experiment**.

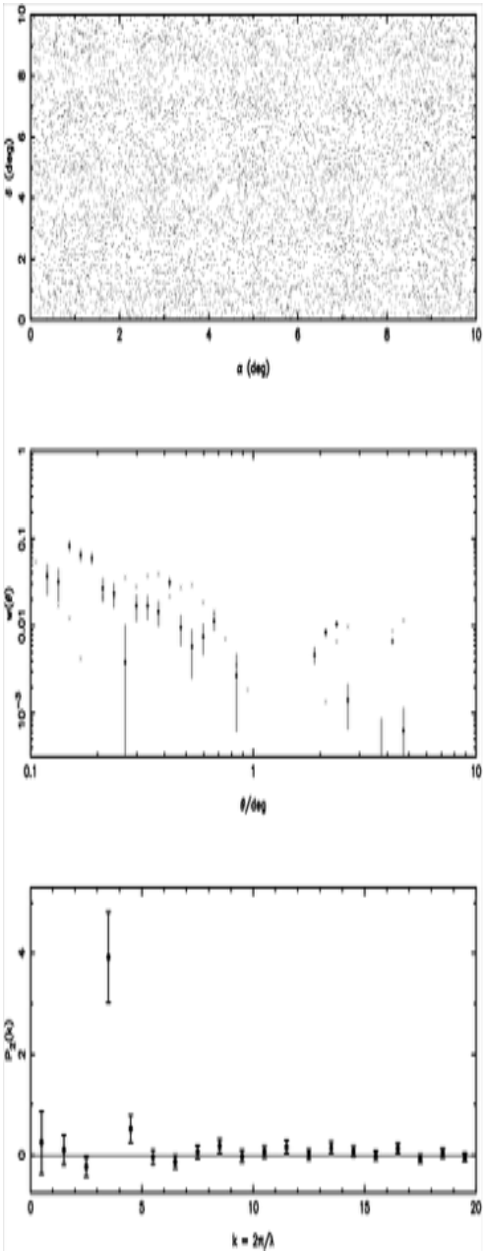




Thus long-wavelength surface density fluctuations (low ℓ) are important in producing angular correlations at small ***θ***.

Not a contradiction! w(***θ)*** is average of the product of the galaxy overdensity at any point with the overdensity at any other point at fixed angular separation - positive contributions to this average are readily produced by long-λ fluctuations.

w(θ) - Example 1 - note angular power spectrum



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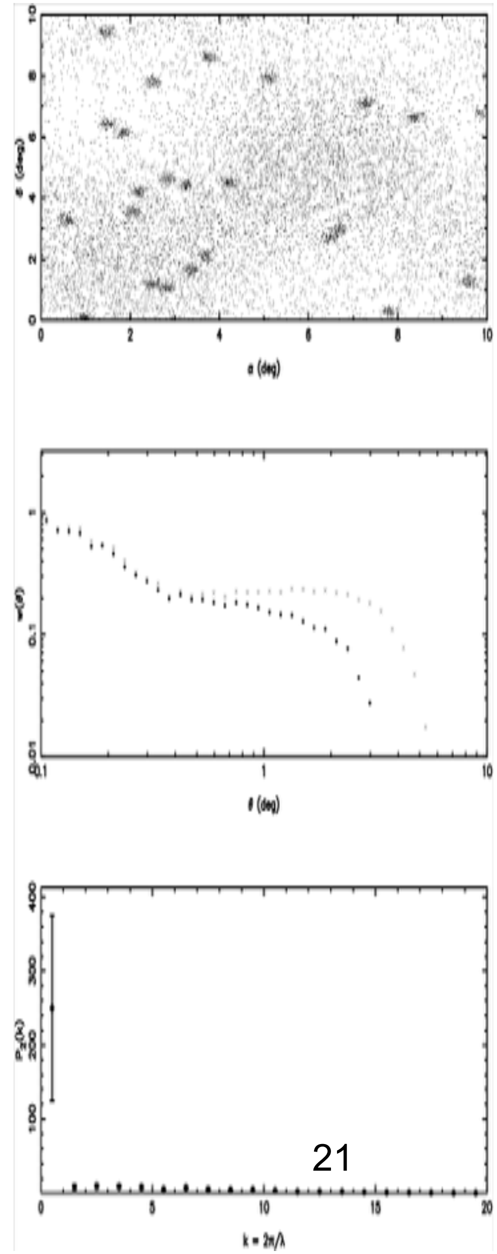
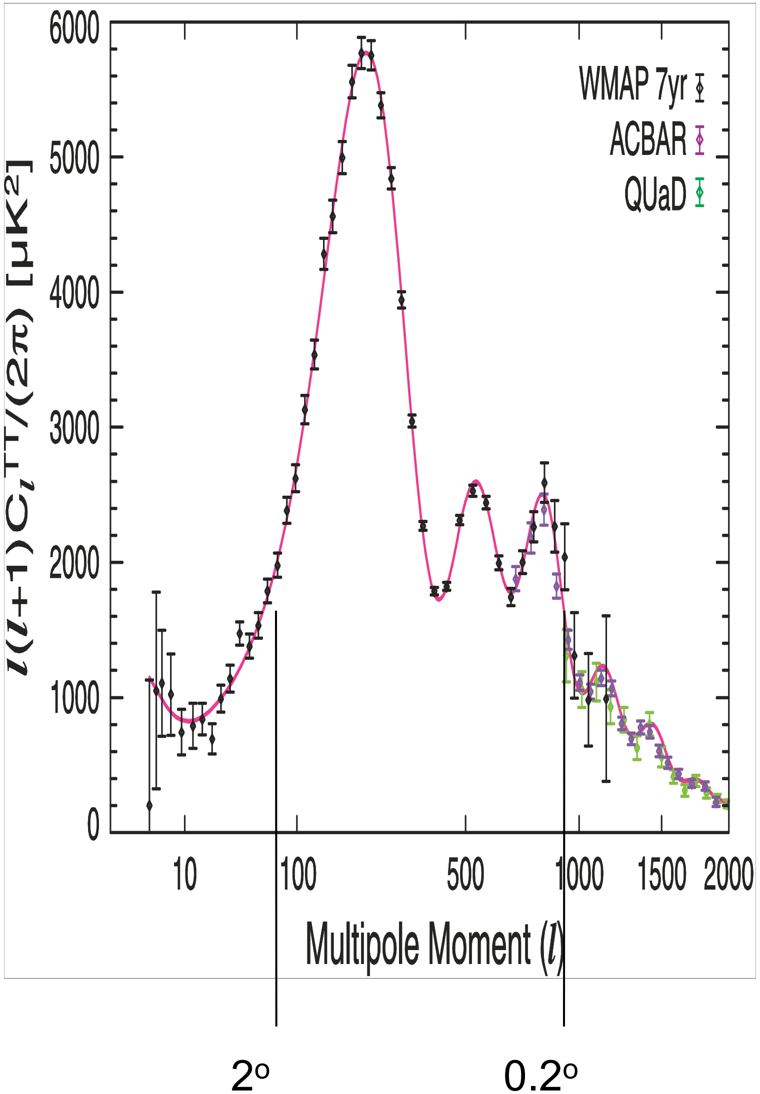


The ‘WMAP+’ CMB Angular Power Spectrum



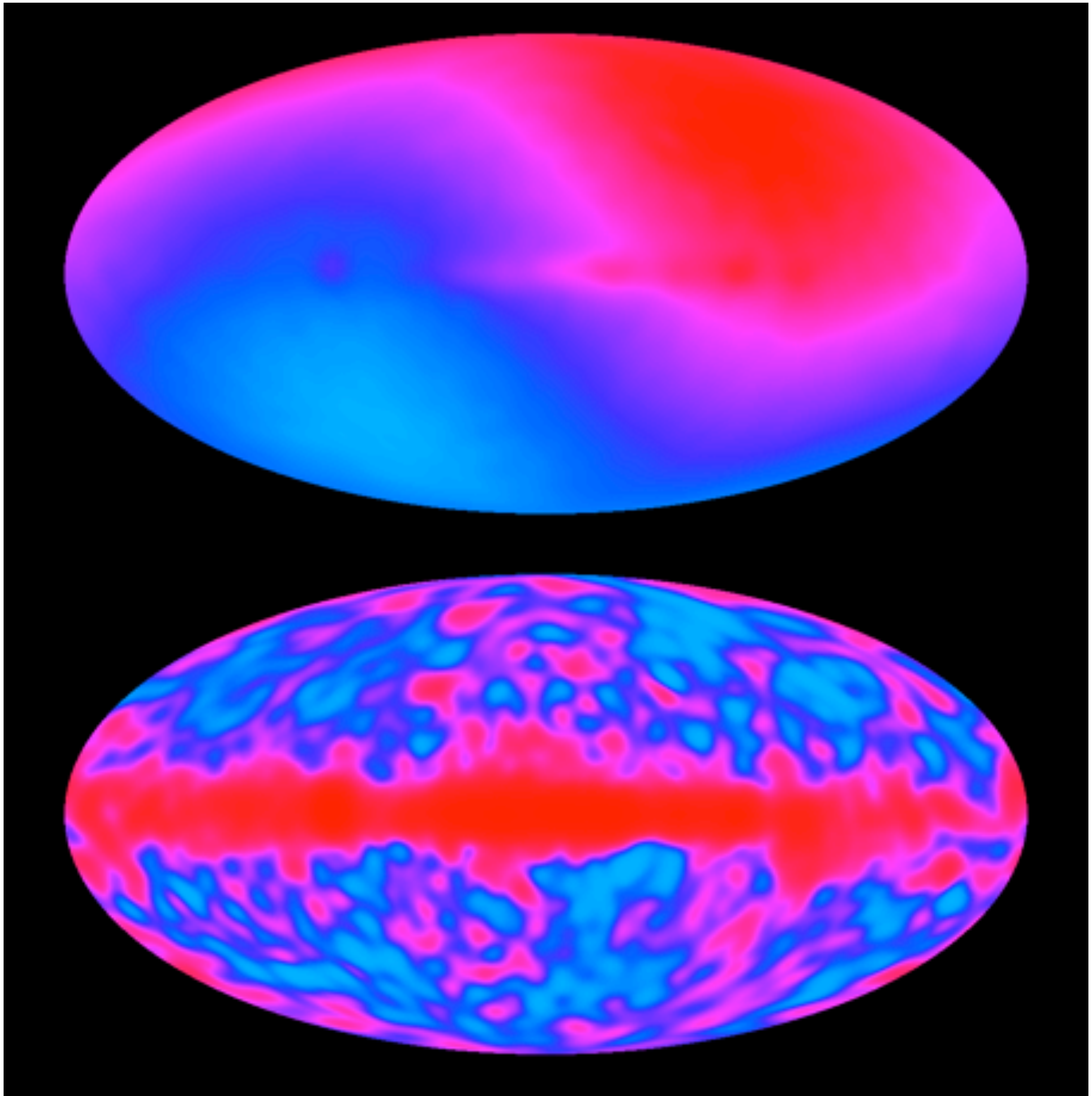
l < 90: scales > horizon size at decoupling (as observed today); **Sachs-Wolfe plateau**; unprocessed primordial fluctuation spectrum. ‘**Cosmic variance**’.

90 < l < 900: acoustic peak region; physics of 3000K plasma of n_e = 300 cm^{-3} responding to DM grav potential fluctuations. 1^{st} – compression; 2^{nd} – rarefaction, 3^{rd} – compression at 2^{nd} harmonic of 1^{st} peak…

l > 900: Silk damping tail, diffusion of photons from the fluctuations and washing out of observed fluctuations by hot and cold regions along line of sight.

END

The CMB Dipole from COBE



NVSS and the Velocity Dipole

The cosmological velocity dipole should be present,
but to now (2002) has not been detected in any class of object

Radio sources are the ideal backdrop candidate
(Ellis & Baldwin 1984)

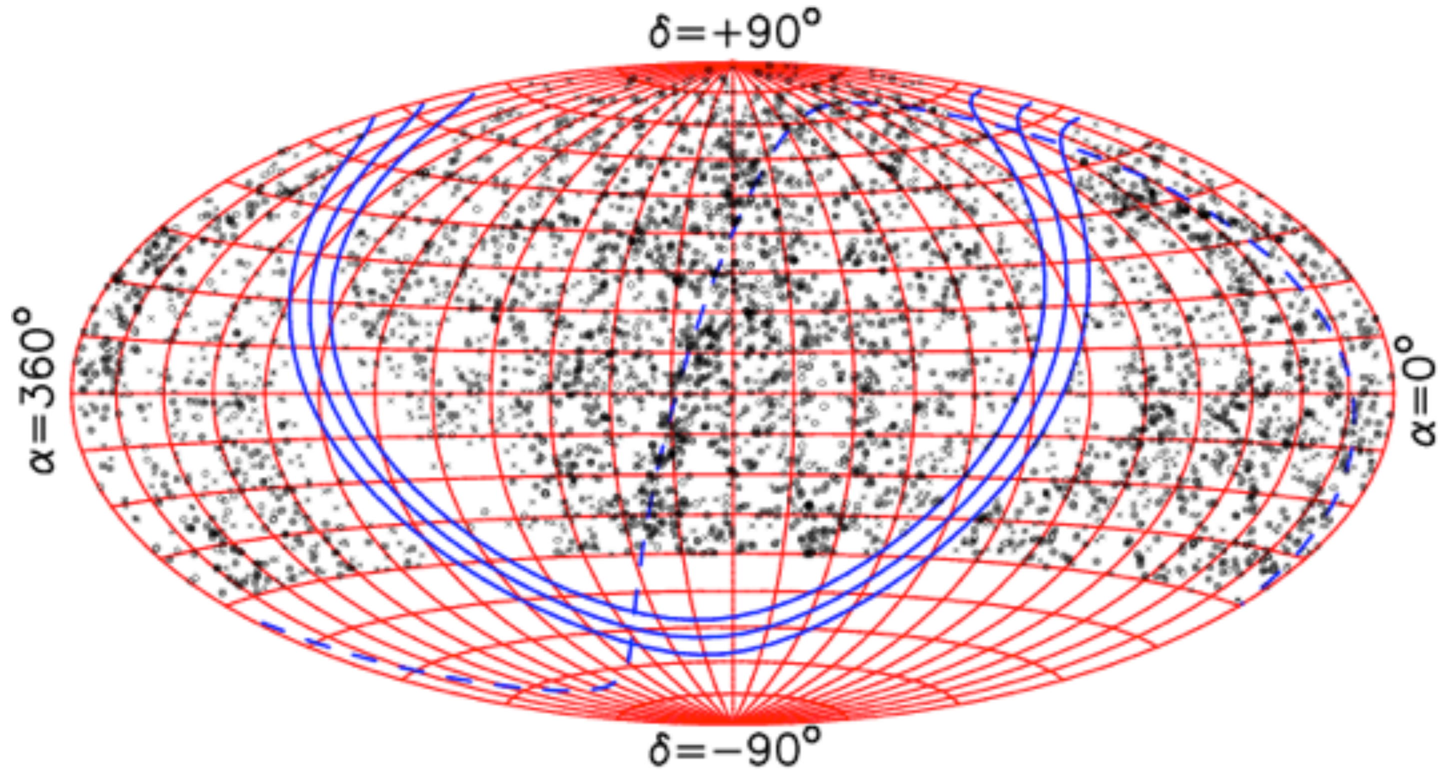
- Measure harmonic coefficients, allowing for unsurveyed region

- Remove `local' sources (RCBG, IRAS PSCz, 2dF)

-> Dipole direction and amplitude A

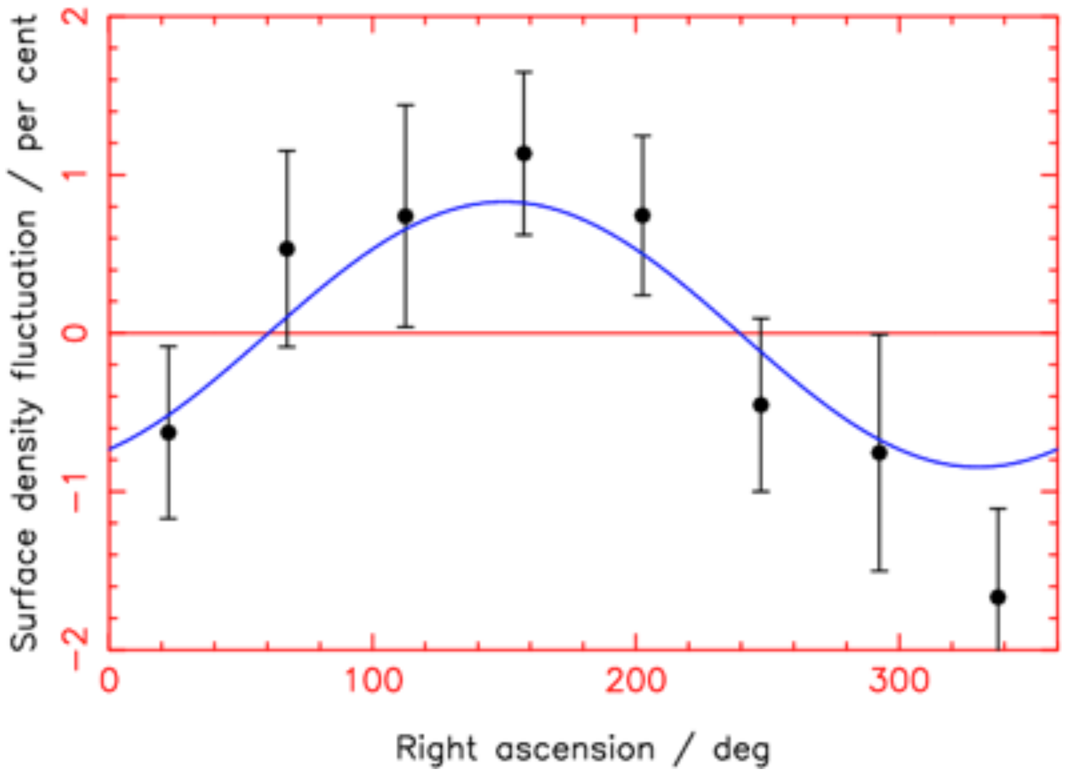
$A = 2(v/c)[2 + x(1 + \alpha)]$ where
x is -exponent of power-law count,
α is –index of power-law spectrum

Sky Distribution of Local NVSS Sources: RCB3, PSCz



$S_{1.4}$ > 10 mJy

NVSS Velocity Dipole



The cosmological (velocity) dipole has been detected in radio galaxies, amplitude and direction consistent with those for CMB

George Ellis: `The result is not unexpected, but it is important nevertheless. The seemingly unlikely standard model, with its simple behaviour at large scales, has passed yet another critical consistency test.'

END